Finally, we mention if above condition C be changed into condition C$'$ in G, the Riemann-Hilbert boundary value problem for system (1) can be discussed by using the similar method. Condition C$'$, namely

1. $A_j, B_j (j = 1, \cdots, n)$ are continuously differentiable in G and $w \in C$, and are analytic function in $|w| < R$ (unknow positive constant);

2. A_j, B_j, and their mixed partial derivative up to $n-1$ order (about different $\bar{z}_k, k \neq j$) are L_p bounded;

3. A_j, B_j and F_j satisfy

$$A_{j\bar{z}_k} = A_{k\bar{z}_j}, B_{j\bar{z}_k} = B_{k\bar{z}_j}, B_j A_k = A_j B_k, \quad j \neq k,$$

$$C_\alpha[F_j(z_1, \cdots, z_n, w_1) - F_j(z_1, \cdots, z_n, w_2), \bar{G}] \leq LC_\alpha[w_1 - w_2, \bar{G}], \quad j, k = 1, \cdots, n.$$

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一阶椭圆型复方程组 Riemann-Hilbert 边值问题

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摘 要

本文以解析函数的边值问题 B 的解的存在性为基础，根据它们的先验估计式及利用参数开拓法，导出了满足条件 C 的多个复变量的一阶拟线性椭圆型复方程组的 Riemann-Hilbert 边值问题的可解条件，并给出了解的积分表达式.

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On Riemann-Hilbert Boundary Value Problem for Complex Elliptic System of First Equations *

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Abstract On the base of the existence theroem of boudary value problem B of analytic function, according to the prior estimates of its boundary value problems solution and by using the method of parameter extansion, their solvability conditions and integral expression of solutions for Riemann-Hilbert boundary value problems of several complex variables and quasilinear elliptic systems that satisfied the so-called condition C of first order equations is established.

Keywords Riemann-Hilbert boundary value problem, quasilinear elliptic system, analytic function, prior estimate.

Classification AMS(1991) 35J55/CCL O175.25

In this paper, we consider the Riemann-Hilbert boundary value problem for elliptic system of first order complex equations

$$\begin{aligned} w_{\bar z_j} &= F_j(z_1,\cdots,z_n,w), \\ F_j &= A_j(z_1,\cdots,z_n,w)w + B_j(z_1,\cdots,z_n,w), \quad j=1,\cdots,n. \end{aligned} \tag{1}$$

In the several cylinder domain $G = G_1\times G_2\times\cdots\times G_n, G_j=\{|z_j|=\rho_j\}, \Gamma=\Gamma_1\times\cdots\times\Gamma_n, \Gamma_j=\{|t_j|=\rho_j\}, 0<\rho_j<\infty$. There is no harm in assuming $\rho_j=1, j=1,\cdots,n$. Suppose that (1) satisfies condition C in G, namely

1. $A_j(z_1,\cdots,z_n,w), B_j(z_1,\cdots,z_n,w), j=1,\cdots,n$ are analytic function in $|w|<R$ (unknow positive constant), and satisfies

$$C_\alpha^{2n-3}[A_j,\bar G]\le L,\quad C_\alpha^{2n-3}[B_j,\bar G]\le L, j=1,\cdots,n,$$

for any $w\in C_\alpha^{n-1}(\bar G)$ and $z\in G$, where $\alpha(0<\alpha<1), L(\ge 0)$ are real constant.

