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Spaces with a σ-Point Finite Base*

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Abstract In this paper it is shown that spaces with a σ-point finite base can be characterized by the images of metric spaces under certain maps.

Key words σ-point finite base, open map, compact map, continuous map.

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All spaces considered in this paper are T_1.

In 1963, S. Hanai[1] proved that a space has a σ-point finite base if and only if it is an image of a metric space under an open, compact and continuous map. L. F. Liu[2] also obtained the same result. The first result in this paper is to point out a counter-example to show the result mentioned above is not true. Next we introduce the concept of inductive open, weak compact and continuous maps, and prove that a space has a σ-point finite base if and only if it is an image of a metric space under an inductive open, weak compact and continuous map.

First of all, we describe a relationship between spaces with a σ-point finite base and images of metric spaces under open, compact and continuous maps. The following Lemma is well known, cf. [3, 4].

Lemma *A* T_2 *space is a perfect space with a σ-point finite base if and only if it is an images of a metric space under an open, compact and continuous map.*

Example There is a regular space X with a σ-point finite base which is not a perfect space.

H. H. Corson and E. Michael[5] has shown that there is a non-perfect, regular space X with a σ-point finite base (Example 6.4 in [5]). By Lemma, X is not an image of a metric space under an open, compact and continuous map. Thus Theorem 2 in [1] and Theorem 1 in [2] are wrong. The mistake of the proof of Theorem 1 in [2] is that the map f constructed in the proof of the necessity can not be a compact map because $f^{-1}(x)$ is only a subset of the compact subspace C_x of the metric space $\prod_{i\in N} A_i$. Hence the following question can be raised: By means of what maps can the relationship between metric spaces and spaces with a

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σ-point finite base be established? Next we shall answer this question by the concept of inductive open, weak compact and continuous maps.

Definition 1 *Let Z be a space. A subset Y of Z is said to be weak compact in Z if the closure of Y in Z is compact in Z.*

Definition 2 *Let g be a map from a space Z onto a space X. g is said to be inductive open, weak compact and continuous if there is a subspace S of Z such that $g|_S$ is an open and continuous map from S onto X, and $g^{-1}(x)\cap S$ is weak compact in Z for each $x\in X$.*

Theorem *A space has a σ-point finite base if and only if it is an image of a metric space under an inductive open, weak compact and continuous map.*

Proof Necessity. By the proof of the necessity of Theorem 1 in [2], there are a metric space $Z(=\prod_{i\in N}A_i)$, a subspace S of Z and an open and continuous map f from S onto X such that there is a compact subspace C_x of Z with $f^{-1}(x)\subset C_x$ for each $x\in X$. Take a point $x_0\in X$. We define a map g from Z onto X by

$$g(z)=\begin{cases} f(z), & z\in S,\\ x_0, & z\in Z\setminus S.\end{cases}$$

It is easy to check that g is an inductive open, weak compact and continuous map from Z onto X.

Sufficiency. Let g be an inductive open, weak compact and continuous map from a metric space Z onto X. Then there is a subspace S of Z such that $g|_S$ is an open, continuous map from S on to X, and $g^{-1}(x)\cap S$ is weak compact in Z for each $x\in X$. Let $f=g|_S$. Since Z is metric, it has a σ-locally finite base $\mathbf{B}$ by the classical Nagata-Smirnov metrization theorem. Denoted $\mathbf{B}$ by $\cup\{\mathbf{B}_n: n\in N\}$, where $\mathbf{B}_n$ is locally finite in Z for each $n\in N$. Put

$$\mathbf{P}=\cup\{\mathbf{P}_n: n\in N\},$$

where $\mathbf{P}_n=\{f(B\cap S): B\in\mathbf{B}_n\}$ for each $n\in N$, then $\mathbf{P}$ is a base of X because f is an open and continuous map from S onto X. For each $x\in X$, $\overline{f^{-1}(x)}$ is compact in Z, then

$$\{\overline{f^{-1}(x)}\cap B: B\in\mathbf{B}_n\}$$

is finite by the locally finiteness of $\mathbf{B}_n$, thus

$$\{\overline{f^{-1}(x)}\cap B\cap S: B\in\mathbf{B}_n\}$$

is finite. Since $f^{-1}(x)$ is closed in S,

$$\overline{f^{-1}(x)}\cap S=f^{-1}(x),$$

hence $\{f^{-1}(x)\cap B\cap S: B\in\mathbf{B}_n\}$ is finite, therefore $\mathbf{P}_n$ is point finite at point x. So, $\mathbf{P}$ is a σ-point finite base for X, i.e., the space X has a σ-point finite base.

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具有 σ-点有限基的空间

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摘 要

本文纠正了Hanai等用开紧连续映射刻画具有σ-点有限基的空间的一个错误, 定义了诱导开的弱紧连续映射建立度量空间与具有σ-点有限基空间的映射联系