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Minimal Surfaces in S^6 with Constant Kähler Angles and Constant Curvature*

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Abstract: In the present paper, we first give some examples of minimal surfaces in the nearly Kähler sphere S^6 with constant Kähler angles and constant curvatures, and then prove two uniqueness theorems.

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1. Introduction

Let S^6 be the unit sphere in $\mathbf{R}^7$. Then there is an almost complex structure $\mathbf{J}$ on S^6 which makes S^6 into a nearly Kähler manifold in the sense that, for any vector field X on S^6,

$$(\nabla_X \mathbf{J})(X) = 0, \tag{1.1}$$

where ∇ denotes the Levi-Civita covariant differentiation related to the standard metric on S^6. In the past years, much progress has been made in the study on minimal surfaces in S^6[1-5]. As a natural extension, we obtained in [6] the following rigidity theorem:

Theorem 1.1[6] *Let M be a complete and minimal immersed surface in S^6 with constant Kähler angle. If the Gauss curvature $K \geq 0$, then either $K \equiv 0$ or $K \equiv 1$.*

Thus it is worthy to find all minimal surfaces in S^6 with constant Kähler angles and constant Gauss curvatures $K = 1$ or $K = 0$. Note that no minimal surfaces exist in S^n (for any n) with constant curvature $K < 0$[7]. In this paper, after some preliminary lemmas, we will give in Section 3 examples of superminimal surfaces in S^6 with constant Kähler angles and constant curvatures $K = 1$ and $K = 0$. Furthermore, we are able to prove theorems showing that these examples are in a sense unique ones, see Theorems 4.2 and 4.3 for detail.

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2 Preliminary Lemmas

Through out this paper, we use the same notations and conventions as in [6]. For example, the range of indices will be as follows:

$$1 \le A, B, C, \cdots \le 6, \quad 1 \le i, j, k, \cdots \le 2,$$
$$3 \le a, b, c, \cdots \le 4, \quad 5 \le \alpha, \beta, \gamma, \cdots \le 6.$$

The multiplication of the Cayley numbers defines one cross-product "$\times$" on $\mathbf{R}^7$ as follow:

$$x \times y = \frac{1}{2}(xy - yx). \tag{2.1}$$

The standard inner product on $\mathbf{R}^7$ can be reformulated as

$$(x, y) = -\frac{1}{2}(xy + yx).$$

It can be shown that [8], the operation $\times$ in (2.1) satisfies the following identity:

$$x \times (y \times z) + (x \times y) \times z = 2(x, z)y - (x, y)z - (y, z)x. \tag{2.2}$$

Furthermore, formula

$$\mathbf{J}_x(X) = x \times X, \quad x \in S^6, \quad X \in T_xS^6 \tag{2.3}$$

defines an almost complex structure $\mathbf{J}$ on S^6, which is also nearly Kähler in the sense of (1.1).

Define a subgroup G_2 of the orthogonal group $O(7)$ by

$$G_2 = \{g \in O(7); g(x \times y) = g(x) \times g(y)\}, \text{ for all } x, y \in \mathbf{R}^7. \tag{2.4}$$

Then G_2 is nothing but the group of ismetries on S^6 preserving the nearly Kähler structure $\mathbf{J}$ [4], [5].

Now let M be an oriented metric surface, that is, an oriented Riemannian manifold of dimension 2, and x be a minimal immersion of M into S^6. For any orthonormal frame $\{e_1, e_2\}$ on M, the Kähler angle of x is the angle θ between $\mathbf{J}(dx(e_1))$ and $dx(e_2)$ on S^6, satisfying $0 \le \theta \le \pi$. [6] In what follows, we write e_i also for $dx(e_i)$ to simplify matters.

