by Lemma 6, proving (i) (b).

By Lemma 7 and by using the argument in the proof of Theorem 2(i), we get that G satisfies the Sylow tower property, proving (ii).

We now prove (iii). Let p be the largest prime divisor of G, and let $P \in \mathrm{Syl}_p(G)$. From (ii) it follows that $P \lhd G$. If $\Phi(G) \neq 1$, then induction implies that $G/\Phi(G)$ is supersolvable and hence G is supersolvable. Therefore, we assume that $\Phi(G) = 1$. Then, since $F \lhd G$, we have $\Phi(P) = 1$ and consequently P is elementary abelian. Since $|G|/\chi(1)$ is a cubefree number for all nonlinear $\chi \in \mathrm{Irr}(G)$, by [2,(6.15)] we have that $|P| \leq p^2$. If P is cyclic, then G is supersolvable since induction implies G/P is supersolvable. Hence we asuume that P is elementary abelian of order p^2. Similarly, we may assume that P is minimal normal in G. By Schur-Zassenhaus theorem, we have that $G = PM$, where M is a p-complement in G. Since P is a minimal normal subgroup of G and P is abelian, M must be a maximal subgroup of G.

Now $M \cong G/P$ and hence M is supersolvable by Lemma 1 and induction. Let Q be a minimal normal subgroup of M. Then $|Q| = q$, a prime. Set $Q = \langle x \rangle$. If $C_G(x) \neq 1$, then $M < N_G(Q)$ and as M is maximal in G, we get that $Q \lhd G$. Thus, since G/Q is supersolvable, G is supersolvable. Therefore, we assume $C_G(x) = 1$. But, this implies that PQ is a Frobenius group of type $(Z_p \times Z_p) \rtimes Z_q$, contradicting our assumption. This completes the proof of (iii). □

Corollary 4 *Let G be a nonabelian group. Assume that for each prime divisor p of $|G|, |G|_p = p$ or $|G|_p = p^n$ with $n \geq 3$. If $|G|/\chi(1)$ is a cubefree number for all nonlinear $\chi \in \mathrm{Irr}(G)$, then G is supersolvable with G' abelian of cubefree order.*

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关于有限群的阶与其特征标的商

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摘　要

设 G 是有限群. D.Chillag 和 M.Herzog 曾考察过下述问题：商集合 $\{|G|/\chi(x)|\chi \in \mathrm{Irr}(G)\}$ 上的某些算术条件如何影响群 G 本身的结构. 本文继续研究这个问题.

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On Character Degree Quotients and Hall π-Subgroups *

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Abstract Let G be a finite group. The question of how certain arithmetical conditions on the set of quotients $\{|G|/\chi(1) : \chi \in \mathrm{Irr}(G)\}$ affect the structure of the group G was studied by D.Chillag and M.Herzog. In this paper, we continue to investigate this question.

Keywords finite groups, character degrees quotients, hall π-subgroups, Frobenius groups.

Classification AMS(1991) 20C15/CCL O152.6

All groups considered are finite groups. Let n be a positive integer. $\pi(n)$ denotes the set of all prime divisors of n. $\pi(G)$ denotes the set of all prime divisors of the order of the group G.

Definition Let G be a group, and let $n > 1$ be a positive integer. n is called a Hall number of G, if there exists a positive integer m such that $|G| = mn$ and $(m, n) = 1$.

Lemma 1 *Let H be either a normal subgroup or a qoutient group of the group G, and let $\theta \in \mathrm{Irr}(H)$. Then there exists $\chi \in \mathrm{Irr}(G)$ such that $|H|/\theta(1)$ divides $|G|/\chi(1)$. Moreover, if θ is nonlinear there exists such χ which is nonlinear. In fact, if H is a normal subgroup of G, any irreducible costituent of θ^G is such a χ.* (See [1,Lemma,p.27] or [2,(11.29)])

Lemma 2 *Let n be a Hall number of the group G. If n divides $\chi(1)$ for every nonlinear $\chi \in \mathrm{Irr}(G)$, then $\pi(n) \cap \pi(G') = \emptyset$, where G' is the commutator subgroup of G.*

Proof Our assumption implies that $|G| = |G : G'| + n^2m$ for some positive integer m. So $|G|(|G'| - 1) = |G'|n^2m$. As n is a Hall number of G, we get that n divides $(|G'| - 1)$ and consequently every prime divisor of n does not divide $|G'|$. Hence we have $\pi(n) \cap \pi(G') = \emptyset$.

