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On Generalized Inverse Vector Valued Continued Fraction Interpolation Splines *

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Abstract The definition of vector valued continued fraction interpolating splines is at first introduced by means of generalized inverse of a vector. In the computation of the interpolating splines, which are of representation of the convergences for Thiele-type continued fraction,the three recurence relation is avioded and a new,effective recursive algorithm is constructed. A sufficient condition for existence is given. Some interpolation resultsincluing uniqueness are given. In the end, a exact interpolation remainder formula is obtained.

Keywords vectors, continued fractions, rational splines.

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1. Introduction

Generalized inverse vector valued rational interpolants have been found wide application in the modal analysis of vibrating structures [1] and the solution of integral equations[2]. Wynn[3] raised the question of rational interpolation of vectors.Graves-Morris[4,5] showed explicitly that the generalized inverse could be used to define vector valued Thiele-type rational interpolants.He put forward that vector valued rational interpolants should satisfy the following principles:

(i) If the kth components of the vector $\vec{v}(x_i)$ are the only non-zero components, then the vector valued interpolant reduces to the corresponding rational fraction interpolant.

(ii) The value of the vector rational interpolant does not depend on the order in which the interpolation points are used to construct the interpolant.

(iii) There is some sence in which a specified rational interpolant is unique.

(iv) The poles of the d components of the interpolant normally occur at common positions in x-plane.

Gu Chuanqing and Zhu Gongqin[6,7,8] applied these principles to multivariate vector valued rational interpolation and approximation.

In this paper, we shall establish above principles for vector valued rational interpolating splines. The method and results in this paper are useful to some numerical analysis

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problems and application problems.

2. Definition and Algorithm

Let distinct real points

$$a = x_1 < x_2 < \cdots < x_n = b$$

and corresponding finite vectors

$$\vec{v}_j, \vec{v}'_j, \cdots, \vec{v}_j^{(k)}$$

which will be interpolated, where $\vec{v}_j^{(i)} = \vec{v}^{(i)}(x_j) \in C^d$, $i = 0, 1, \cdots, k$, $j = 1, 2, \cdots, n$.

Consider the following rational spline interpolation problem:

Find vector valued rational functions

$$\vec{R}(x) = \vec{R}_j(x) = \frac{\vec{N}(x)}{D(x)}, \quad x \in [x_j, x_{j+1}] = I$$

such that $\vec{N}(x) = (N_1(x), N_2(x), \cdots, N_d(x))$ is a vector polynomial and D(x) is a real scalar polynomial and satisfy the following conditions:

$$\vec{R}_j^{(i)}(x_j) = \vec{v}_j^{(i)}, \ \vec{R}_j^{(i)}(x_{j+1}) = \vec{v}_{j+1}^{(i)}, \quad i = 0, 1, \cdots, k, \ j = 1, 2, \cdots, n; \tag{1}$$

$$D(x) \mid |\vec{N}(x)|^2, \tag{2}$$

where "|" is a entire division sign;

$$\deg \vec{N}(x) = \max(\deg N_1(x), \cdots, \deg N_d(x)) \leq M, \deg D(x) = m, \tag{3}$$

where $M = 2k + 1$.

As in [4], the generalized inverse of a vector is defined by

$$\vec{v}^{-1} = \frac{1}{\vec{v}} = \frac{\vec{v}^*}{|\vec{v}|^2}. \tag{4}$$

The convergences of Thiele-type osculatory continued fraction are expressed as

$$\vec{R}_j(x) = \vec{b}_{j,0} + \frac{x - x_j}{\vec{b}_{j,1}} {}_{+\cdots+} \frac{x - x_j}{\vec{b}_{j,k}} {}_{+} \frac{x - x_j}{\vec{b}_{j+1,0}} {}_{+} \frac{x - x_{j+1}}{\vec{b}_{j+1,1}} {}_{+\cdots+} \frac{x - x_{j+1}}{\vec{b}_{j+1,k}}. \tag{5}$$

By means of the generalized inverse (4),the coefficient algorithm of (5) is introduced as follows:

