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Periodic Solution of Nonautonomous System with Continuous Time Delays*

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We consider the nonautonomous cooperative system with continuous time delay

$$\dot{x}(t) = x(t)\left[a_1(t) - a_2(t)x(t) + a_3(t)\int_{-\tau}^{0} k_1(s)x(t+s)\,ds - a_4(t)\int_{-\tau}^{0} k_2(s)y(t+s)\,ds\right]$$
$$\dot{y}(t) = y(t)\left[b_1(t) - b_2(t)y(t) + a_3(t)\int_{-\tau}^{0} k_3(s)y(t+s)\,ds - b_4(t)\int_{-\tau}^{0} k_4(s)x(t+s)\,ds\right] \quad (1)$$

where $a_i(t), b_i(t)$ $(i = 1, 2, 3, 4)$ are assuned to be continuous, positive and ω-periodic functions; and $x(t), y(t)$ are the density of species; $k_i(s)$ $(i = 1, 2, 3, 4)$ denote nonnegative piecewise continuous defined in $[-\tau, 0]$ (there $0 < \tau < +\infty$) and normalized such that $\int_{-\tau}^{0} k_i(s)\,ds = 1$. Let $f^L = \inf\{f(t) \mid t \in R\}$, $f^M = \sup\{f(t), t \in R\}$, for a continuous and bounded function $f(t)$. We denote

$$C^+ = \{(\varphi_1, \varphi_2) \mid \varphi_i(\theta) \geq 0, \theta \in [-\tau, 0], (\varphi_1(0), \varphi_2(0)) > 0\}.$$

Given $\sigma \in R$ and $\varphi = (\varphi_1, \varphi_2) \in C^+$, it is easy to that (1) have a respective unique solution $x(\sigma, \varphi)(t), y(\sigma, \varphi)(t)$ through (σ, φ) at $t = \sigma$. Moreover, $x(\sigma, \varphi)(t) > 0$, $y(\sigma, \varphi)(t) > 0$ for all $t \in [\sigma, T]$, where $[\sigma, T]$ is the maximal existence interval of the solution. Such solutions of system (1) are called positive solutions.

The following theorem sets forth the principal result of this paper.

Theorem *Suppose that system* (1) *satisfies* $a_2^L > a_3^M$, $b_2^L > b_3^M$. *Then system* (1) *has a unique positive* ω-*periodic solution which is globally asymptotically stable.*

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