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The Commutative Ring of a Symmetric Design Is Not Associative when $\lambda \neq 1$ *

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Let X denote the set of points of a symmetric (v, k, λ)-design with $n = k - \lambda$. Adjoin a formal symbol I to X to yield a set X^*. Let R denote the free Z-module whose basis is formed by the elements of X^*. If B is a block, we denote the element $\sum_{b \in B} b$ of R simply by B. We obtain a multiplication on R by defining the product of the basis elements by the following rules, and then extending bilinearly:

(i) If a and b are distinct points of X, then

$$ab = B_1 + B_2 + \cdots + B_\lambda - n\lambda^2 I,$$

where $B_1, B_2, \cdots, B_\lambda$ are the Blocks containing a and b;

(ii) If $a \in X, a^2 = na$;

(iii) $Ix = x = xI$ for all $x \in X^*$.

The main conclusion of [1] is that R is associative, but I think it may be not true if $\lambda \neq 1$. In fact, we can consider a special case as follows.

$ac = B_1 + B_2 + \cdots + B_\lambda - n\lambda^2 I$, where $a \neq c, B_1, B_2, \cdots, B_\lambda$ are the Blocks containing a and c. Here $(aa)c = n(ac) = n(B_1 + B_2 + \cdots + B_\lambda) - n^2\lambda^2 I$, and the coefficient of I is $(-n^2\lambda^2)$; $a(ac) = aB_1 + aB_2 + \cdots + aB_\lambda - n\lambda^2 a$, and the coefficient of I is $(-n\lambda^2)(k-1)\lambda$. Obviously, it follows that $(aa)c = a(ac) \Longleftrightarrow \lambda = 1$.

Certainly, the result for $\lambda = 1$ is not the aim of [1] since it belons to [2].

References

[1] Prince A R. *The commutative ring of a symmetric design* [J]. Discrete Mathematics, 1990, **80**: 101–103.

[2] Prince A R. *A commutative ring associated with any finite projective plane* [J]. J. Algebra, 1986, **99**: 295–303.

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