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Some Errors in the Paper "Programming with Semilocally Convex Functions"*

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Abstract In this paper, we illustrate some errors[1] with some concrete counterexample.

Keywords locally starshaped set, semilocally convex functions on a locally starshaped set, S - semilocally convex functions on a locally starshaped set, separation theorem, theorem of the alternatives

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In [1], T. Weir derives a theorem of the alternatives for semilocally convex functions defined on locally starshaped sets. This result is applied to constrained minimization problems to obtain optimality conditions and duality theorems. But the results obtained are all erroneous. In this paper, we list all errors with some counterexamples.

R^n will denote n - dimensional Euclidean space with Euclidean norm $|\cdot|$, $R^+ = [0, +\infty)$, A set X in R^n is a convex cone if $X + X \subseteq X$ and $\alpha X \subseteq X$ for all $\alpha \in R^+$.

A set C of R^n is locally starshaped at $x_0 \in C$ if corresponding to x_0 and each $x \in C$ there exists a maximal positive number $a(x_0, x) \leq 1$ such that $wx + (1-w)x_0 \in C$ for $0 < w < a(x_0, x)$; the set C is said to be locally starshaped if it is locally starshaped at each of its points.

Let C be a locally starshaped set in R^n, A scalar valued function $f: C \to R$ is called similocally convex on C if corrsponding to each $x, y \in C$ there exists a positive number $d(x, y) \leq a(x, y)$ such that

$$f(wx + (1-w)y) \leq wf(x) + (1-w)f(y), \quad 0 < w < d(x, y).$$

Let C be a locally starahaped set in R^n and let S be a convex cone in R^m, A vector valued function $f: C \to R^m$ is called S - similocally convex on C if corresponding to each $x, y \in C$ there exists a positive number $d(x, y) \leq a(x, y)$ such that

$$wf(x) + (1-w)f(y) - f(wx + (1-w)y) \in S, \quad 0 < w < d(x, y).$$

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Firstly, T. Weir established the following quite strong result for locally starshaped set in R^n.

Lemma 3.2[1] *Let C be a locally starshaped set in R^n, then* $\mathrm{cl}(c)$ *is convex.*

The following example shows that the Lemma 3.2 is false.

Example 1 In R^1. Let $C = (1, 2) \cup (3, 4)$, then C is a locally starshaped set in R^1, but $\mathrm{CL}\ (C) = [1, 2] \cup [3, 4]$ is not convex set in R^1.

In R^2. Let

$$C = \{(x, y) \in R^2 | x^2 + y^2 < 1 \quad \text{or} \quad (x - 2)^2 + (y - 2)^2 < 1\},$$

then C is a locally starshaped set in R^2, but

$$\mathrm{CL}\ (C) = \{(x, y) \in R^2 | x^2 + y^2 \leq 1 \quad \text{or} \quad (x - 2)^2 + (y - 2)^2 \leq 1\}$$

is not a convex in R^2.

Remark 1 The above example shows that a set C is locally starshaped set, but $\mathrm{CL}\ (C)$ is not necessarilly a locally starshaped set.

Remark 2 By the definetion of a locally starshaped sets, we can obtain that each open set in R^n is a locally starshaped set.

Secondly, T. Weir obtained the following separation theorem which is used for establishing the theorem of the alternatives.

Lemma 3.3[1] *Let S be a locally starshaped set in R^n and let T be a convex set in R^n with non-empty interior, if S and T are disjoint, then there exists a non-zero continuous linear functional P defined on R^n and a scalar β such that* $\sup\{P(x): x \in T\} \leq \beta \leq \inf\{P(x): x \in S\}$.

To prove Lemma 3.3, T. Weir used the wrong Lemma 3.2 so that the proof is false. The following example shows that the Lemma 3.3 itself is erroneous.

Example 2 In R^1. Let $S = (1, 2) \cup (5, 6)$, $T = (3, 4)$ then S and T satisfy the reqirements of the Lemma 3.3. But there doesn't exist any non-zero continuous linear functional P and scalar β which satisfy the requirements of the Lemma 3.3.

In R^2. Let

$$S = \{(x, y) \in R^2 | (x - 2)^2 + y^2 < 1 \quad \text{or} \quad (x + 2)^2 + y^2 < 1\},$$

$$T = \{(x, y) \in R^2 | x \in (-1, 1) \quad \text{and} \quad y \in R^1\},$$

then S and T satisfy the requirements of the Lemma 3.3. But there doesn't exist any non-zero continuous linear functional P and scalar β which satisfy the requirements of the Lemma 3.3.

The following is a theorem of the alternatives for semilocally convex functions defined on locally starshaped sets. This theorem is an important result in [1].

Theorem 3.5 *Let S be a convex cone with non-empty interior in R^m. Let T be a locally starshaped set in R^n and $f: T \to R^m$ be S-semilocally convex, then exactly one of the following two sys-*

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tems has a solution:

(1) $-f(x) \in \text{int}(S)$;

(2) $(p * f)(T) \subseteq R^+$, $0 \neq p \in S^*$.

To prove Theorem 3.5, T. Weir used the wrong Lemma 3.3 so that the proof is false. The following example shows that the Theorem 3.5 itself is erroneous.

Example 3 Let

$$S = R_+^2 \subseteq R^2, \ T = \{(x, y) \in R^2 \mid (x - 2)^2 + (y - 2)^2 < 1$$

or

$$(x - 2)^2 + (y + 2)^2 < 1\} \subseteq R^2,$$

$f: T \to R^2$ defined as $f(x) = x, \forall x \in T$, then f is S-semilocally convex function, T and S fulfill the requirements of Theorem 3.5, but the results of Theorem 3.5 is not correct.

The first system has not solution, since $f(T) = T$ and $(-T) \cap \text{int}(S) = \emptyset$, The second system has not any solution. Otherwise, there exists a $0 \neq p = (p_1, p_2) \in S^*$ such that $p_1 x + p_2 y \geq 0$, $\forall (x, y) \in T$. This implies $p_1 = p_2 = 0$, a contradiction.

The results following Theorem 3.5 in [1] are also not correct since the error of the Theorem 3.5, we omit the counterexample here.

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References

[1] T. Weir, *Programming with semilocally convex functions*, J. of Math. Anal. Appl., **168**(1992), 1—12.

关于"Programming with Semilocally Convex Function"一文的错误

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摘 要

"Programming with Semilocally Convex Function"[1]一文对定义在局部星形集上的局部凸函数给出了二择一定理, 并应用到约束最小化问题, 得到了优化条件和共轭定理, 本文用具体反例说明这些结果是错误的.

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