Lemma 3 *Let K be a normal subgroup of the group C. If $K' < G'$, then exists a linear character θ of K such that θ^G has a nonlinear irreducible constituent. In particular, we have that $|K| \mid (|G|/\chi(1))$ for some nonlinear $\chi \in \mathrm{Irr}(G)$.*

Proof Since $K' < G'$, K has a linear character θ such that $G' \not\leq \mathrm{Ker}(\theta)$([(2,(2,23)]). If every irrducible constituent of θ^G is linear, then $G' \leq \mathrm{Ker}(\theta^G) \leq \mathrm{Ker}(\theta)$([2,(2,21),(2,23),(5,11)]),

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a contradiction. Thus θ^G has a nonlinear irreducible constituent $\chi \in \mathrm{Irr}(G)$. Since $\theta(1) = 1$, by Lemma 1, we get that $|K| = (|K|/\theta(1)) \mid (|G|/\chi(1))$. □

Theorem 1 *Let G be a nonabelian group. Let $\pi \subset \pi(G)$ and $\pi \neq \pi(G)$. Assume that $\pi(|G|/\chi(1)) \subseteq \pi$ for all nonlinear $\chi \in \mathrm{Irr}(G)$. Then the following statements are true:*

(i) $G = HK$, where H is a cyclic Hall π'-subgroup of G and K is a normal Hall π-subgroup of G.Moreover, $G' \leq K$ and $M = G'H$ is a Frobenius group with Frobenius kernel G'.

(ii) $F(G)' < G', F(G) = \times_{p\in\pi} O_p(G)$. Moreover, if $p \nmid (|G|/\chi(1))$ for some nonlinear $\chi \in \mathrm{Irr}(G)$, then $O_p(G) = 1$.

(iii) G' has a complement in G.

(iv) If $K' < G'$, then there exists a nonlinear $\chi \in \mathrm{Irr}(G)$ such that $|K| \mid |G|/\chi(1)$.

Proof Let p be any prime in π'. By the hypothesis, we have that $p|\chi(1)$ for all nonlinear $\chi \in \mathrm{Irr}(G)$. Thus, by [2,(12,2)], we get that $G = HK$, where H is a Hall π'-subgroup of G and K is a normal Hall π-subgroup of G. By Lemma 2 and the hypothesis, G' is a π-group, and hencce $G' \leq K$ and $H \cong G/K$ is abelian.

Let $M = G'H$. If $M' < G'$, then M is a π-group by Lemma 3 and hence $\pi = \pi(G)$, a contradiction. Thus $M' = G'$. In particular, M is nonablian.

By Lemma 1 and the hypothesis, $|M|/\theta(1)$ is a π-number for all nonlinear $\theta \in \mathrm{Irr}(M)$. Let $h \in H - 1.h$ is π'-element. Then we have that $\theta(h) = 0$ for all nonlinear $\theta \in \mathrm{Irr}(M)$ ([2,(8.17)]). By the second orthogonality relation we get that $|C_M(h)| = |M : M'| = |M : G'| = |H|$ for all $1 \neq h \in H$. Thus every element of $H - 1$ acts fixed point freely on G', and so $M = G'H$ is a Frobenius group with the Frobenius kernel G' and a Frobenius complement H. Since H is abelian, by [3, Theorem 3.1, p.339], H is cyclic. This completes the proof of (i).

Let N be any minimal normal subgroup of G. From (i) we know that N is either a π-group or an abelian π'-group. If N is an abelian π'-group, then N is a π-group by [2,(6.15)] and the hypothesis, a contradiction. Hence N is a π-group. This implies that $F(G) = \times_{p\in\pi} O_p(G)$. Let $p \in \pi$. If there exists a nonlinear $\chi \in \mathrm{Irr}(G)$ such that $p \nmid (|G|/\chi(1))$, we claim that $O_p(G) = 1$. Indeed, if $O_p(G) \neq 1$ then $Z(O_p(G)) \neq 1$, and so $p \mid (|G|/\chi(1))$ by [2,(6,15)], a contradiction. Suppose that $F(G)' = G'$. Then $G/F(G)$ is abelian and H acts trivially on $F(G)$ (See:[3,Theorem 3.5, p.180], [4,3.24 Satz, p.272], [4,Aufgaben,9), p.275]) forcing $H = 1$ ([4,4.25 Satz, p.277]). But then G is a π-group and $\pi = \pi(G)$, a contradiction. Thus $F(G)' < G'$. This completes the proof of (ii).

