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The Necessary and Sufficient Conditions of Equivalence of a Class of Real Singular Germs*

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Abstract Given a reflexive Banach space X. In the ring of C^∞ function germs $\epsilon(X)$, any real singular germ f in $\epsilon(X)$ whose second Frechet derivative at origin $f''(0) = T$ is a Fredholm operator is isomorphic to a germ of the form $\frac{1}{2} < Tz, z > + g(v)$. If we replace g by a g_1 which is isomorphic to g, we clearly obtain a germ in $\epsilon(X)$ which is isomorphic to the original one. However, is true converse of this proposition? In this paper, we will show that the converse is also true.

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1. Introduction and conclusions

In singularity theory and catastrophe theory of germs of C^∞ functions, splitting lemma is a very important theorem:

Proposition A (Splitting lemma) *Assume that the ring of germs of C^∞ functions at $O \in \mathbf{R}^n$ is denoted by $\epsilon(n)$. If $f \in \epsilon(n)$ is a singular germ and its Hessian matrix at O has rank r, then f is right equivalent to germ of form:*

$$Q(x_1, x_2, \cdots, x_r) + g(x_{r+1}, \cdots, x_n), \tag{1}$$

where $Q(x_1, x_2, \cdots, x_r)$ is a non-degenerate quadratic form in variables $x_1, x_2, \cdots, x_r$, $g \in M_3(n-r)$. $M_3(n-r)$ is an ideal consisting of those germs in $x_{r+1}, \cdots x_n$ whose function value and all the partial derivatives of order less than or equal to 2 at $o \in \mathbf{R}^{n-r}$ are all zero. ([1] p. 45, p. 155)

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There are similar conclusions for certain class of real singular germs which are defined at O on a Hilbert space or Banach space and their second Frechet derivarives are a Fredholm operator. ([2]p. 361-369, [3]p. 91-95)

Now, we only state corresponding conclusions for this class of germs at origin on a Banach space as follows:

Proposition B *Assume that X is a reflexive Banach space and $\epsilon(X)$ denotes the ring of germs of C^∞ functions at $O \in X$. If $f \in \epsilon(X)$, $f(0) = 0$, $f'(0) = 0$, $f''(0) = T$ is a Fredholm operator, $V = \ker T$ (finite dimension), $X = Z \oplus V$, then there exists $g \in M_3(V)$ such that f is right equivalent to germ:*

$$\frac{1}{2} < Tz, z > + g(v) = Q(z) + g(v), \tag{2}$$

where $Q(z)$ is a non-degenerate quadratic fom in z.

Obviously, in (1) and (2), if we replace g by g_1 which is right equivalent to g, then easily know that $Q + g_1$ is right equivalent to $Q + g$. However, its converse: If $Q + g_1$ is right equivalent to $Q + g$, then is g_1 right equivalent to g? In [1] p. 155, J. Martinet mentioned this problem, but it was not proved in this literature. In the published literatures, we seldom see something concerning with it. As a note, we will give a necessary and sufficient condition of right equivalence and contact equivalence for this class of sigular germs, so as to illustrate the relationship between right equivalence (or contact equivalence) of $g_1(v)$ and $g(v)$ in $\epsilon(V)$.

Theorem 1 *Assume that X is a reflexive Banach space, $T: X \to X^*$ is a Fredholm operator, $\ker T = V$, $X = Z \oplus V$, then function germ $G: (z, v) \to \frac{1}{2} < Tz, z > + g(v)$ is right equivalent to germ $G_1: (z, v) \to \frac{1}{2} < Tz, z > + g_1(v)$ in $\epsilon(X)$, if and only if $g(v)$ is right equivalent to $g_1(v)$ in $\epsilon(V)$.*

Theorem 2 *Assume that X is a reflexive Banach space, $T: X \to X^*$ is a Fredholm operator, $\ker T = V$, $X = Z \oplus V$, then function germ $G: (z, v) \to \frac{1}{2} < Tz, z > + g(v)$ is contact equivalent to germ $G_1: (z, v) \to \frac{1}{2} < Tz, z > + g_1(v)$ in $\epsilon(X)$, if and only if $g(v)$ is contact equivalent to $g_1(v)$ in $\epsilon(V)$. where $g, g_1 \in M_3(V)$.*

2 Prilminaries

The proof of above conclusions is based on an application of following implicit function theorem:

Lemma 1 (Implicit function theorem) *Let E, F, G be three Banach spaces, f a continuously differentiable mapping of an open subset A of $E \times F$ into G. Let (x_0, y_0) be a point of A such that $f(x_0, y_0) = 0$ and that the partial derivative $D_2 f(x_0, y_0)$ be a linear homeomorphism of F onto G. Then, there is an open neighborhood U of x_0 in E such that for every open connected neighborhood U of x_0 contained in U_0, there is a unique continous mapping u of U into F such that $u(x_0) = y_0$, $(x, u(x)) \in A$ and $f(x, u(x)) = 0$ for any $x \in U$. Furtheremore, u is continuously differentiable in U, and its derivative is given by*

$$u'(x) = -(D_2 f(x, u(x))^{-1} \circ (D_1 f(x, u(x)).$$

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(*It can be seen* [4] *p.* 270, (10. 2 1)) (*Note: If* $f \in C^\infty$, *then* $u \in C^\infty$)

Because $T: X \to X^*$ is a Fredholm operator, $\ker T = V$, thereby V is a finite dimensional linear subspace in X. Therefore there is a projective operator P_V from X to V, thereby $(I - P_V)$ is also a projective operator. ([5], p. 112, theorem 2)

Writing $Z = R(I - P_V)$ -the range of $(I - P_V)$. Then $X = V \oplus Z$ and Z is a topological supplement of V ([8], p. 102-103). $X^* = V^* \oplus Z^*$ and $Z^* \cong V^\perp, V^* \cong Z^\perp$. Where, $V^\perp = \{x^* \in X^* \mid x \in V \Rightarrow \langle x^*, x \rangle = 0\}$. $Z^\perp$ is similar. ([5], p. 107, Theorem 3). $Z^* = TX = TZ$.

Lemma 2 *Let* $\Phi: (X, 0) \to (X, 0)$, $(v, z) \mapsto \Phi(v, z)$ *be a diffeomorphism germ, then there are* $\varnothing: (X, 0) \to (V, 0)$, $(v, z) \mapsto \varnothing(v, z)$ *and* $\Psi: (X, 0) \to (Z, 0)$, $(v, z) \mapsto \Psi(v, z)$ *such that* $\Phi(v, z) = \varnothing(v, z) + \Psi(v, z)$.

Proof Because $X = V \oplus Z$, and Z is a topological supplement of V in X. We need take only $\varnothing(v, z) = P_V \Phi(v, z)$, $\Psi(v, z) = (I - P_V)\Phi(v, z)$.

3. The proof of theorems

Assume that $G(v, z) = \frac{1}{2}\langle Tz, z \rangle + g(v)$ is right equivalent to $G_1(v, z) = \frac{1}{2}\langle Tz, z \rangle + g_1(v)$, then there are $\varnothing(v, z)$ and $\Psi(v, z)$ in lemma 2 such that $Q(z) + g(v) = Q(\Psi(v, z)) + g_1(\varnothing(v, z))$. Namely $G(v, z) = G_1(\varnothing(v, z), \Psi(v, z))$, where, $Q(z) = \frac{1}{2}\langle Tz, z \rangle$, $\Phi = \varnothing + \Psi: (X, 0) \to (X, 0)$ is a local diffeomorphism.

The proof of theorems follows directly from the following two lemmas:

Lemma 3 *Assume that* $G(v, z) = Q(z) + g(v)$ *is right equivalent to* $G_1(v, z) = Q(z) + g_1(z)$. *then*

$$\begin{pmatrix} 0 & 0 \\ 0 & T \end{pmatrix} = \begin{pmatrix} \frac{\partial \varnothing}{\partial v}(0, 0) & \frac{\partial \varnothing}{\partial z}(0, 0) \\ \frac{\partial \Psi}{\partial v}(0, 0) & \frac{\partial \Psi}{\partial z}(0, 0) \end{pmatrix}^t \begin{pmatrix} 0 & 0 \\ 0 & T \end{pmatrix} \begin{pmatrix} \frac{\partial \varnothing}{\partial v}(0, 0) & \frac{\partial \varnothing}{\partial z}(0, 0) \\ \frac{\partial \Psi}{\partial v}(0, 0) & \frac{\partial \Psi}{\partial z}(0, 0) \end{pmatrix}.$$

Furthermore, $\frac{\partial \Psi}{\partial z}(0, 0): Z \to Z$ *is a linear homeomorphism.* $\frac{\partial \varnothing}{\partial v}(0, 0): V \to V$ *is a linear homeomorphism.*