2. A_j, B_j and $F_j(j=1,\cdots,n)$ satisfy

$$A_{j\bar z_k}=A_{k\bar z_j}, B_{j\bar z_k}=B_{k\bar z_j}, B_jA_k=A_jB_k,\ j\ne k$$

$$C_\alpha^{n-1}[F_j(z_1,\cdots,z_n,w_1)-F_j(z_1,\cdots,z_n,w_2),\bar G]\le LC_\alpha^{n-1}[w_1-w_2,\bar G],\quad j,k=1,\cdots,n.$$

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The so-called Riemann-Hilbert boundary value problem (problem A) for (1) is to find a solution $w(z_1,\cdots,z_n)\in C_\alpha^{n-1}(\bar{G})$ for the complex system (1) satisfying the boundary condition

$$\mathrm{Re}[\overline{\lambda(t_1,\cdots,t_n)}w(t_1,\cdots,t_n)]=r(t_1,\cdots,t_n),\quad t\in\Gamma, \tag{2}$$

where $C_\alpha^{n-1}[r,\Gamma]\le d,|\lambda|=1,C_\alpha^{n-1}[\lambda,\Gamma]\le d,\alpha(0<\alpha<1),\alpha(\ge 0)$ are real constant. We introduce

$$K_j=\frac{1}{2\pi}\Delta_{\Gamma_j}\arg\lambda(t_1,\cdots,t_n),\quad j=1,\cdots,n,$$

and call $(k_1,\cdots,k_n)$ is the index for $\lambda(t_1,\cdots,t_n)$.

However, problem A is not always solvable for $k_i<0(i=1,\cdots,n$ at least one hold). The solution of problem A is a unique indefinitely for $k_i\ge 0,i=1,\cdots,n$. Hence we propose the modified Riemann-Hilbert problem B for complex system with the boundary condition

$$\begin{aligned}&\mathrm{Re}[\overline{\lambda(t_1,\cdots,t_n)}w(t_1,\cdots,t_n)]\\&=r(t_1,\cdots,t_n)+[H(t_1,\cdots,t_n)+h(t_1,\cdots,t_n)]e^{-s_2(t_1,\cdots,t_n)},t\in\Gamma,\end{aligned} \tag{3}$$

$$H(t_1,\cdots,t_n)=\mathrm{Re}\sum_{(\alpha_1,\cdots,\alpha_n)\in E_1}H_{\alpha_1,\cdots,\alpha_n}t_1^{\alpha_1}\cdots t_n^{\alpha_n},t\in\Gamma,$$

$$h(t_1,\cdots,t_n)=\begin{cases}-\mathrm{Re}\displaystyle\sum_{(\alpha_1,\cdots,\alpha_n)\in E_1'}H_{\alpha_1,\cdots,\alpha_n}t_1^{\alpha_1}\cdots t_n^{\alpha_n},k_i\ge 0,i=1,\cdots,n,\\ \mathrm{Re}\displaystyle\sum_{(\alpha_1,\cdots,\alpha_n)\in E_2}h_{\alpha_1,\cdots,\alpha_n}t_1^{\alpha_1}\cdots t_n^{\alpha_n},k_i<0,i=1,\cdots,n,\\ \mathrm{Re}\displaystyle\sum_{\alpha_i=0}^{-k_i-1}h_{\alpha_i}(t_1,\cdots,t_{i-1},t_{i+1},\cdots,t_n)t_i^{\alpha_i},\\ k_1\ge 0,\cdots,k_{i-1}\ge 0,k_i<0,k_{i+1}\ge 0,\cdots,k_n\ge 0,\end{cases}$$

$$\begin{aligned}E_1&=\{(-\infty,+\infty),\cdots,(-\infty,+\infty)\}-\{[0,+\infty),\cdots,[0,+\infty)\}\\&\quad-\{(-\infty,0],\cdots,(-\infty,0]\},\\E_1'&=\{(\alpha_1,\cdots,\alpha_n)|(\alpha_1,\cdots,\alpha_n)\in E_1,\text{ and }|\alpha_i|\le k_i,i=1,\cdots,n\},\\E_2&=\{[0,+\infty),\cdots,[0,+\infty)\}-\{[-k_1,+\infty),\cdots,[-k_n,+\infty)\},\end{aligned}$$