Starting from any orthonormal frame field $\{e_1, e_2\}$ on M, we can construct along x an orthonormal frame field $E = \{e_i, e_a, e_\alpha\}$ on S^6, such that the following are satisfied in case $\theta \ne 0, \pi$ [6]:

$$\begin{cases} \mathbf{J}(e_1) = e_2\cos\theta + e_3\sin\theta, \quad \mathbf{J}(e_2) = -e_1\cos\theta + e_4\sin\theta, \\ \mathbf{J}(e_3) = -e_1\sin\theta - e_4\cos\theta, \quad \mathbf{J}(e_4) = -e_2\sin\theta + e_3\cos\theta, \\ (\nabla_{e_1}\mathbf{J})(e_2) = 3D\, e_5\sin\theta, \quad \mathbf{J}(e_5) = e_6, \end{cases} \tag{2.5}$$

where e_α's are free of the choice of e_i and globally defined, while e_a's depend on e_i. Such a frame field E, or the corresponding $\{x; e_i, e_a, e_\alpha\}$, of S^6 along x will be called "*special*" in the sequel, where x denotes the position vector in $\mathbf{R}^7$.

Let $\{\omega_A\}$ be the dual frame field of $\{e_A\}$, and ω_{AB} the components of the Levi-Civita connection of S^6 with respect to the special frame field E along x. Put

$$\omega_{ia} = \sum h_{ij}^{a}\omega_j, \quad \omega_{i\alpha} = \sum h_{ij}^{\alpha}\omega_j,$$

then one can prove:

Lemma 2.1[6] *If* $\sin\theta \neq 0$, *then the following identities hold for the immersion* x:

$$\begin{aligned} &\omega_{16} + \omega_{25}\cos\theta + \omega_{35}\sin\theta = -\omega_2\sin\theta, \\ &\omega_{26} - \omega_{15}\cos\theta + \omega_{45}\sin\theta = \omega_1\sin\theta, \end{aligned} \tag{2.6}$$

$$\omega_{15} = \omega_{26}\cos\theta + \omega_{36}\sin\theta, \omega_{25} = -\omega_{16}\cos\theta + \omega_{46}\sin\theta, \tag{2.7}$$

$$d\theta = \omega_{14} - \omega_{23}, (\omega_{13} + \omega_{24})\cos\theta + (\omega_{34} - \omega_{12})\sin\theta = 0. \tag{2.8}$$

Lemma 2.2[6] *If* x *is minimal with constant Kähler angle* $\theta \neq 0, \pi$, *then along* x, *the following identities hold*:

$$\omega_{34} = \omega_{12}, \quad \omega_{13} + \omega_{24} = 0, \quad \omega_{36} = 0, \tag{2.9}$$

$$[(h_{11}^5)^2 + (h_{12}^5) + (h_{11}^6)^2 + (h_{12}^6)^2]\cos\theta = 2(h_{11}^5h_{12}^6 - h_{12}^5h_{11}^6). \tag{2.10}$$

Using (2.2), (2.3) and (2.5), it is easy to prove another lemma:

Lemma 2.3 *For any immersion* x *of* M *into* S^6, *if the Kähler angle* $\theta \neq 0, \pi$, *then the following equalities hold*:

$$e_5 \times e_6 = x, e_3 \times e_4 = \csc\theta x \times (e_1 \times e_4) - \csc\theta\cot\theta x + \cot^2\theta e_1 \times e_2, \tag{2.11}$$

$$\begin{aligned} &e_3 \times e_5 = -\csc\theta x \times (e_1 \times e_5) - \cot\theta e_2 \times e_5, \\ &e_4 \times e_5 = -\csc\theta x \times (e_2 \times e_5) + \cot\theta e_1 \times e_5, \end{aligned} \tag{2.12}$$

$$\begin{aligned} &e_3 \times e_6 = -\sin\theta e_1 \times e_5 - \cos\theta e_4 \times e_5, \\ &e_4 \times e_6 = -\sin\theta e_2 \times e_5 + \cos\theta e_3 \times e_5, \end{aligned} \tag{2.13}$$

where $\csc\theta = \dfrac{1}{\sin\theta}$

3 Examples

Now we give two examples of minimal surfaces in S^6, which are of constant curvatures and constant Kähler angles

Example 3.1 Let $S^2 = \{(x_1, x_2, x_3) \in \mathbf{R}^3; x_1^2 + x_2^2 + x_3^2 = 1\}$, $\theta \in [0, \pi]$. Define x: $S^2 \to S^6 \subset \mathbf{R}^7$ as

$$x(x_1, x_2, x_3) = (x_1, x_2, x_3\cos\theta, x_3\sin\theta, 0, 0, 0). \tag{3.1}$$

