Journal of Mathematical Research & Exposition
Vol. 17, No. 3, 371-374, Aug. 1997

Some Linear Isomorphism Theorems for Certain 2nth-Order Nonsymmetric Differential Operator with Parameter*

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Abstract This paper provides several regularity theorems for certain $2n$th-order nonsymmetric differential operator A_λ with parameter, from which we can illustrate the stability behaviors in both directions of a class of flying vehicles in their moving prodesses

Key words differential operator, regularity, linear isomorphism, stability.

Classification AMS(1991) 47H, 47N20/CCL O177.91

Let A_λ be a general $2n$th-order nonsymmetric differential operator as follows:

$$A_\lambda = \frac{d^{2n}}{dx^{2n}} + \sum_{i=1}^{2n-1} P_i \frac{d^{2n-i}}{dx^{2n-i}} + N_\lambda I: K \subset C^{2n}[0,1] \to A_\lambda K \subset C[0,1],\ n \geq 2, \tag{1}$$

where $N_\lambda = \lambda^2 + \frac{\lambda}{V_0} N(x)$, $N(x)$ and $P_i = P_i(x)$ $(i = 1, \cdots, 2n-1) \in C[0,1]$ which are always real functions, $\lambda \in C$, I is the identity mapping and

$$K = \begin{cases} K_{(i_1, \cdots, i_n)} = \{y(x) \in C^{2n}[0,1] \mid y^{(i_k)}_{x=0,1} = 0, k = 1, \cdots, n\}, \\ \qquad 0 \leq i_1 < i_2 < \cdots < i_n \leq 2n, \\ C_C^{2n}[0,1], \text{ and } K_1 = K_{(0, \cdots, n-1)},\ K_2 = K_{(n, n+1, \cdots, 2n-1)}. \end{cases} \tag{2}$$

We know that the regularity points of A_λ for the parameter λ can illustrate the stability behaviors in both direction of a class of flying vehicles[1]. The papers [2, 3] have considered some special cases of this operator for both $n = 2$ and 3. This paper deals with the regularity of A_λ overall by giving a series of linear isomorphism classes respectively for the following three type of operators:

$$\left.\begin{aligned} &(A_K): A_\lambda: (K, \|\cdot\|_{n+k}) \to (A_\lambda K, \|\cdot\|_{-(n+k)}), k = 0, 1, \cdots, n \\ &(B_K): A_\lambda: (K, \|\cdot\|_{2n}) \to (A_\lambda K, \|\cdot\|_{L^2}), \\ &(C_K): A_\lambda: (K, \|\cdot\|_{C^{2n}}) \to (A_\lambda K, \|\cdot\|_C). \end{aligned}\right\} \tag{3}$$

* Received Sep. 1, 1994.

where

$$\|y\|_m = \Big(\sum_{i=0}^{m} \|y^{(i)}\|_{L^2}^2\Big)^{\frac{1}{2}}, m = 0, \cdots, 2n,$$

$$\|y\|_{C^{2n}} = \sum_{i=0}^{2n} \|y^{(i)}\|_C \text{ for each } y \in C^{2n}[0, 1],$$

and $\|\cdot\|_{-m}$ is the dual norm of $\|\cdot\|_m$ for every $m = 1, \cdots, 2n$.

The main results have been obtained by considering all possible functional constructions concerning A_λ, and all the extension isomorphisms of the operators in (3), using the methods of partial differential equations and functional analysis. In the following theorems, the alphabet a is a positive constant dependent only on $2n$ but not on $m = 0, 1, \cdots, 2n$.

Theorem 1 *Let* $k = 0, P_i \in C^{n-i}[0, 1], i = 1, \cdots, n-1$. *Let*

$$\delta = \Big[\sum_{i=1}^{n-1} \sum_{K=0}^{n-i} \binom{n-i}{k} \cdot \|P_i^{(n-K-i)}\|_C + \|P_n\|_C + 1\Big]^{-1},$$

$$\epsilon_0^* \triangleq \epsilon_0^*(\epsilon_1, \cdots, \epsilon_{n-1}) = \Big[a \sum_{K=1}^{n-1} \Big(\frac{1}{\delta} \sum_{i=1}^{n-K} \binom{n-i}{k} \|P_i^{(n-K-i)}\|_C + \|P_{2n-K}\|_C + \epsilon_K\Big) + 1\Big]^{-1},$$

$$C_0(\epsilon_0, \epsilon_1, \cdots, \epsilon_{n-1}) = \Big\{\lambda \in C \,\Big|\, \min_{x \in [0, 1]} (-1)^n \mathrm{Re} N_\lambda \geq \frac{1}{2\delta} \sum_{i=1}^{n} \|P_i^{(n-i)}\|_C$$

$$+ \frac{1}{2} \sum_{j=n+1}^{2n-1} \|P_j\|_C + \frac{1}{2}(\epsilon_0^{*-1} - 1)(\epsilon_0^*)^{-(n-1)} + \frac{1}{2}\epsilon_0\Big\},$$

where $\epsilon_0, \epsilon_1, \cdots, \epsilon_{n-1} \geq 0$. *Then the* (A_0) *type operator* A_λ *in* (3) *is a linear isomorphism for each* $\lambda \in C_0(\overbrace{0, \cdots, 0}^{n})$ *when* $K = K_1$ *and for each* $\lambda \in C_0(\overbrace{0, \cdots, 0}^{n})$ *when* $K = K_2$ *respectively.*

