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A Localization of Dirac's Theorem for Hamiltonian Graphs*

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Abstract New sufficient conditions for Hamiltonian graphs are obtained in this paper, which generalize Fan's theorem and Bedrossian et al's result[1].

Keywords Hamiltonian graph, subgraphs pair, maximal cycle, induced subgraph.

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1. Terminologies and Notations

Let G be a simple graph. Its vertex set is denoted by $V(G)$ and edge set by $E(G)$. For $\forall S \subset V(G)$, the induced subgraph by S in G is denoted by $\langle S\rangle_G$. We use $G_1 \to G$ to denote that G_1 is an induced subgraph of G, and use $C(G)$, $w(G)$ to denote the closure to G and the component number of G, respectively.

For given graph H and integer k, the vertex set $D_k(S)$ is defined as

$$D_k(S) = \{x \mid d_G(x) \geq k, x \in S \text{ and } S \subset V(G), \langle S\rangle_G \cong H\}$$

We use $D_k(H)$ denoting the vertex set $D_K(S)$ with minimum elements

If $K \subset V(G)$, $L \subset V(G)$, the notation $N_K(L)$ denote the set of vertices in K which are adjacent to some vertices in L. The claw and modified claw are denoted by $K_{1,3}$, Z_1, respectively.

The end-sum graph $sP \oplus G$ is defined as follows:

Definition 1.1 *For any distinct* $x_1, x_2, \cdots, x_s \in V(G)$ *and* $y_1, y_2, \cdots, y_s \in V(G)$ $(x_i \neq y_i)$, *and* $P_1, P_2, \cdots, P_s$ *are paths with length* ≥ 3 *and* $End(P_i) = \{u_i, v_i\}$, *the end-sum graph* $sP \oplus G$ *is defined as*

$$V(sP \oplus G) = \bigcup_{i=1}^{s} V(P_i) \cup V(G) \setminus \left(\bigcup_{j=1}^{s} End(P_i)\right).$$

$$E(sP \oplus G) = \bigcup_{i=1}^{s} E(P_i) \cup E(G) \cup \{x_iu, y_iv \mid u_iu \in E(P_i) \text{ and } v_iv \in E(P_i), 1 \leq i \leq s\} \setminus \{u_iu, v_iv \mid u_iu \in E(P_i), v_iv \in E(P_i), 1 \leq i \leq s\}.$$

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A direct description of the end-sum graph is shown in Fig. 1.

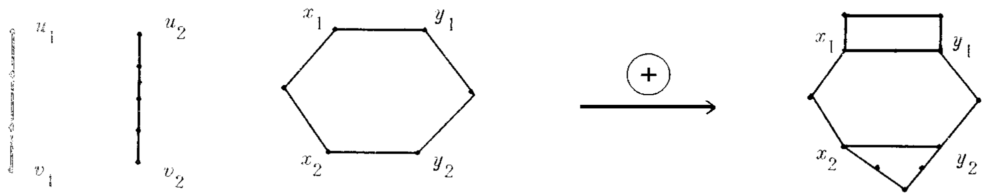

Fig. 1

It is obvious that the connectivity of $sP \oplus G$ is 2.

2 Main Results

Choosing C as the maximal cycle containing all the vertices of degree $\geq n/2$ in the closure graph $C(G)$, we get

Lemma 2.1 *Let G be a 2-connected non-hamiltonian graph with order $n \geq 3$, and $H_1, H_2, \cdots, H_s$ be the components of $C(G) \setminus C$. If $|D_{n/2}(K_{1,3})| \geq 2$ and one of the following condition is satisfied*

i) $|D_{n/2}(Z_1)| \geq 2$;

ii) *if* $Z_1 \to G$, $xy \in E(\overline{Z_1})$, *then* $\max\{d_G(x), d_G(y)\} \geq n/2$.

Then for $\forall i, 1 \leq i \leq s$, $N_C(H_i) = 2$ *and* $N_{H_i}(C) = 2$. □

The main result of this paper is the following theorem.

Theorem *Let G be a 2-connected graph with order $n \geq 3$. If $|D_{n/2}(K_{1,3})| \geq 2$ and one of the following conditions is satisfied*

i) $|D_{n/2}(Z_1)| \geq 2$;

ii) *if* $Z_1 \to G$, $xy \in E(\overline{Z_1})$, *then* $\max\{d_G(x), d_G(y)\} \geq n/2$.

Then G is hamiltonian unless $C(G) = sP \oplus H$, where C is a maximal cycle containing all the vertices of degree $\geq n/2$, $H = \langle V(C) \rangle_G$ and $s = w(C(G) \setminus C)$, and $\forall x_k \in V(P_i) \cap V(C)$, $x_k u_k \in E(P_i)$, $1 \leq i \leq s$, $\langle \{x_k^-, x_k, x_k^+, u_k\} \rangle_G \cong K_{1,3}$. □

Corollary 2.1 *Let G be a 2-connected graph with order $n \geq 3$. If $|D_{n/2}(K_{1,3})| \geq 2$ and $|D_{n/2}(Z_1)| \geq 2$, and the edge-connectivity $K_1(G) \geq 3$, then G is hamiltonian.*

Corollary 2.2[1] *Let G be a 2-connected graph of order $n \geq 3$. If for each pair of nonadjacent x, y in an induced claw of G or induced modified claw of G, $\max\{d_G(x), d_G(y)\} \geq n/2$, then G is hamiltonian.*

Corollary 2.3[2] *Let G be a 2-connected graph with order $n \geq 3$. If $|D_{n/2}(K_{1,3})| \geq 3$ and $|D_{n/2}(Z_1)| \geq 2$, then G is hamiltonian.* □

Note that the closure of the graphs considered in our theorem, not liking that in the Corollary 2.2 and 2.3, are not all $K_{1,3}$-free. Here we also present two conjectures.

Conjecture 2.1 *Let G be a 3-connected graph with order $n \geq 3$. If $|D_{n/2}(K_{1,3})| \geq 2$ and*

$|D_{n/2}(Z_1)| \geq 1$, *then G is hamiltonian.*

Conjecture 2.2 *Let G be a 1-tough graph with order* $n \geq 3$. *If* $|D_{n/2}(K_{1,3})| \geq 2$ *and* $|D_{n/2}(Z_1)| \geq 1$, *then G is hamiltonian.*

References

[1] F. Bedrossian, G. Chen and R. H. Schelp, *A generalization of Fan's condition for hamiltonicity, pancyclicity and hamiltonian connectedness*, Discrete Math., **115**(1993), 39-50.

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Dirac 定理的局部化与 Hamilton 图

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摘 要

设 G 为一个 n 阶 2-连通图, $n \geq 3$. 若 $|D_{n/2}(K_{1,3})| \geq 2$ 且满足下述条件之一:

i) $|D_{n/2}(K_{1,3}+e)| \geq 2$,

ii) 若 $K_{1,3}+e \to G$, $xy \bar{\in} E(K_{1,3}+e)$, 则 $\max\{d_G(x), d_G(y)\} \geq n/2$,

则 G 是一个 Hamiltonian 图或其闭包为 $sP \oplus H$, 这里 $sP \oplus H$ 是一类极小 2-边连通图.