Let χ be any nonlinear irreducible character of G. Since $|G|/\chi(1)$ is a π-number and H is a π'-group, we have $\chi(h) = 0$ for all $1 \neq h \in H$ ([2,(8.17)]). By the second orthogonality relation we get that $|C_G(h)| = |G : G'|$ for every $h \in H - 1$. On the other hand, since $G'H$ is a Frobenius group with the Frobenius kernel G' and a Frobenius complement H, we have that $G' \cap C_G(h) = 1$ for all $1 \neq h \in H$. Hence $C_G(h)$ is a complement to G' in G, completing the proof of (iii).

Finally, from Lemma 3 it follows that (iv) is true. □

Corollary 1 Let G be a nonabelian group, and let $\pi = \pi(G')$. Assume that $\pi \neq \pi(G)$. If $|G|/\chi(1) = |G'|$ for all nonlinear $\chi \in \mathrm{Irr}(G)$, then

(i) $G = G'H$ is a Frobenius group with the kernel G' and a cyclic complement H, or

(ii) $G = KH$, where K is a normal Hall π-subgroup of G and H is a cyclic Hall π'-subgroup of G. Moreover, $K' = G'$, $K'H$ is a Frobenius group with the Frobenius kernel K' and a complement H, and K' has a complement in G.

Remark Let $G = G'H$ be a Frobenius group with the kernel G'. Assume that G' is abelian. Then, by [2,(6.34)], we have that $|G|/\chi(1) = |G'|$ for all nonlinear $\chi \in \mathrm{Irr}(G)$.

Lemma 4 *Let χ and θ be nonlinear irreducible characters of G. Assume that $|G|/\chi(1) = n$ and $|G|/\theta(1) = m$. Then $(n,m) \neq 1$.*

Proof n and m are positive integers ([2,(3,11)]). Suppose that $(n,m) = 1$. Then $nm \mid |G|$ and so $nm \leq |G|$. On the other hand, since $\chi(1)^2 < |G|$ we get that $n^2 > |G|$. Similarly, $m^2 > |G|$. Hence we get that $mn < n^2$ and $mn < m^2$. From it follows that $n < m$ and $m < n$, a contradiction. □

We note that [1,Theorem 2.A, p.25] is an immediate corollary of Lemma 4 and Theorem 1. [1,Theorem 2.B, p.25] is also a consequence of Theorem 1.

Lemma 5 *Let G be a nonabelian p-group, p prime. Then there exists a nonlinear $\chi \in \mathrm{Irr}(G)$ such that $p^2 \mid (|G|/\chi(1))$.*

Proof $|G| = p^n, n \geq 3$. Suppose that $p^2 \nmid (|G|/\chi(1))$ for all nonlinear $\chi \in \mathrm{Irr}(G)$. Then, $p^{n-1}|\chi(1)$ for all nonlinear $\chi \in \mathrm{Irr}(G)$. Therefore, we have that $|G| = |G:G'| + \Sigma\{\chi^2(1) : \chi \in \mathrm{Irr}(G), \chi(1) > 1\} = |G:G'| + p^{2n-2}m$, where m is some positive integer. Then since $1 < G' < G$, we get that $n-2 \leq -1$, and so $n = 1$, a contradiction. □

Theorem 2 *Let G be a nonabelian group. Suppose that $|G|/\chi(1)$ is a squarefree number for all nonlinear $\chi \in \mathrm{Irr}(G)$. Then the following statements are true:*

(i) $G = HK$, where H is an abelian Hall π'-subgroup of G and K is a normal Hall π-subgroup of G; π satisfies the following condition: for any $p \in \pi$ and $q \in \pi'$, $p > q$. Moreover, $K \neq 1$ and K is of squarefree order.

