Journal of Mathematical Research & Exposition
Vol. 18, No. 3, 326 - 328, August 1998

Note on the Paper "Representation of Set Valued Operators" *

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Abstract The aim of this paper is to show that one main theorem in [1] is not correct by a counterexample and to give its correction.

Keywords set valued operator, support function, measurability.

Classification AMS(1991) 28A45, 46G10/CCL O177.2

Integral representation theory turns out to be the appropriate analytical tool in several applied fields like optimization, optimal control, mathematical economics, and has been studied by many authors for various functions and operators (see Papageorgiou[1], Diestel and Uhl[2], etc). Among these, the representation of set valued operators was investigated by Papageorgiou in [1]. One of main theorems is (Theorem 4.1 in [1]):

Theorem A *If $\Phi:[0,T]\to P_fc(X)$ is measurable and for all $x^*\in X^*$ and $t_0,t_1\in[0,T]$ we have*

$$|\sigma_{\Phi(t_1)}(x^*)-\sigma_{\Phi(t_0)}(x^*)|\le\|x^*\|\cdot|t_1-t_0|,$$

then there exists $F:[0,T]\to P_fc(X)$ scalarly integrable s. t.

$$\Phi(t_1)=\Phi(t_0)+cl\int_{t_0}^{t_1}F(t)\,dt,\ for\ all\ t_0,t_1\in[0,T].$$

The following example shows that the above Theorem A does not hold. Recall, first, some definitions and symbols. Let (Ω,Σ,μ) be a complete finite measure space and X a separable reflexive Banach space with dual space X^*.

Let

$$\sigma_A(x^*)=\sup_{x\in A}\langle x^*,x\rangle,$$

* Received October 15, 1995. Supported by N. S. F. of Hebei Province.

the support function of A, and

$$P_f(c) = \{A \subset X : \text{nonempty, closed (convex)}\}.$$

Let us give the example:

Example Let $X = R$, $\Phi(t) = [t, T]$. It is easy to see that $\Phi(t)$ is measurable and closed convex valued. For $f \in X^* = R$, we have

$$\sigma_{\Phi(t)}(f) = \begin{cases} Tf, & f \geq 0, \\ tf, & f < 0 \end{cases}$$

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Therefore, for all $f \in X^*$, we have

$$|\sigma_{\Phi(t_0)}(f) - \sigma_{\Phi(t_1)}(f)| \leq |t_1 - t_0| \, |f|,$$

that is, $\Phi(t)$ satisfies the whole conditions of Theorem A. But for all $t_1 > t_0$, $t_1, t_0 \in [0, T]$, there is no set $A \subset R$ such that

$$\Phi(t_1) = \Phi(t_0) + A,$$

it follows that the conclusion of Theorem A is not correct.

We give the following Theorem B as the correction of Theorem A.

Theorem B *If $\Phi : [0, T] \to P_{fc}(X)$ is measurable and satisfies:*

i) for all $t_1 > t_0$, $t_1, t_0 \in [0, T]$, there exists $A \subset X$ such that $\Phi(t_1) = \Phi(t_0) + A$,

ii) for all $x^* \in X^*$ and $t_0, t_1 \in [0, T]$ we have

$$|\sigma_{\Phi(t_1)}(x^*) - \sigma_{\Phi(t_0)}(x^*)| \leq \|x^*\| \cdot |t_1 - t_0|,$$

then there exists $F : [0, T] \to P_{fc}(X)$ scalarly integrable s. t.

$$\Phi(t_1) = \Phi(t_0) + cl \int_{t_0}^{t_1} F(t)\, dt, \text{ for all } t_0, t_1 \in [0, T].$$

Proof In the proof of Theorem A in [1], the author infers that $x^* \to \phi(x^*, t)$ is sublinear for $t \in [0, T] \setminus N_{x^*}$, where $\lambda(N_{x^*}) = 0$, from $m_{x^*}(A) = \int_A \phi(x^*, t)\, dt$. But it does not hold. Now if Φ satisfies the condition i), we can conclude that $x^* \to \phi(x^*, t)$ is sublinear for $t \in [0, T] \setminus N_{x^*}$, where $\lambda(N_{x^*}) = 0$. By a lifting argument as in the proof of Theorem 3.1 in [1], we can get that $x^* \to \Phi(x^*, t) = \rho[\phi(x^*, t)]$ is sublinear for all $t \in [0, T]$. Also it is continuous. So applying Hormander's theorem we can find $F : \Omega \to P_{fc}(X)$ s.t. $\Phi(x^*, t) = \sigma_{F(t)}(x^*)$. So $F(\cdot)$ is scalarly integrable. Hence we have

$$\sigma_{\Phi(t_1)}(x^*) = \sigma_{\Phi(t_0)}(x^*) + \int_{t_0}^{t_1} \sigma_{F(t)}(x^*)\, dt,$$

$$\sigma_{\Phi(t_1)}(x^*) = \sigma_{\Phi(t_0)}(x^*) + \sigma_{\int_{t_0}^{t_1} F(t)\, dt}(X^*).$$

Since this is true for all $x^* \in X^*$, we conclude that

$$\Phi(t_1) = \Phi(t_0) + cl \int_0^1 F(t)\, dt, \text{ for all } t_0, t_1 \in [0, T].$$

The above differentiation result for multifunctions extends the single valued result of Gelfand and the earlier multivalued results of Artstein[3] and Hermes[4].

References

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关于“集值算子的表示”一文的注记

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摘　要

本文通过一个反例说明[1]中一个主要定理是错误的,并给出其修正结果.