Theorem 2 *Let* $0 < k < n, N(x) \in C^K[0, 1], P_i \in C^{n-K-i}[0, 1], 1 \leq i \leq n-k-1$, *and* $K = C_C^{2n}(0, 1)$. *Let*

$$\delta = \Big[1 + 2\sum_{i=1}^{n-k-1} \sum_{j=0}^{n-K-i} \binom{n-k-i}{j} \|P_i^{(n-K-i-j)}\|_C + 2\|P_{n-K}\|_C\Big]^{-1},$$

and

$$\epsilon_K^* = \Big[a\Big(\frac{2}{\delta} \sum_{j=1}^{n-K-1} \sum_{i=1}^{n-K-i} \binom{n-k-i}{j} \|P_i^{(n-K-i-j)}\|_C + 3\sum_{j=1}^{n+K-1} \|P_{2n-j}\|_C$$

$$+ \frac{1}{\delta} \sum_{i=1}^{n-K} \|P_i^{(n-K-i)}\|_C + \frac{1}{\delta}\Big) + 1\Big]^{-1},$$

$$C_K(\epsilon_0, \epsilon_1, \cdots, \epsilon_{K-1})$$

$$= \{\lambda \in C \mid \min_{x\in[0,1]}[(-1)^n \mathrm{Re} N_\lambda - \frac{1}{2}(a+1)\sum_{j=1}^{K-1}\binom{k}{j}\|N_\lambda^{(K-j)}\|_C - \frac{1}{2}\|N_\lambda^{(K)}\|_C]$$

$$\geq \frac{1}{2\delta}\sum_{i=1}^{n-K}\|P_i^{(n-K-i)}\|_C + \frac{1}{2}\sum_{j=1}^{n+K-1}\|P_{2n-j}\|_C + \frac{a}{2}\sum_{j=1}^{K-1}\epsilon_j$$

$$+ \frac{1}{2}\epsilon_0 + \frac{1}{2}(\epsilon_K^{*-1} - 1)(\epsilon_K^*)^{-(n+K-1)}\},$$

where $\epsilon_i \geq 0\ (i = 0, \cdots, k-1)$. *Then the* (A_K) *type operator* A_λ *in* (3) *is a linear isomorphism for each* $\lambda \in C_K(\overbrace{\frac{1}{2}, \cdots, \frac{1}{2}}^{k})$.

Theorem 3 *Let* $k = n$ *and* $N(x) \in C^n[0,1]$.

(*i*) *For* $K = K_1$. *Let*

$$\delta_1 = (\sum_{i=1}^{2n-1}\|P_i\|_C + 1)^{-1}, \epsilon_n^{(1)} = (\frac{a}{\delta_1}\sum_{j=n+1}^{2n-1}\|P_{2n-j}\|_C + 1)^{-1},$$

and

$$C_n^{(1)} = \{\lambda \in C \mid \min_{x\in[0,1]}[(-1)^n \mathrm{Re} N_\lambda - \frac{1}{2}\sum_{j=0}^{n-1}(1 + (\frac{1}{4})^{n-j})\binom{n}{j}\|N_\lambda^{(n-j)}\|_C]$$

$$\geq \frac{1}{2\delta_1}\sum_{j=1}^{n}(\frac{1}{4})^{n-j}\|P_{2n-j}\|_C + \frac{1}{2}(\epsilon_n^{(1)-1} - 1)(\epsilon_n^{(1)})^{-(n-1)}\},$$

then the (A_n), (B_n) *and* (C_n) *type operators* A_λ *in* (3) *are all linear isomorphisms for each* $\lambda \in C_n^{(1)}$;

(*ii*) *For* $K = K_2$. *Let*

$$\delta_2 = \delta_1, \epsilon_n^{(2)} = \epsilon_n^{(2)}(\epsilon_1, \cdots, \epsilon_{2n-1}) = [2a\sum_{j=1}^{2n-1}(\frac{1}{\delta_2}\|P_{2n-j}\|_C + \epsilon_j) + 1]^{-1},$$

$$C_n^{(2)}(\epsilon_0, \epsilon_1, \cdots, \epsilon_n)$$

$$= \{\lambda \in C \mid \min_{x\in[0,1]}[(-1)^n \mathrm{Re} N_\lambda - \frac{1}{2}(a+1)\sum_{j=1}^{n-1}\binom{n}{j}\|N_\lambda^{(n-j)}\|_C] - \frac{1}{2}\|N_\lambda^{(n)}\|_C]$$

$$\geq \frac{\delta_2}{2}\sum_{i=1}^{2n-1}\|P_i\|_C + \frac{1}{2\delta_2} + \frac{1}{2}(\epsilon_n^{(2)-1} - 1) - 1)(\epsilon_n^{(2)})^{-(2n-1)} + \epsilon_0\},$$

$$\text{where } \epsilon_i \geq 0,\ i = 0, \cdots, 2n-1.$$

Then all operators A_λ *for* $k = n$ *in* (3) *are linear isomorphisms for each* $\lambda \in C_n^{(2)}(\overbrace{\frac{1}{4}, \cdots, \frac{1}{4}}^{n}, \overbrace{0, \cdots, 0}^{n})$.

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带参数2n阶非对称微分算子的一些线性同构定理

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摘　要

本文给出了一类带参数2n阶非对称微分算子A_λ的一些正则性定理, 籍此可刻画一类飞行器在其运行过程中的双向平稳行为