(ii) G is supersolvable with G' cyclic of squarefree order; however G can not be nilpotent.

(iii) $F(G)$ is a cyclic group of squarefree order.

Proof (a) First, we show that for the smallest prime divisor p of $|G|, G$ is p-nilpotent. In facct, if $|G|_p \geq p^2$, by the hypothesis we have that $p|\chi(1)$ for all nonlinear $\chi \in \mathrm{Irr}(G)$, and so G is p-nilpotent ([2,(12.2)]); If $|G|_p = p$, then G is also p-nilpotent ([4,2.8 Sats, p.420]). Let L be the normal p-complement, and let q be the smallest prime divisor of $|L|$. If L is abelian then L is q- nilpotent; If L is nonabelian, by Lemma 1, $|L|/\lambda(1)$ is a squarefree number for all nonlinear $\lambda \in \mathrm{Irr}(L)$, and so L is q- nilpotent. Repeatting the above argument, we claim that G satisfies the Sylow tower property.

Let p be,the largest prime divisor of $|G|$, and let P be a Sylow p- subgroup of G. Then $P \lhd G$. By Lemma 1 and Lemma 5, P is abelian. So P is a cyclic group of order p by [2, (6.15)].

By Schur-Zassenhaus theorem, $G = PT$, where T is a p-complement in G. If T is abelian, then the proof of (i) is completed. We suppose that T is nonabelian. Then since $T \cong G/P$, by Lemma 1 $|T|/\lambda(1)$ is a squarefree number for all nonlinear $\lambda \in \mathrm{Irr}(T)$. Let

q be the largest prime divisor of $|T|$, and let Q be a Sylow q-subgroup of T. The above argument shows that Q is a cyclic group of order q. Let B be a q-complement in T. We have that $G = PQB$. Since G satisfies the Sylow tower property, $PQ \lhd G$. If B is abelian, the proof of (i) is completed. If B is nonabelian, we repeat the above argument again until we meet an abelina Hall subgroup. This completes the proof of (i).

(b) Note that a group of squarefree order is supersolavable, and that a split extension of a group of squarefree order by a supersolvable group is also supersolvable. Thus, from (i), (ii) is obtained.

(c) Let $P \in \mathrm{Syl}_p(F(G))$. By Lemma 1 and Lemma 5 we get that P is abelian. Thus, by [2, (6.15)] P is a cyclic group of order p and consequently $F(G)$ is a group of squarefree order, proving (iii). □

From the argument of Theorem 2, we obtain the following:

Corollary 2 *Let G be a nonabelian group, and let $\pi(G) = \{p_1, p_2, \cdots, p_n\}$, where $p_1 < p_2 < \cdots < p_n$. Assume that $p_i^2 \not| |G|/\chi(1), i = 1, 2, \cdots, n-1$, for all nonlinear $\chi \in \mathrm{Irr}(G)$. Then G satisfies the Sylow tower property. In particular, G is solvable. In addition, G/P is supersolvable, where $P \in \mathrm{Syl}_{p_n}(G)$, and $dl(G) \le dl(P) + 2$.*

Corollary 3 *Let G be a nonabelian group, and let $\pi(G) = \{p_1, p_2, \cdots, p_n\}$, where $p_1 < p_2 < \cdots < p_n$. Assume that $p_i^2 \not| |G|/\chi(1), i = 1, 2, \cdots, m$, where $1 \le m \le n$, for all nonlinear $\chi \in \mathrm{Irr}(G)$. Then $G = HK, H$ is a Hall $\{p_1, \cdots, p_m\}$-subgroup of G and K is a normal Hall $\{p_{m+1}, \cdots, p_n\}$-subgroup of G. Moreover, H is supersolvable and $dl(G) \le dl(K) + 2$.*

Theorem 3 *Let G be a nonsolvable group. Assume that $2^3 \not| (|G|/\chi(1))$ for all nonlinear $\chi \in \mathrm{Irr}(G)$. Then G has exactly one chief factor which is nonabelian, and this nonabelian chief factor is $\mathrm{PSL}(2, q), q > 3, q \equiv 3$ or 5(mod 8). In paricular, if G is a nonabelian simple group, then G is isomorphic to $\mathrm{PSL}(2, q), q > 3, q \equiv 3$ or 5 (mod 8).*

Proof In the following proof, the solvability of groups of odd order is used.