where $H_{\alpha_1,\cdots,\alpha_n}(|\alpha_i|=1,2,\cdots,i=1,\cdots,n),h_{\alpha_1,\cdots,\alpha_n}(\alpha_i=1,\cdots,-k_i-1,i=1,\cdots,n$ at least one hold) are unknown complex constants to be determined appropriatly, $h_{0,\cdots,0}$ is an unknow real constant, $h_{\alpha_i}(t_1,\cdots,t_{i-1},t_{i+1},\cdots,t_n)$ is an unknow complex functions on $\Gamma_1\times\cdots\times\Gamma_{i-1}\times\Gamma_{i+1}\times\cdots\times\Gamma_n,h_0(t_1,\cdots,t_{i-1},t_{i+1},\cdots,t_n)$ is an unknow real functions, $s_2(t_1,\cdots,t_n)$ can be found in [4], when $k_i\ge 0,i=1,\cdots,n$. We also require that the solution $w(z_1,\cdots,z_n)$ satisfies $m=2^n(k_1+1)\cdots(k_n+1)-(2^n-1)$ point condition

$$\mathrm{Im}[\overline{\lambda(a_{1j},\cdots,a_{nj})}w(a_{1j},\cdots,a_{nj})]=b_j,\quad j=1,\cdots,m, \tag{4}$$

where $a_j=(a_{1j},\cdots,a_{nj})$ is distinct point on Γ,b_j is real constant given.

In [4], we have given the necessary and sufficient condition on solvability and established expression of solution of the same problem of analytic function. In the following, we first give the estimates of solution of Dirichlet boundary value problem E of analytic function with boundary condition

$$\begin{aligned}&\mathrm{Re}\Phi(t_1,\cdots,t_n)=r(t_1,\cdots,t_n)+H(t_1,\cdots,t_n)+h(t_1,\cdots,t_n),\\&\mathrm{Im}\Phi(a_{1j},\cdots,a_{nj})=b_j,\quad j=1,\cdots,m,\end{aligned}$$

where r, H, h, are as stated in (3), and problem B of analytic function with boundary condition (3).

Theorem 1 *Suppose that the solution $\Phi_1(z_1,\cdots,z_n)$ of problem E satisfies the condition: $|\mathrm{Im}\Phi_1(0,\cdots,0)|\le d<\infty$, we have the estimate*

$$C_\alpha^{n-1}[\Phi_1(z_1,\cdots,z_n),\bar{G}]\le M_1(\alpha,d),\tag{5}$$

where $M_1(\alpha,d)$ is a non-negative number depending only on α and d.

Proof From the Schwarz formula of analytic function and by the Sohotski-Plemelj formula, we have

$$\begin{aligned}&\Phi_1(z_1,\zeta_2,\cdots,\zeta_n)\\&=\frac{1}{2\pi i}\int_{\Gamma_1}\frac{1}{t_1-z_1}[\frac{1}{(2\pi i)^{n-1}}\int_{\Gamma_2}\cdots\int_{\Gamma_n}\frac{2r(t_1,\cdots,t_n)}{\prod_{i=2}^n(t_i-\zeta_i)}dt_2\cdots dt_n\\&+\sum_{\lambda=2}^{n-1}\sum{}^*\frac{1}{2^\lambda(2\pi i)^{n-1-\lambda}}\int_{\Gamma_{j_1}}\cdots\int_{\Gamma_{j_{n-\lambda}}}\frac{2r(t_1,\zeta_2,\cdots,t_{j_1},\cdots,t_{j_{n-\lambda}},\cdots,\zeta_n)}{\prod_{k=1}^{n-\lambda}(t_{j_k}-\zeta_{j_k})}dt_{j_1}\cdots dt_{j_{n-\lambda}}\\&+\frac{1}{2^{n-1}}2r[z_1,\zeta_2,\cdots,\zeta_n)]dt_1-\frac{1}{(2\pi i)^n}\int_{\Gamma_1}\cdots\int_{\Gamma_n}\frac{r(t_1,\cdots,t_n)}{\prod_{i=1}^n t_i}dt_1\cdots dt_n+ic,\\&\qquad\zeta_j\in\Gamma_j,j=2,\cdots,n,\end{aligned}$$