1th. Compute $\vec{b}_{j,i}$, $i = 0, 1, \cdots, k$.

$$\vec{g}_{j,0}(x) = \vec{v}(x), \ \vec{b}_{j,0} = \vec{g}_{j,0}(x_j);$$

$$\vec{g}_{j,1}(x) = (\frac{d}{dx}\vec{g}_{j,0}(x))^{-1}, \quad \vec{b}_{j,1} = \vec{g}_{j,1}(x_j);$$

$$\vec{h}_{j.i}(x) = i(\frac{d}{dx}\vec{g}_{j.i-1}(x))^{-1}, \quad \vec{b}_{j.i} = \vec{h}_{j.i}(x_j); i \geq 2,$$

$$\vec{g}_{j.i}(x) = \vec{h}_{j.i}(x) + \vec{g}_{j.i-2}(x).$$

2th. Compute $\vec{b}_{j+1.i}$, $i = 0, 1, \cdots, k$.

$$\vec{r}_{j.i}(x) = \frac{1}{(-\vec{b}_{j.i})} {}_{+} \frac{x - x_j}{(-\vec{b}_{j.i-1})} {}_{+ \cdots +} \frac{x - x_j}{(-\vec{b}_{j.1})} {}_{+} \frac{x - x_j}{(\vec{v}(x) - \vec{b}_{j.0})}, i = 1, 2, \cdots, k,$$

$$\vec{r}_{j.i+1}(x) = (-\vec{b}_{j.i+1} + (x - x_j)\vec{r}_{j.i}(x))^{-1}, \quad i = 1, 2, \cdots, k-1,$$

$$\vec{g}_{j+1.0}(x) = (x - x_j)\vec{r}_{j.k}(x), \quad \vec{b}_{j+1.0} = \vec{g}_{j+1.0}(x_{j+1});$$

$$\vec{g}_{j+1.1}(x) = (\frac{d}{dx}\vec{g}_{j+1.0}(x))^{-1}, \quad \vec{b}_{j+1.1} = \vec{g}_{j+1.1}(x_{j+1});$$

$$\vec{h}_{j+1.i}(x) = i(\frac{d}{dx}\vec{g}_{j+1.i}(x))^{-1}, \quad \vec{b}_{j+1.i} = \vec{h}_{j+1.i}(x_{j+1}); \quad i \geq 2,$$

$$\vec{g}_{j+1.i}(x) = \vec{h}_{j+1.i}(x) + \vec{g}_{j+1.i-2}(x).$$

As the same as the scalar case [9], the continued fraction (5) has the property that the determination of $\vec{b}_{l.i}$ is independent of all $b's$ that follow.

In (5), let

$$\vec{G}_{s.t}(x) = \vec{b}_{j.0} + \frac{x - x_j}{\vec{b}_{j.1}} {}_{+ \cdots +} \frac{x - x_s}{\vec{b}_{s.t}}, \tag{6}$$

$$\vec{H}_{s.t}(x) = \vec{b}_{s.t+1} + \frac{x - x_s}{\vec{b}_{s.t+2}} {}_{+ \cdots +} \frac{x - x_{j+1}}{\vec{b}_{j+1.k}}, s = j, t = 0, 1, \cdots, k-1, \tag{7}$$

where

$$\vec{H}_{j.k}(x) = \vec{b}_{j+1.0} + \frac{x - x_{j+1}}{\vec{b}_{j+1.1}} {}_{+ \cdots +} \frac{x - x_{j+1}}{\vec{b}_{j+1.k}}. \tag{8}$$

Theorem 2.1 *Let all $\vec{b}_{l.i}$ in $\vec{R}_j(x)$ as in (5) exist. If the following conditions are satisfied:*

(i) For $i = 1, 2, \cdots, k$, $\vec{b}_{j.i} \neq \vec{0}$, for $i = 0, 1, \cdots, k$, $\vec{b}_{j+1.i} \neq \vec{0}$;

(ii) $\vec{H}_{j.k}(x_j) \neq \vec{0}$,

then $\vec{R}_j(x)$ as in (5)exist such that the interpolating condition (1) hold.