Proof Because $G(v, z) = G_1(\Phi(v, z)) = G_1(\varnothing(v, z), \Psi(v, z)) = Q(\Psi(v, z)) + g_1(\varnothing(v, z))$. From differential calculus on a Banach space, by twice differentiating at the origin, we have

$$DG(v, z) = \left(\frac{\partial G(v, z)}{\partial v}, \frac{\partial G(v, z)}{\partial z} \right) = \left(\frac{\partial g(v)}{\partial v}, \frac{\partial Q(z)}{\partial z} \right) = (g'(v), Tz),$$

$$D^2 G(v, z) = \begin{pmatrix} \frac{\partial^2 g(v)}{\partial v^2} & \frac{\partial^2 g(v)}{\partial z \partial v} \\ \frac{\partial^2 Q(z)}{\partial v \partial z} & \frac{\partial^2 Q(z)}{\partial z^2} \end{pmatrix} = \begin{pmatrix} g''(v) & 0 \\ 0 & T \end{pmatrix}.$$

Note $g \in M_3(v)$, thereby $g''(0) = 0$. Therefore

$$D^2 G(0, 0) = \begin{pmatrix} 0 & 0 \\ 0 & T \end{pmatrix}.$$

On the other hand

$$DG_1(\Phi(v, z)) = (DG_1 \circ \Phi(v, z)) \cdot D\Phi(v, z)$$

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$$= \left(\frac{\partial g_1}{\partial v} \circ \varnothing(v,z), \frac{\partial Q}{\partial z} \circ \psi(v,z) \right) \begin{pmatrix} \frac{\partial \varnothing}{\partial v}(v,z) & \frac{\partial \varnothing}{\partial z}(v,z) \\ \frac{\partial \psi}{\partial v}(v,z) & \frac{\partial \psi}{\partial z}(v,z) \end{pmatrix}.$$

Therefore

$$\begin{aligned} D^2 G_1(\Phi(v,z)) &= D(DG_1 \circ \Phi(v,z)) \bullet D\Phi(v,z) + (DG_1 \circ \Phi(v,z)) \bullet D^2\Phi(v,z) \\ &= (D^2 G_1 \circ \Phi(v,z)) \bullet (D\Phi(v,z))^2 + (DG_1 \circ \Phi(v,z)) \bullet D^2\Phi(v,z) \\ &= \begin{pmatrix} \frac{\partial^2 g_1}{\partial v^2} \circ \varnothing(v,z) & \frac{\partial^2 g_1}{\partial z \partial v} \circ \varnothing(v,z) \\ \frac{\partial^2 Q}{\partial v \partial z} \circ \psi(v,z) & \frac{\partial^2 Q}{\partial z^2} \circ \psi(v,z) \end{pmatrix} \begin{pmatrix} \frac{\partial \varnothing}{\partial v}(v,z) & \frac{\partial \varnothing}{\partial z}(v,z) \\ \frac{\partial \psi}{\partial v}(v,z) & \frac{\partial \psi}{\partial z}(v,z) \end{pmatrix}^2 + \\ &\quad \left(\frac{\partial g_1}{\partial v} \circ \varnothing(v,z), \frac{\partial Q}{\partial z} \circ \psi(v,z) \right) \bullet D \begin{pmatrix} \frac{\partial \varnothing}{\partial v}(v,z) & \frac{\partial \varnothing}{\partial z}(v,z) \\ \frac{\partial \psi}{\partial v}(v,z) & \frac{\partial \psi}{\partial z}(v,z) \end{pmatrix}. \end{aligned}$$

From this, we easily know

$$\begin{aligned} D^2 G_1(0,0) &= \begin{pmatrix} 0 & 0 \\ 0 & T \end{pmatrix} \begin{pmatrix} \frac{\partial \varnothing}{\partial v}(0,0) & \frac{\partial \varnothing}{\partial z}(0,0) \\ \frac{\partial \psi}{\partial v}(0,0) & \frac{\partial \psi}{\partial z}(0,0) \end{pmatrix}^2 \\ &= \begin{pmatrix} \frac{\partial \varnothing}{\partial v}(0,0) & \frac{\partial \varnothing}{\partial z}(0,0) \\ \frac{\partial \psi}{\partial v}(0,0) & \frac{\partial \psi}{\partial z}(0,0) \end{pmatrix}^* \begin{pmatrix} 0 & 0 \\ 0 & T \end{pmatrix} \begin{pmatrix} \frac{\partial \varnothing}{\partial v}(0,0) & \frac{\partial \varnothing}{\partial z}(0,0) \\ \frac{\partial \psi}{\partial v}(0,0) & \frac{\partial \psi}{\partial z}(0,0) \end{pmatrix} \\ &= \begin{pmatrix} \frac{\partial \varnothing}{\partial v}(0,0) & \frac{\partial \varnothing}{\partial z}(0,0) \\ \frac{\partial \psi}{\partial v}(0,0) & \frac{\partial \psi}{\partial z}(0,0) \end{pmatrix}^t \begin{pmatrix} 0 & 0 \\ 0 & T \end{pmatrix} \begin{pmatrix} \frac{\partial \varnothing}{\partial v}(0,0) & \frac{\partial \varnothing}{\partial z}(0,0) \\ \frac{\partial \psi}{\partial v}(0,0) & \frac{\partial \psi}{\partial z}(0,0) \end{pmatrix}, \end{aligned}$$