where $c=\mathrm{Im}\Phi_1(0,\cdots,0),\sum^*$ denote summation about subscript $j_1,\cdots,j_n$ different from each other. $2\le j_1<j_2<\cdots<j_{n-\lambda}\le n-1, z_1\in G_1$. It is easy to see that $\Phi_1(z_1,\zeta_2,\cdots,\zeta_n)$ is analytic function in G_1, and according to Privalov theorem, $\Phi_1(z_1,\zeta_2,\cdots,\zeta_n)$ satisfy the estimate

$$C_\alpha^{n-1}[\Phi_1(z_1,\zeta_2,\cdots,\zeta_n),\bar{G}_1]\le M_2(\alpha,d),$$

similarly, we have

$$\begin{aligned}&C_\alpha^{n-1}[\Phi_1(\zeta_1,z_2,\zeta_3,\cdots,\zeta_n),\bar{G}_2]\le M_3(\alpha,d),\\&\qquad\cdots\\&C_\alpha^{n-1}[\Phi_1(\zeta_1,\cdots,\zeta_{n-1},z_n),\bar{G}_n]\le M_{n+1}(\alpha,d).\end{aligned}$$

Let $M_1=\max[M_2,\cdots,M_{n+1}]$. By the maximum modulas principle for analytic functions, (5) can be derived.

By using Theorem 1, we may prove the following theorem.

Theorem 2 *The solution $\Phi(z_1,\cdots,z_n)$ of problem B for analytic function satisfies the estimate*

$$C_\alpha^{n-1}[\Phi(z_1,\cdots,z_n),\bar{G}] \le M_1'(\alpha,d,k), \tag{6}$$

where k is index $(k_1,\cdots,k_n)$.

Now, we consider the Riemann-Hilbert boundary value problems for complex system of equation (1) in several cylinder domain.

Theorem 3 *The solution of problem B for equation (1) has the form*

$$w(z_1,\cdots,z_n) = [\Phi(z_1,\cdots,z_n) + \tilde{T}G]e^{is(z_1,\cdots,z_n)}, \tag{7}$$

where $\Phi(z_1,\cdots,z_n)$ is an analytic function, $s(z_1,\cdots,z_n)$ is stated in [4], and

$$\begin{cases} G_j = F_j e^{-is}, \quad j=1,\cdots,n, \\ \tilde{T}G = \sum\limits_{\lambda=1}^{n}(-1)^{\lambda-1}\sum{}^* J_{j_1,\cdots,j_\lambda}G_{j_1,\bar{\zeta}_{j_2},\cdots,\bar{\zeta}_{j_\lambda}}, \\ J_{j_1,\cdots,j_\lambda}G_{j_1,\bar{\zeta}_{j_2},\cdots,\bar{\zeta}_{j_\lambda}} \\ \quad = (-\dfrac{1}{\pi})^\lambda \iint_{D_{j_1}}\cdots\iint_{D_{j_\lambda}} \dfrac{G_{j_1,\bar{\zeta}_{j_2},\cdots,\bar{\zeta}_{j_\lambda}}(z_1,\cdots,\zeta_{j_1},\cdots,\zeta_{j_\lambda},\cdots,z_n)}{\prod_{k=1}^{\lambda}(\zeta_{j_k}-z_{j_k})} d\sigma_{\zeta_{j_1}}\cdots d\sigma_{\zeta_{j_\lambda}}, \end{cases}$$