Then for any local isothermal coodinates (u, v) on S^2,

$$\mathbf{J}\frac{\partial x}{\partial u} = \Big(x_2\frac{\partial x_3}{\partial u}\cos\theta - x_3\frac{\partial x_2}{\partial u}\cos\theta, x_3\frac{\partial x_1}{\partial u}\cos\theta - x_1\frac{\partial x_3}{\partial u}\cos\theta,$$

$$x_1\frac{\partial x_2}{\partial u} - x_2\frac{\partial x_1}{\partial u}, 0, x_1\frac{\partial x_4}{\partial u} - x_4\frac{\partial x_1}{\partial u}, 0, 0\Big),$$

$$\Big(\mathbf{J}\frac{\partial x}{\partial u}, \frac{\partial x}{\partial v}\Big) = \Big\langle\cos\theta\Big(x_2\frac{\partial x_3}{\partial u} - x_3\frac{\partial x_2}{\partial u}, x_3\frac{\partial x_1}{\partial u} - x_1\frac{\partial x_3}{\partial u},$$

$$x_1\frac{\partial x_2}{\partial u} - x_2\frac{\partial x_1}{\partial u}\Big), \Big(\frac{\partial x_1}{\partial v}, \frac{\partial x_2}{\partial v}, \frac{\partial x_3}{\partial v}\Big)\Big\rangle.$$

where $\langle\cdot, \cdot\rangle$ denotes the standard inner product in $\mathbf{R}^3$. If we put

$$\lambda = \Big|\frac{\partial x}{\partial u}\Big| = \Big|\frac{\partial x}{\partial v}\Big| \text{ and } e_1 = \frac{1}{\lambda}\frac{\partial x}{\partial u},\ e_2 = \frac{1}{\lambda}\frac{\partial x}{\partial v},$$

then by the fact that S^2 is a unit sphere in $\mathbf{R}^3$, we can write

$$(\mathbf{J}e_1, e_2) = \cos\theta \tag{3.2}$$

Thus, the Kähler angle of x equals identically to θ. Clearly, $K \equiv 1$.

Example 3.2 Let θ be as in Example 3.1, and define (cf. [5])

$$x_\theta(u, v) = \frac{1}{\sqrt{3}}\Big(\cos\big(\tfrac{1}{3}\theta + \sqrt{2}v\big), \cos\big(\tfrac{1}{3}\theta + \sqrt{\tfrac{3}{2}}u - \tfrac{1}{\sqrt{2}}v\big),$$

$$\cos\big(\tfrac{1}{3}\theta - \sqrt{\tfrac{3}{2}}u - \tfrac{1}{\sqrt{2}}v\big), 0, -\sin\big(\tfrac{1}{3}\theta + \sqrt{2}v\big),$$

$$-\sin\big(\tfrac{1}{3}\theta + \sqrt{\tfrac{3}{2}}u - \tfrac{1}{\sqrt{2}}v\big), -\sin\big(\tfrac{1}{3}\theta - \sqrt{\tfrac{3}{2}}u - \tfrac{1}{\sqrt{2}}v\big)\Big),$$

$$(u, v) \in \mathbf{R}^2. \tag{3.3}$$

It is easily verified that x_θ is a minimal immersion of the flat $\mathbf{R}^2$ into S^6, i.e., $K \equiv 0$. Let

$$e_1 = \frac{\partial x_\theta}{\partial u},\quad e_2 = \frac{\partial x_\theta}{\partial v}.$$

By further calculations, we can find the special frame field $\{e_1, e_2, e_3, e_4, e_5, e_6\}$ of x_θ. In particular we have

$$\mathbf{J}e_1 = \Big(\sqrt{\tfrac{2}{3}}\sin\big(\tfrac{2}{3}\theta - \sqrt{2}v\big), -\tfrac{1}{\sqrt{6}}\sin\big(\tfrac{2}{3}\theta - \tfrac{3}{\sqrt{6}}u + \tfrac{1}{\sqrt{2}}v\big),$$

$$-\tfrac{1}{\sqrt{6}}\sin\big(\tfrac{2}{3}\theta + \tfrac{3}{\sqrt{6}}u + \tfrac{1}{\sqrt{2}}v\big), 0, -\sqrt{\tfrac{2}{3}}\cos\big(\tfrac{2}{3}\theta - \sqrt{2}v\big),$$