Let $|G|_2 = 2^n$. Then $n \ge 2$, and if $n = 2$ then Sylow 2-subgroups of G are elementary abelian of order 4. We claim that $n = 2$. Indeed, if $n \ge 3$, then by the hypothesis $2|\chi(1)$ for all nonlinear $\chi \in \mathrm{Irr}(G)$. Thus, by [2,(12.2)] G is 2-nilpotent and consequently G is solvable, a contradiction. Since $|G|_2 = 4, G$ has exactly one nonabelian chief factor and it must be a nonabelian simple group.

By Lemma 1 we may assume without loss of generality that G is nonabelian simple group. Since Sylow 2-subgroups of G are elementary abelian of order 4, by [3, Theorem, p.485], noting that $\mathrm{PSL}(2,4) \cong A_5 \cong \mathrm{PSL}(2,5)$, we get that $G \cong \mathrm{PSL}(2, q), q > 3, q \equiv 3$ or 5 (mod 8). □

Let $G = G'H$ be a Frobenius group with the Frobenius kernel G' and a complement H. If the commutator subgroup G' is elementary abelian of order p^2 and H is cyclic of order q, where p and q are distinct prime numbers., then G is called a Frobenius group of type $(Z_p \times Z_p) \rtimes Z_q$, we have that $|G|/\chi(1) = p^2$ for all nonlinear $\chi \in \mathrm{Irr}(G)$ (2,(6.34)).

Lemma 6 *Let G be a nonabelian p-group, p prime. If $p^3 \not| (|G|/\chi(1))$ for all nonlinear $\chi \in \mathrm{Irr}(G)$, then G is of order p^3.*

Proof Similar to the proof of Lemma 5. □

Lemma 7 *Let G be a nonabelian group, and let p be the smallest prime divisor of $|G|$. Assume that $|G|_p = p^2$. If A_4 is not involved in G, then G is p-nilpotent.*

Proof We use induction on the order of G. Let H be a proper subgroup of G. If $|H|_p \leq p$, then H is p-nilpotent by [4, 2.8 Satz, p.420]. If $|H|_p = p^2$, then H is p-nilpotent by induction. Thus, if G is not p-nilpotent then G is minimal non p-nilpotent, and consequently a Frobenius group of type $(Z_p \times Z_p) \rtimes Z_q$ is involved in G by [4,5.4 Satz, p.434]. Since p is the smallest prime divisor of $|G|$, we get that $p = 2$ and $q = 3$, so that A_4 is involved in G, a contradiction. □

Theorem 4 *Let G be a nonabelian group, and let $\pi = \{p | p \in \pi(G)$ and $|G|_p \geq p^3\}$. Suppose that $\pi \neq \emptyset$. If $|G|/\chi(1)$ is a cubefree number for all nonlinear $\chi \in \mathrm{Irr}(G)$, then the following staements are true:*

(i) G is either of the following two types:

(a) $G = HK$, where H is an abelian Hall π-subgroup of G and K is a normal Hall π'-subgroup of G of cubefree order. Moreover, H is not normal in G and $F(G)$ is of cubefree order.

(b) $G = PK$, where P is a Sylow p-subgroup of G for some prime p, and K is the normal p-complement in G. Moreover, P is nonabelian of order p^3 and K is of cubefree order. P is not normal in G unless K is abelian.

(ii) If $|G|_2 \neq 4$, then G satisfies the Sylow tower property. If $|G|_2 = 4$ and A_4 is no involved in G, then G satisfies the Sylow tower property.

(iii) If Frobenius groups of type $(Z_p \times Z_p) \rtimes Z_q$ are not involved in G, then G is supersolvable and G' is abelian of cubefree order.

Proof Let p be an arbitrary prime number in π. By hypothesis $p | \chi(1)$ for all nonlinear $\chi \in \mathrm{Irr}(G)$ and so G is p- nilpotent. From this it follows that $G = HK$, where H is a Hall π subgroup of G and K is a normal Hall π'-subgroup of G. Clearly, K is of cubefree order and $|H|_p \geq p^3$ for every prime divisor p of $|H|$.