$\sum^*$ is the same as upper explanation, and $1\le j_1<j_2<\cdots<j_\lambda\le n$. For example, when $n=3$, then

$$\begin{aligned} &\tilde{T}G(z_1,\cdots,z_n) \\ &= -\frac{1}{\pi}\iint_{G_1}\frac{G_1(\zeta_1,z_2,z_3)}{\zeta_1-z_1}d\sigma_{\zeta_1} - \frac{1}{\pi}\iint_{G_2}\frac{G_2(z_1,\zeta_2,z_3)}{\zeta_2-z_2}d\sigma_{\zeta_2} - \frac{1}{\pi}\iint_{G_3}\frac{G_3(z_1,z_2,\zeta_3)}{\zeta_3-z_3}d\sigma_{\zeta_3} \\ &-\frac{1}{\pi^2}\iint_{G_1}\iint_{G_2}\frac{G_{1\bar{\zeta}_2}(\zeta_1,\zeta_2,z_3)}{(\zeta_1-z_1)(\zeta_2-z_2)}d\sigma_{\zeta_1}d\sigma_{\zeta_2} - \frac{1}{\pi^2}\iint_{G_1}\iint_{G_3}\frac{G_{1\bar{\zeta}_3}(\zeta_1,z_2,\zeta_3)}{(\zeta_1-z_1)(\zeta_3-z_3)}d\sigma_{\zeta_1}d\sigma_{\zeta_3} \\ &-\frac{1}{\pi^2}\iint_{G_2}\iint_{G_3}\frac{G_{2\bar{\zeta}_3}(z_1,\zeta_2,\zeta_3)}{(\zeta_2-z_2)(\zeta_3-z_3)}d\sigma_{\zeta_2}d\sigma_{\zeta_3} \\ &-\frac{1}{\pi^3}\iint_{G_1}\iint_{G_2}\iint_{G_3}\frac{G_{1\bar{\zeta}_2\bar{\zeta}_3}(\zeta_1,\zeta_2,\zeta_3)}{(\zeta_1-z_1)(\zeta_2-z_2)(\zeta_3-z_3)}d\sigma_{\zeta_1}d\sigma_{\zeta_2}d\sigma_{\zeta_3}. \end{aligned}$$

Proof It is not difficult to prove that

$$U(z_1,\cdots,z_n) = w(z_1,\cdots,z_n)e^{-is(z_1,\cdots,z_n)}$$

satisfies the system

$$U_{\bar{z}_j} = G_j(z_1,\cdots,z_n,U),$$

$$G_j = F_j(z_1,\cdots,z_n,Ue^{is(z_1,\cdots,z_n)})e^{-is(z_1,\cdots,z_n)},$$

and the boundary condition

$$\mathrm{Re}[t_1^{-k_1},\cdots,t_n^{-k_n}U(t_1,\cdots,t_n)] = R(t_1,\cdots,t_n) + H(t_1,\cdots,t_n) + h(t_1,\cdots,t_n), \quad t\in\Gamma,$$

where $R(t_1, \cdots, t_n) = r(t_1, \cdots, t_n)e^{s_2(t_1, \cdots, t_n)}, s_2(t_1, \cdots, t_n)$ is as stated as in [4], It can be verified that $\Phi(z_1, \cdots, z_n)$ satisfies the complex system

$$\Phi_{\bar{z}_j} = (U - \tilde{T}G)_{\bar{z}_j} = 0, \quad j = 1, \cdots, n,$$

This shows that $\Phi(z_1, \cdots, z_n)$ is an analytic function in G. So we have the conclusion.

By using the property of pompieu operator, it is easy to verify the next theorem 4.

Theorem 4 *Let the system (1) satisfy condition C, then solution of problem B for (1) hold estimate*

$$C_\alpha^{n-1}[w(z_1, \cdots, z_n), \bar{G}] \le M_2'(\alpha, d, L, k), \tag{8}$$

$$C_\alpha^{n-1}[\Phi(z_1, \cdots, z_n), \bar{G}] \le M_3'(\alpha, d, L, k), \tag{8$'$}$$

where $\Phi(z_1, \cdots, z_n) = we^{-is} - \tilde{T}G, \alpha, d, L, k$ *are as stated in above.*