Proof As $\vec{H}_{j.0}(x_j) = \vec{b}_{j.1} \neq \vec{0}$, $\vec{H}_{j.1}(x_j) = \vec{b}_{j.2} \neq \vec{0}$, thus

$$\vec{R}_j(x) = \vec{b}_{j.0} + \frac{x - x_j}{\vec{b}_{j.1}} {}_{+} \frac{x - x_j}{\vec{H}_{j.1}(x)},$$

$$\vec{G}_{j.1}(x) = \vec{b}_{j.0} + \frac{x - x_j}{\vec{b}_{j.1}}$$

exist respectively and

$$\vec{R}_j(x_j) = \vec{G}_{j.1}(x_j) = \vec{v}_j, \vec{R}'_j(x_j) = \vec{G}'_{j.1}(x_j) = 1/\vec{b}_{j.1} = \vec{v}'_j.$$

As

$$\vec{H}_{j,0}(x_j) = \vec{b}_{j,1} \neq \vec{0}, \cdots, \vec{H}_{j,k-1}(x_j) = \vec{b}_{j,k} \neq \vec{0}$$

and

$$\vec{H}_{j,k}(x_j) \neq \vec{0},$$

thus

$$\vec{R}_j(x) = \vec{b}_{j,0} + \frac{x - x_j}{\vec{b}_{j,1}}{}_{+\cdots+}\frac{x - x_j}{\vec{b}_{j,k}}{}_{+}\frac{x - x_j}{\vec{H}_{j,k}(x)}$$

and

$$\vec{G}_{j,k}(x) = \vec{b}_{j,0} + \frac{x - x_j}{\vec{b}_{j,1}}{}_{+\cdots+}\frac{x - x_j}{\vec{b}_{j,k}}$$

exist respectively. By Thiele's formula[10],have

$$\vec{R}_j^{(i)}(x_j) = \vec{G}_{j,k}^{(i)} = \vec{v}_j^{(i)}, \quad i = 0, 1, \cdots, k.$$

Substituting $\vec{b}_{j+1,0} = (x_{j+1} - x_j)\vec{r}_{j,k}(x_{j+1})$ into $\vec{G}_{j+1,0}(x)$ and noting $\vec{H}_{j+1,0}(x_{j+1}) = \vec{b}_{j+1,1} \neq \vec{0}$, it is not difficult to find that

$$\begin{aligned}\vec{R}_j(x_{j+1}) &= \vec{G}_{j+1,0}(x_{j+1}) = \vec{b}_{j,0} + \frac{x_{j+1} - x_j}{\vec{b}_{j,1}}{}_{+\cdots+}\frac{x_{j+1} - x_j}{\vec{b}_{j,k}}{}_{+}\frac{x_{j+1} - x_j}{(x_{j+1} - x_j)\vec{r}_{j,k}(x_{j+1})} \\ &= \vec{b}_{j,0} + \frac{x_{j+1} - x_j}{(x_{j+1} - x_j)(\vec{v}(x_{j+1}) - \vec{b}_{j,0})^{-1}} = \vec{v}_{j+1}. \end{aligned} \tag{9}$$

By means of (9), using Thiele's formula [10] again for

$$\vec{G}_{j+1,k}(x) = \vec{b}_{j,0} + \frac{x - x_j}{\vec{b}_{j,1}}{}_{+\cdots+}\frac{x - x_{j+1}}{\vec{b}_{j+1,k}},$$

it follows that

$$\vec{R}_j^{(i)}(x_{j+1}) = \vec{G}_{j+1,k}^{(i)}(x_{j+1}) = \vec{v}_{j+1}^{(i)}, \quad i = 0, 1, \cdots, k.$$

Example 2.1 Let $\vec{v}(x) = (x, x^2, x^3)$, $x_1 = 0, x_2 = 1, k = 1$.