Suppose that H is abelian. If $H \lhd G$, by [2, (6.15)] $|H|_p \leq p^2$ for every prime divisor p of $|H|$, a contradiction. Thus H is non-normal in G. Since $O_p(G)$ is abelian for each prime divisor p of $|G|$, by [2,(6.15)] we have that $|O_p(G)| \leq p^2$ for every prime divisor p of $|G|$. From this we get that $F(G)$ is abelian of cubefree order, proving (i) (a).

Suppose next that H is nonabelian. Let p be an arbitrary prime divisor of $|H|$ and let $P \in \mathrm{Syl}_p(H)$. Since $H \cong G/K$, by hypothesis and Lemma 1 $|H|/\lambda(1)$ is a cubefree number for all nonlinear $\lambda \in \mathrm{Irr}(H)$ and hence H is p-nilpotent by [2,(12.2)]. Thus H is nilpotent and $P \lhd H$. P is not abelian; otherwise, by [2,(6.15)], we would get that $|P| \leq p^2$, a contradiction. Therefore, by [2,(3.11),(4.21)] we get that $|\pi| = 1$ and $H = P$. Suppose that P is normal in G and K is nonabelian. Then $G = HK = P \times K$ and there exists a nonlinear $\lambda \in \mathrm{Irr}(K)$. Hence by [2,(3.11) and (4.21)] there exists a nonlinear $\chi \in \mathrm{Irr}(G)$ such that $p^3 \mid |P| \mid (|G|/\chi(1))$, a contradiction. Therefore, P is not normal in G unless K is abelian.

Since $|H|/\lambda(1)$ is a cubefree number for all nonlinear $\lambda \in \mathrm{Irr}(H)$, $H = P$ is of order p^3

by Lemma 6, proving (i) (b).

By Lemma 7 and by using the argument in the proof of Theorem 2(i), we get that G satisfies the Sylow tower property, proving (ii).

We now prove (iii). Let p be the largest prime divisor of G, and let $P \in \mathrm{Syl}_p(G)$. From (ii) it follows that $P \lhd G$. If $\Phi(G) \neq 1$, then induction implies that $G/\Phi(G)$ is supersolvable and hence G is supersolvable. Therefore, we assume that $\Phi(G) = 1$. Then, since $F \lhd G$, we have $\Phi(P) = 1$ and consequently P is elementary abelian. Since $|G|/\chi(1)$ is a cubefree number for all nonlinear $\chi \in \mathrm{Irr}(G)$, by [2,(6.15)] we have that $|P| \leq p^2$. If P is cyclic, then G is supersolvable since induction implies G/P is supersolvable. Hence we asuume that P is elementary abelian of order p^2. Similarly, we may assume that P is minimal normal in G. By Schur-Zassenhaus theorem, we have that $G = PM$, where M is a p-complement in G. Since P is a minimal normal subgroup of G and P is abelian, M must be a maximal subgroup of G.

Now $M \cong G/P$ and hence M is supersolvable by Lemma 1 and induction. Let Q be a minimal normal subgroup of M. Then $|Q| = q$, a prime. Set $Q = \langle x \rangle$. If $C_G(x) \neq 1$, then $M < N_G(Q)$ and as M is maximal in G, we get that $Q \lhd G$. Thus, since G/Q is supersolvable, G is supersolvable. Therefore, we assume $C_G(x) = 1$. But, this implies that PQ is a Frobenius group of type $(Z_p \times Z_p) \rtimes Z_q$, contradicting our assumption. This completes the proof of (iii). □

Corollary 4 *Let G be a nonabelian group. Assume that for each prime divisor p of $|G|$, $|G|_p = p$ or $|G|_p = p^n$ with $n \geq 3$. If $|G|/\chi(1)$ is a cubefree number for all nonlinear $\chi \in \mathrm{Irr}(G)$, then G is supersolvable with G' abelian of cubefree order.*

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关于有限群的阶与其特征标的商

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摘 要

设 G 是有限群. D.Chillag 和 M.Herzog 曾考察过下述问题：商集合 $\{|G|/\chi(x) | \chi \in \mathrm{Irr}(G)\}$ 上的某些算术条件如何影响群 G 本身的结构. 本文继续研究这个问题.