Proof Let the solution $w(z_1, \cdots, z_n)$ be substituted into the system (1) and the boundary condition (3), seting that

$$\begin{cases} \tilde{T}A(z_1, \cdots, z_n) = \sum_{\lambda=1}^{n}(-1)^{\lambda-1} \sum{}^* J_{j_1, \cdots, j_\lambda} A_{j_1, \bar{\zeta}_{j_2}, \cdots, \bar{\zeta}_{j_\lambda}}, \\ B_j' = B_j e^{-\tilde{T}A}, \quad j = 1, \cdots, n, \\ \tilde{T}B' = \sum_{\lambda=1}^{n}(-1)^{\lambda-1} \sum{}^* J_{j_1, \cdots, j_\lambda} B'_{j_1 \bar{\zeta}_{j_2}, \cdots, \bar{\zeta}_{j_\lambda}}. \end{cases}$$

By the condition C, we can prove

$$(\tilde{T}A)_{\bar{z}_j} = A_j, \quad (\tilde{T}B')_{\bar{z}_j} = B_j',$$

$$B'_{j\bar{z}_k} = B_{j\bar{z}_k} \cdot e^{-\tilde{T}A} - B_j e^{-\tilde{T}A} \cdot (\tilde{T}A)_{\bar{z}_k} = B_{k\bar{z}_j} \cdot e^{-\tilde{T}A} - B_k A_j e^{-\tilde{T}A} = B'_{k\bar{z}_j},$$

and function

$$V(z_1, \cdots, z_n) = w(z_1, \cdots, z_n)e^{-\tilde{T}A} - \tilde{T}B'$$

satisfies the system

$$\begin{aligned} V_{\bar{z}_j} &= w_{\bar{z}_j} \cdot e^{-\tilde{T}A} - we^{-\tilde{T}A}A_j - B_j e^{-\tilde{T}A} \\ &= (A_j w + B_j)e^{-\tilde{T}A} - we^{-\tilde{T}A}A_j - B_j e^{-\tilde{T}A} = 0, \quad z \in G, j = 1, \cdots, n. \end{aligned}$$

Hence $V(z_1, \cdots, z_n)$ is an analytic function in G. And $w(z_1, \cdots, z_n)$ can be expressed as

$$w(z_1, \cdots, z_n) = V(z_1, \cdots, z_n) + \tilde{T}B']e^{\tilde{T}A}. \tag{9}$$

By condition C and property of pompieu operator, we get estimate

$$C_\alpha^{n-1}[\tilde{T}A, \bar{G}] \le M_4'(\alpha, L), \quad C_\alpha^{n-1}[\tilde{T}B', \bar{G}] \le M_5'(\alpha, L). \tag{10}$$

Moreover, analytic function $V(z_1,\cdots,z_n)$ satisfies the boundary condition

$$\begin{aligned}&\mathrm{Re}[\overline{\lambda(t_1,\cdots,t_n)}e^{\tilde{T}A}V(t_1,\cdots,t_n)] = r(t_1,\cdots,t_n) - \mathrm{Re}[\overline{\lambda(t_1,\cdots,t_n)}\tilde{T}B'e^{\tilde{T}A}]\\ &+[H(t_1,\cdots,t_n)+h(t_1,\cdots,t_n]e^{-s_2(t_1,\cdots,t_n)},\quad t\in\Gamma.\end{aligned} \tag{11}$$

From Theorem 2, and (10), we have

$$C_\alpha^{n-1}[V(z_1,\cdots,z_n),\bar{G}] \le M_6'(\alpha,d,L,k). \tag{12}$$

Conbining (9), (10), (11), the estimate (8) can be derived. It hold that (8)$'$ by using expression of function $\Phi(z_1,\cdots,z_n)$ again. This complete the proof.