$$\tfrac{1}{\sqrt{6}}\cos\big(\tfrac{2}{3}\theta - \tfrac{3}{\sqrt{6}}u + \tfrac{1}{\sqrt{2}}v\big), \tfrac{1}{\sqrt{6}}\cos\big(\tfrac{2}{3}\theta + \tfrac{3}{\sqrt{6}}u + \tfrac{1}{\sqrt{2}}v\big)\Big),$$

$$e_2 = \Big(-\sqrt{\tfrac{2}{3}}\sin\big(\tfrac{1}{3}\theta + \sqrt{2}v\big), \tfrac{1}{\sqrt{6}}\sin\big(\tfrac{1}{3}\theta + \sqrt{\tfrac{3}{2}}u - \tfrac{1}{\sqrt{2}}v\big),$$

$$\tfrac{1}{\sqrt{6}}\sin\big(\tfrac{1}{3}\theta - \sqrt{\tfrac{3}{2}}u - \tfrac{1}{\sqrt{2}}v\big), 0, -\sqrt{\tfrac{2}{3}}\cos\big(\tfrac{1}{3}\theta + \sqrt{2}v\big),$$

$$\tfrac{1}{\sqrt{6}}\cos\big(\tfrac{1}{3}\theta + \sqrt{\tfrac{3}{2}}u - \tfrac{1}{\sqrt{2}}v\big), \tfrac{1}{\sqrt{6}}\cos\big(\tfrac{1}{3}\theta - \sqrt{\tfrac{3}{2}}u - \tfrac{1}{\sqrt{2}}v\big)\Big).$$

Therefore

$$(\mathbf{J}e_1, e_2) = \cos\theta,$$

i. e. , x_θ has constant Kähler angle θ. The components for the second fundamental form of x_θ are as follows:

$$h^3_{11} = - h^4_{12} = - h^4_{21} = - h^3_{22} = \frac{1}{\sqrt{2}},$$

$$h^4_{11} = h^4_{22} = h^3_{12} = h^3_{21} = 0, h^5_{ij} = h^6_{ij} = 0, \quad i, j = 1, 2.$$

4 Uniqueness Theorems

In this section, we prove two uniqueness theorems.

Theorem 4.1 Let M be a connected metric surface, x, $\tilde{x}$ be two minimal immersions of M into S^6 with constant Kähler angle θ, $\tilde{\theta}$ respectively, $0<\theta, \tilde{\theta}<\pi$. Suppose that, with respect to two special frame fields along x and x determined by a same local frame field on M, the connection matrices ω, $\tilde{\omega}$ of $\mathbf{R}^7$ equal to each other. Then x is G_2-congruent to x *if and only if* $\tilde{\theta} = \theta$.

Proof Let $\{x; e_i, e_a, e_\alpha\}$ and its tilde denote the two special frame fields of x, x just mentioned.

If G is a subgroup of $O(7)$, Then, the term of x being *G-congruent to* x is equivalent to $x = g \cdot x$ for some $g \in G$. Since G_2 is the automorphism group of the nearly Kähler sphere S^6, the "only if" part of the proposition is trivial.

For the "if" part, we need a fundamental theorem of Riemann geometry. By this theorem and the assumption, there is some $g \in O(7)$ such that, at any point of M, the special frame fields of x and x are g-related, i. e. ,

$$x = g \cdot x, \tilde{e}_A = g \cdot e_A, 1 \le A \le 6. \tag{4.1}$$

Since the Kähler angles of x and x are a same constant, we see from (2.3), (2.5) and (4.1) that,

$$\mathbf{J}(g(e_A)) = g\mathbf{J}(e_A), \tag{4.2}$$

or, equivalently,

$$x \times \tilde{e}_A = g(x \times e_A). \tag{4.3}$$

Take differentiations of (4.3), we get

$$\sum \omega \tilde{e}_i \times \tilde{e}_A + \sum \omega_{AB} x \times \tilde{e}_B = \sum \omega g(e_i \times e_A) + \sum \omega_{AB} g(x \times e_B).$$