$$\vec{g}_{1,0}(x) = \vec{v}(x), \quad \vec{b}_{1,0} = (0,0,0);$$

$$\vec{g}_{1,1}(x) = (1, 2x, 3x^2)/(1 + 4x^2 + 9x^4), \quad \vec{b}_{1,1} = (1,0,0);$$

$$\vec{r}_{1,1}(x) = (-x(1 + x^2), 1, x)/x(1 + x^2),$$

$$\vec{g}_{2,0}(x) = x\vec{r}_{1,1}(x), \quad \vec{b}_{2,0} = (1, -1/2, -1/2);$$

$$\vec{g}_{2,1}(x) = (-(1 + x^2)^2, -2x, 1 - x^2)/(2 + 2x^2 + x^4), \quad \vec{b}_{2,1} = (-4/5, -2/5, 0).$$

$$\vec{R}_1(x) = (0,0,0) + \frac{x}{(1,0,0)}{}_{+}\frac{x}{(-1, 1/2, 1/2)}{}_{+}\frac{x - 1}{(-4/5, -2/5, 0)}$$

hold

$$\vec{R}_1^{(i)}(x_1) = \vec{v}_1^{(i)},\ \vec{R}_1^{(i)}(x_2) = \vec{v}_2^{(i)},\ i = 0,1.$$

Obviously,the condition of Theorem 2.1 is satisfied,

$$\vec{H}_{1.1}(x_1) = (2,0,-1/2) \neq \vec{0}.$$

Example 2.2 Let $\vec{v}(x) = (e^x, x, 1), x_1 = 0, x_2 = 1, k = 1.$

$$\vec{g}_{1.0}(x) = \vec{v}(x),\quad \vec{b}_{1.0} = (1,0,1);$$

$$\vec{g}_{1.1}(x) = (e^x,1,0)/(1+e^{2x}),\quad \vec{b}_{1.1} = (1/2,1/2,0);$$

$$\vec{r}_{1.1}(x) = (1+x+e^x, 1-x-e^x, 0)/(e^x - x - 1),$$

$$\vec{g}_{2.0}(x) = x\vec{r}_{1.1}(x),\quad \vec{b}_{2.0} = (-1,-e/(2-e),0);$$

$$\vec{g}_{2.1}(x) = (-1,f(x),0)/(1+f(x)),\quad \vec{b}_{2.1} = (-1,f(x_1),0)/(1+f^2(x_1));$$

where

$$f(x) = x(e^x + x + 1)/(e^x - x - 1)^2,\quad f(x_1) = (e+2)/(e-2)^2.$$

$$\vec{R}_1(x) = (1,0,1) + \frac{x}{(1/2,1/2,0)} {}_{+} \frac{x}{(-1,-e/(2-e),0)} {}_{+} \frac{x-1}{(-1,f(x_1),0)/(1+f^2(x_1))}$$

hold

$$\vec{R}_1^{(i)}(x_1) = \vec{v}_1^{(i)}, \vec{R}_1^{(i)}(x_2) = \vec{v}_2^{(i)}, i = 0,1.$$

3. Uniqueness

Lemma 3.1 ([4,p.335]) *Let all vectors $\vec{b}_{l.j} \in C^d$ and $x_i \in R$. Define $\vec{R}_j(x)$ as in (5) by a tail-to-head rationalization using the generalized inverse(4),then a vector polynomial $\vec{N}(x)$ and a real polynomial $D(x)$ exist such that*

(i) $\vec{R}_j(x) = \vec{N}(x)/D(x)$;

(ii) $D(x) \mid |\vec{N}(x)|^2$.