Next, by using the prior estimates of its boundary value problem solution $w(z_1,\cdots,z_n)$ and the method of parameter extension, we can prove the existence of solution of problem B for the system (1), namely

Theorem 5 *Suppose that the system (1) satisfies Condition C. Then problem B has a solution.*

Proof We consider the complex system with the parameter $P(0\le P\le 1)$

$$w_{\bar{z}_j} - PF_j(z_1,\cdots,z_n,w) = Q_j(z_1,\cdots,z_n),\quad j=1,\cdots,n, \tag{13}$$

where $Q_j(z_1,\cdots,z_n)\in C_\alpha^{n-1}(\bar{G})$, and $Q_{j\bar{z}_k} = Q_{k\bar{z}_j}$, we can find $P_0(0\le P_1\le 1)$ it let problem B of system (13) solvable for any above function $Q_j(z_1,\cdots,z_n)$. The system (13) can be rewritten as

$$w_{\bar{z}_j} - P_0F_j(z_1,\cdots,z_n,w) = (P-P_0)F_j(z_1,\cdots,z_n,w)+Q_j(z_1,\cdots,z_n),\quad j=1,\cdots,n. \tag{14}$$

Thus by successive iteration, we obtain a sequence of solution $w_n(z_1,\cdots,z_n), n=1,2,\cdots$. It belong to space $C_\alpha^{n-1}(\bar{G})$. Choosing $\delta=\frac{1}{2(M_7'+1)}$, it can be derived that when $|P-P_0|\le\delta$, there exist an integer N, so that

$$C_\alpha^{n-1}[w_{n+1}-w_n,\bar{G}] \le \frac{1}{2}C_\alpha^{n-1}[w_n-w_{n-1},\bar{G}] \le\cdots\le \frac{1}{2^N}C_\alpha^{n-1}[w_1,\bar{G}],$$

when $m,n>N$,

$$C_\alpha^{n-1}[w_n-w_m,\bar{G}] \le \frac{1}{2^N}\sum_{j=1}^{\infty}\frac{1}{2^j}C_\alpha^{n-1}[w_1,\bar{G}] = \frac{1}{2^N}C_\alpha^{n-1}[w_1,\bar{G}].$$

We have

$$C_\alpha^{n-1}[w_n-w_m,\bar{G}]\to 0.$$

Since the Banach space $C_\alpha^{n-1}(\bar{G})$ is complete, there is a function $w_*(z_1,\cdots,z_n)\in C_\alpha^{n-1}(\bar{G})$ such that $C_\alpha^{n-1}[w_n-w_*,\bar{G}]\to 0$ as $n\to\infty$, and $w_*(z_1,\cdots,z_n)$ is a solution of problem B for (13). So it follows that when $P=0,\delta,\ ,[\frac{1}{\delta}]\delta,1$, problem B for (13) are solvable. In particular, when $P=1$ and $Q_j(z_1,\cdots,z_n)=0, j=1,\cdots,n$, problem B for (13), i.e., (1) is solvable. So we have the conclusion.

Finally, we mention if above condition C be changed into condition C′ in G, the Riemann-Hilbert boundary value problem for system (1) can be discussed by using the similar method. Condition C′, namely

1. $A_j, B_j (j = 1, \cdots, n)$ are continuously differentiable in G and $w \in C$, and are analytic function in $|w| < R$ (unknow positive constant);
2. A_j, B_j, and their mixed partial derivative up to $n-1$ order (about different $\bar{z}_k, k \neq j$) are L_p bounded;
3. A_j, B_j and F_j satisfy

$$A_{j\bar{z}_k} = A_{k\bar{z}_j}, B_{j\bar{z}_k} = B_{k\bar{z}_j}, B_j A_k = A_j B_k, \quad j \neq k,$$

$$C_\alpha[F_j(z_1, \cdots, z_n, w_1) - F_j(z_1, \cdots, z_n, w_2), \bar{G}] \leq L C_\alpha[w_1 - w_2, \bar{G}], \quad j, k = 1, \cdots, n.$$

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一阶椭圆型复方程组 Riemann-Hilbert 边值问题

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摘 要

本文以解析函数的边值问题 B 的解的存在性为基础，根据它们的先验估计式及利用参数开拓法，导出了满足条件 C 的多个复变量的一阶拟线性椭圆型复方程组的 Riemann-Hilbert 边值问题的可解条件，并给出了解的积分表达式.