Thus by (4.3), we have

$$g(e_i \times e_A) = \tilde{e}_i \times \tilde{e}_A = g(e_i) \times g(e_A). \tag{4.4}$$

Now, we can use Lemma 2.3 with (4.3) and (4.4) to get

$$\begin{cases} g(e_3\times e_4) = g(e_3)\times g(e_4),\ g(e_5\times e_6) = g(e_5)\times g(e_6),\\ g(e_3\times e_5) = g(e_3)\times g(e_5),\ g(e_4\times e_5) = g(e_4)\times g(e_5),\\ g(e_3\times e_6) = g(e_3)\times g(e_6),\ g(e_4\times e_6) = g(e_4)\times g(e_6). \end{cases} \tag{4.5}$$

Fix one point o on M. $\{x; e_i, e_a, e_\alpha\}$ at o is a standard basis for $\mathbf{R}^7$, and all the relations from (4.3) to (4.5) are equivalent to $g\in G_2$ (cf. the definition (2.4) of G_2). □

Now we are in a position to prove our uniquness theorems.

First, if the Gauss curvature $K\equiv 1$, then x is totally geodesic. Thus the following theorem is a direct consequence of Theorem 4.1.

Theorem 4.2 Let M be a complete and connected metric surface, and x be a minimal immersion of M into S^6 with constant Kähler angle $\theta\in(0,\pi)$. If the Gauss curvature $K\equiv 1$, then up to a G_2-congruence, x exists uniquely.

Similarly, we have for the case $K\equiv 0$,

Theorem 4.3 Let $\mathbf{R}^2$ be the 2-plane with the standard flat metric, and x be a superminimal immersion of $\mathbf{R}^2$ into S^6 with constant Kähler angle $\theta\in(0,\pi)$. Then, up to a G_2-congruence and a rotation on $\mathbf{R}^2$, x exists uniquely.

Proof Let (u, v) be the canonical coordinates on $\mathbf{R}^2$ such that the standard flat metric is written by $ds^2 = du^2 + dv^2$. Set $e_1 = \frac{\partial}{\partial u}$, $e_2 = \frac{\partial}{\partial v}$ and thus $\{\omega_1, \omega_2\} = \{du, dv\}$. Therefore by (2.9),

$$\omega_{34} = \omega_{12}\equiv 0. \tag{4.6}$$

Note that a rotation on $\mathbf{R}^2$ does not change the standard metric and the complex structure, it makes no change on the Kähler angle of x.

Because of Proposition 4.1, we need only to show that the connection matrix of $\mathbf{R}^2$, with respect to the "special" frame field along x, can be uniquely determined. To this end, let $E = \{e_i, e_a, e_\alpha\}$ be the special frame field along x determined by e_1 and e_2, and h^a_{ij}, h^α_{ij} the components of the second fundamental form of x with respect to E.

Set

$$H_a = h^a_{11} - \sqrt{-1}\,h^a_{12},\quad H_\alpha = h^\alpha_{11} - \sqrt{-1}\,h^\alpha_{12},$$

and

$$f = \left|\sum_a H_a^2 + \sum_\alpha H_\alpha^2\right|^2.$$

Then f is a globally defined, and x is called superminimal if $f\equiv 0$.

Since $H_a^2\equiv 0$ by the minimality and (2.9), $f\equiv 0$ if and only if $\sum_\alpha H_\alpha^2\equiv 0$, which is equivalent to

$$(h^5_{11})^2 + (h^6_{11})^2 = (h^5_{12})^2 + (h^6_{12})^2,\ h^5_{11}h^5_{12} = -h^6_{11}h^6_{12}. \tag{4.7}$$

Suitably choose the frame $\{e_1, e_2\}$, we can assume that $h^6_{12} = 0$, thus by (2.10) and (4.7),

$$h^5_{ij} = h^6_{ij} = 0, \tag{4.8}$$

which is independent of the choice of $\{e_1, e_2\}$. (4.8) together with (2.6) and (2.7), shows that

$$\omega_{35} = -\omega_2, \omega_{45} = \omega_1, \omega_{36} = \omega_{46} = 0. \tag{4.9}$$