Definition 3.1 *A vector valued rational fractions $\vec{R}_j(x) = \vec{N}(x)/D(x)$ is said to be of $[l/m]$ if $\deg\{\vec{N}(x)\} \leq l$ and $\deg\{D(x)\} = m$.*

Lemma 3.2 ([4,p.336]) *Let $\vec{R}_j(x) = \vec{N}(x)/D(x)$ as in (5), $M = 2k+1$. Then $\vec{R}_j(x)$ is of $[M/(M-1)]$.*

Definition 3.2 *A vector valued rational fractions $\vec{R}_j(x) = \vec{N}(x)/D(x)$ as in (5) is defined as a k order generalized inverse continued fraction interpolating spline (GICFS) in $[x_j, x_{j+1}]$ if $D(x)$ is real, $D(x) \mid |\vec{N}(x)|^2$, $\vec{R}_j(x)$ is of $[M/(M-1)]$ and the interpolation condition (1) hold.*

Lemma 3.3[11] *For $D(x_j) \neq 0$, $k = 0,1,\cdots,l$, $j = 1,2,\cdots,n$,*

$$\frac{d^k}{dx^k}\left[\frac{\vec{N}(x_j)}{D(x_j)}\right] = \vec{v}_j^{(k)}$$

is equivalent to

$$\vec{N}^{(k)}(x_j) = [\vec{v}(x_j)D(x_j)]^{(k)}.$$

Theorem 3.4 (Uniqueness) *Let any two GICFS* $\vec{R}_j(x) = \vec{N}_j(x)/D_j(x)$ *and* $\vec{R}_l(x) = \vec{N}_l(x)/D_l(x)$ *satisfy:*

(i) $\vec{R}_j^{(i)}(x_j) = \vec{R}_l^{(i)}(x_j) = \vec{v}_j^{(i)}, \quad \vec{R}_j^{(i)}(x_{j+1}) = \vec{R}_l^{(i)}(x_{j+1}) = \vec{v}_{j+1}^{(i)}, \quad i = 0, 1, \cdots, k,$

(ii) $\vec{R}_j(x)$ *and* $\vec{R}_l(x)$ *are of the same type (Lemma 3.2),*

then

$$\vec{N}_j(x)D_l(x) \equiv \vec{N}_l(x)D_j(x).$$

Proof Suppose

$$\deg\{\vec{N}_j\} \le M, \deg\{D_j\} = M-1, D_j \mid |\vec{N}_j|^2, \tag{10}$$

$$\deg\{\vec{N}_l\} \le M, \deg\{D_l\} = M-1, D_l \mid |\vec{N}_l|^2. \tag{11}$$

Let $t(x)$ be the greatest common factor of $D_j(x)$and $D_l(x)$ and define $d_j(x), d_l(x)$ and a vector polynomial $\vec{T}(x)$,respectively,as follows

$$D_j(x) = t(x)d_j(x), D_l(x) = t(x)d_l(x), \tag{12}$$

$$\vec{T}(x) = \vec{N}_j(x)d_l(x) - \vec{N}_l(x)d_j(x). \tag{13}$$

By the condition (i),it is known that $D_j(x_s) \neq 0, D_l(x_s) \neq 0, s = j, j+1$. Appling Leibniz theorem and Lemma 3.3 to (13),it is derived that

$$\begin{aligned}
\vec{T}^{(i)}(x_s) &= \sum_{t=0}^{i} C_i^t[\vec{N}_j^{(t)}(x_s)d_l^{(i-t)}(x_s) - \vec{N}_l^{(t)}(x_s)d_j^{(i-t)}(x_s)] \\
&= \sum_{t=0}^{i} C_i^t[(D_j(x_s)\vec{v}(x_s))^{(t)}d_l^{(i-t)}(x_s) - (D_l(x_s)\vec{v}(x_s))^{(t)}d_j^{(i-t)}(x_s)] \\
&= [(D_j(x_s)d_l(x_s) - D_l(x_s)d_j(x_s))\vec{v}(x_s)]^{(i)} = \vec{0}, \\
&\qquad s = j, j+1, \quad i = 0, 1, \cdots, k.
\end{aligned} \tag{14}$$

In terms of (14),let

$$W(x) = (x - x_j)^{k+1}(x - x_{j+1})^{k+1}, \tag{15}$$

where $\deg\{W\} = 2k + 2 = M + 1$.