Next, we are to determine h_{ij}^{a}'s By Gaussian equation and the flatness,

$$\sum_{i,j,a} (h_{ij}^{a})^2 = 2. \tag{4.10}$$

Due to (2.9), (4.10) becomes

$$(h_{11}^{3})^2 + (h_{12}^{3})^2 = \frac{1}{2}.$$

Thus there is a function t, such that

$$h_{11}^{3} = \frac{1}{\sqrt{2}}\cos t, h_{12}^{3} = \frac{1}{\sqrt{2}}\sin t, h_{11}^{4} = \frac{1}{\sqrt{2}}\sin t, h_{12}^{4} = -\frac{1}{\sqrt{2}}\cos t \tag{4.11}$$

Therefore,

$$\omega_{13} = \frac{1}{\sqrt{2}}(\omega_1 \cos t + \omega_2 \sin t), \omega_{23} = \frac{1}{\sqrt{2}}(\omega_1 \sin t - \omega_2 \cos t). \tag{4.12}$$

Take external differentiations of (4.12) using (4.6) and (4.8), we get

$$\mathrm{d}t \wedge \omega_1 = \mathrm{d}t \wedge \omega_2 = 0,$$

These are equivalent to $\mathrm{d}t = 0$, that is, $t = $ const. In particular, h_{ij}^{a} are all constant on $\mathbf{R}^2$.

To find the value of t, we note first that, if the frame $\{e_1, e_2\}$ is changed to $\{\tilde{e}_1, \tilde{e}_2\}$ by

$$\tilde{e}_1 = e_1 \cos \varphi + e_2 \sin \varphi, \tilde{e}_2 = -e_1 \sin \varphi + e_2 \cos \varphi, \tag{4.13}$$

Then $\{e_3, e_4\}$ must change into

$$\tilde{e}_3 = e_3 \cos \varphi + e_4 \sin \varphi, \tilde{e}_4 = -e_3 \sin \varphi + e_4 \cos \varphi. \tag{4.14}$$

Using (4.13) and (4.14), we derive that

$$\tilde{h}_{11}^{3} = h_{11}^{3} \cos 3\varphi + h_{12}^{3} \sin 3\varphi, \tilde{h}_{12}^{3} = -h_{11}^{3} \sin 3\varphi + h_{12}^{3} \cos 3\varphi.$$

If $h_{12}^{3} \neq 0$, we can choose φ suitably, such that $\tilde{h}_{12}^{3} = 0$. Without loss of generality, we assume $h_{12}^{3} = 0$ and $\cos t = 1$. So that (4.11) becomes

$$h_{11}^{3} = -h_{12}^{4} = \frac{1}{\sqrt{2}}, \text{ and } h_{12}^{3} = h_{11}^{4} = 0. \tag{4.15}$$

Combine formulas obtained, one sees that, with a rotation on $\mathbf{R}^2$ if necessary, the connection matrix of $\mathbf{R}^7$ along x can be written as follows:

$$\begin{pmatrix}
0 & -\mathrm{d}u & -\mathrm{d}v & 0 & 0 & 0 & 0 \\
\mathrm{d}u & 0 & 0 & -\frac{1}{\sqrt{2}}\mathrm{d}u & \frac{1}{\sqrt{2}}\mathrm{d}v & 0 & 0 \\
\mathrm{d}v & 0 & 0 & \frac{1}{\sqrt{2}}\mathrm{d}v & \frac{1}{\sqrt{2}}\mathrm{d}u & 0 & 0 \\
0 & \frac{1}{\sqrt{2}}\mathrm{d}u & -\frac{1}{\sqrt{2}}\mathrm{d}v & 0 & 0 & \mathrm{d}v & 0 \\
0 & -\frac{1}{\sqrt{2}}\mathrm{d}v & -\frac{1}{\sqrt{2}}\mathrm{d}u & 0 & 0 & -\mathrm{d}u & 0 \\
0 & 0 & 0 & -\mathrm{d}v & \mathrm{d}u & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}, \tag{4.16}$$

which is uniquely determined. □

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S^6 中具有常数 Kähler 角和常数曲率的极小曲面

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摘　要

本文给出了近 Kähler 球面 S^6 中具有常数 Kähler 角和常数曲率的极小曲面的例子, 同时证明了两个唯一性定理