Define again

$$\vec{T}(x) = W(x)\vec{S}(x).$$

By means of (10),(11),(13) and

$$|\vec{T}|^2 = |\vec{N}_j|^2 d_l^2 + |\vec{N}_l|^2 d_j^2 - 2d_j d_l Re(\vec{N}_j \cdot \vec{N}_l^*),$$

it is derived that

$$d_j \mid |\vec{T}|^2, d_l \mid |\vec{T}|^2. \tag{16}$$

Using the interpolating condition (i) in (16),get

$$d_j \mid |\vec{S}|^2, d_l \mid |\vec{S}|^2. \tag{17}$$

Unless $\vec{S}(x) \equiv \vec{0}$, it follows from (17) that

$$\deg\{|\vec{S}|^2\} \geq \deg\{d_j\} + \deg\{d_j\} = 2(M - \deg\{t\} - 1). \tag{18}$$

But from (10),(11),(13) and (15), it is got again that

$$\deg\{|\vec{S}|^2\} \leq 2(M - 2 - \deg\{t\}). \tag{19}$$

Obviously,(18) and (19) show a contradiction,hence,$\vec{S}(x) \equiv \vec{0}$.

4. Remainder Formula

Theorem 4.1 *Let $\vec{R}_j(x) = \frac{\vec{N}(x)}{D(x)}$ is a GICFS, $D(x) \neq 0, x \in I$. If $\vec{v}(x)$ is of $M+1$ order continuous derivative in $[x_j, x_{j+1}] = I$,then for any $x \in I$,hold*

$$\vec{v}(x) - \vec{R}_j(x) = \frac{W(x)}{(M+1)!}\frac{d^{M+1}}{dx^{M+1}}(D(x)\vec{v}(x))|_{x=\xi},$$

where $\xi \in (x_j, x_{j+1}), M = 2k+1$.

Proof Define two vector functions,respectively,as follows

$$\vec{Z}(t) = \vec{v}(t) - \frac{\vec{N}(t)}{D(t)} - \frac{\vec{Q}(x)W(t)}{D(t)}, \tag{20}$$

$$\vec{F}(t) = D(t)\vec{v}(t) - \vec{N}(t) - \vec{Q}(x)W(t). \tag{21}$$

Note $D(t) \neq 0$ for $t \in [x_j, x_{j+1}]$. In terms of the construction of (20), it is easily obtained that

$$\vec{Z}(x) = \vec{0}, \vec{Z}^{(i)}(x_s) = \vec{0}, i = 0, 1, \cdots, k, s = j, j+1.$$

Appling Lemma 3.3 to (20), it is got again that

$$\vec{F}(x) = \vec{0}, \vec{F}^{(i)}(x_s) = \vec{0}, i = 0, 1, \cdots, k, s = j, j+1. \tag{22}$$

In (22), total zero number is equal to $(M+1)+1 = M+2$. Thus,by means of the Rollm theorem of vector functions,it follow from (21) that there exists $\xi \in (x_j, x_{j+1})$ such that

$$\vec{F}^{(M+1)}(\xi) = \vec{0}.$$

According to Lemma 3.2, $\deg\{\vec{N}\} \leq \vec{0}$, it is found that

$$\frac{d^{M+1}}{dt^{M+1}}(\vec{N}(t)) = \vec{0}. \tag{23}$$

It is obviously known from (15) that $\deg\{W^{(M+1)}\} = (M+1)!$. Substituting (23) and $\deg\{W^{(M+1)}\} = (M+1)!$ into (21), it is got from (21) that

$$\vec{Q}(x) = \frac{1}{(M+1)!}\frac{d^{M+1}}{dx^{M+1}}(D(x)\vec{v}(x))|_{x=\xi}.$$

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关于广义逆的向量连分式插值样条

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摘 要

本文首次引入了关于广义逆的向量有理插值样条的概念．这类插值样条具有 Thiele 型连分式的截断分式的表现形式．在它的构造过程中，不必用到连分式的三项递推关系，本文得到的新的有效的系数算法具有递推运算的特点．存在性的一个充分条件得以建立．包括唯一性在内的有关插值问题的某些结果得到证明．最后，本文给出了一个精确的插值误差公式．