where

$$[D\Phi(0,0)]^* = \begin{pmatrix} \frac{\partial \varnothing}{\partial v}(0,0) & \frac{\partial \varnothing}{\partial z}(0,0) \\ \frac{\partial \psi}{\partial v}(0,0) & \frac{\partial \psi}{\partial z}(0,0) \end{pmatrix}^* : X^* \to X^*$$

is the conjugate operator of $D\Phi(0,0): X \to X$. ([6]p. 168, Theorem 6 and p. 172 Lemma 1)

From $D^2 G(0,0) = D^2 G_1(0,0)$, we get

$$\begin{pmatrix} 0 & 0 \\ 0 & T \end{pmatrix} \begin{pmatrix} \frac{\partial \varnothing}{\partial v}(0,0) & \frac{\partial \varnothing}{\partial z}(0,0) \\ \frac{\partial \psi}{\partial v}(0,0) & \frac{\partial \psi}{\partial z}(0,0) \end{pmatrix}^t \begin{pmatrix} 0 & 0 \\ 0 & T \end{pmatrix} \begin{pmatrix} \frac{\partial \varnothing}{\partial v}(0,0) & \frac{\partial \varnothing}{\partial z}(0,0) \\ \frac{\partial \psi}{\partial v}(0,0) & \frac{\partial \psi}{\partial z}(0,0) \end{pmatrix}.$$

Computting the above right side:

$$\begin{pmatrix} 0 & 0 \\ 0 & T \end{pmatrix} = \begin{pmatrix} \left(\frac{\partial \psi}{\partial v}(0,0)\right)^* T \frac{\partial \psi}{\partial v}(0,0) & \left(\frac{\partial \psi}{\partial v}(0,0)\right)^* T \frac{\partial \psi}{\partial z}(0,0) \\ \left(\frac{\partial \psi}{\partial z}(0,0)\right)^* T \frac{\partial \psi}{\partial v}(0,0) & \left(\frac{\partial \psi}{\partial z}(0,0)\right)^* T \frac{\partial \psi}{\partial z}(0,0) \end{pmatrix}.$$

Compare and note $T: Z \to Z^*$ is a symmetric linear homeomorphism associated with Q, therefore, $\frac{\partial \psi}{\partial z}(0,0): Z \to Z$ is a linear homeomorphism, $\frac{\partial \psi}{\partial v}(0,0) = 0$, thereby $\frac{\partial \varnothing}{\partial v}(0,0): V \to V$ is also a linear homeomorphism. Which ends the proof.

Lemma 4 *There exists a neighborhood of O in V and a unique C^∞ mapping $\gamma_1: U \to Z$, $\gamma_2: U \to Z$ for $G(v,z)$, $G_1 \circ \Phi(v,z)$ respectively such that $\gamma_1(0) = 0$, $\gamma_2(0) = 0$, and $\forall v \in V$ and $\forall z \in Z$, we always have $< G'(v, \gamma_1(v)), z > = 0$, $< G_1' \circ \Phi(v, \gamma_2(v)), z > = 0$.*

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Furthermore, $\gamma_1'(0) = 0, \gamma_2'(0) = 0$.

Proof We will only prove the relevant results for $G_1 \circ \Phi(v, z)$. $G(v, z)$ is similar.

From II, we know that $Z^* = TX = TZ, X^* = V^* \oplus Z^*$ and $V^* \cong Z^\perp$. Hence, V^* is not only a topological supplement of Z^*, but also the annihilator of Z. ($V^* \cong Z^\perp$)

Let P_{Z^*} be the continuous projection of X^* onto Z^* ($= TX$), then $\ker P_{Z^*} = V^*$.

Consider mapping

$$h: V \times Z \to TX, (v, z) \mapsto P_{Z^*} G_1' \circ \Phi(v, z).$$

Then $h(v, z) = 0$ is equivalent to $G_1' \circ \Phi(v, z) \in V^*$. Namely, $\forall \bar{z} \in Z, < G_1' \circ \Phi(v, z), \bar{z} > = 0$.

Note $h(v, z) = P_{Z^*} T \Psi(v, z)$, hence $\frac{\partial \Psi}{\partial z}(0, 0) = P_{Z^*} T \frac{\partial \Psi}{\partial z}(0, 0)$.

From Lemma 3, we know that $\frac{\partial \Psi}{\partial z}(0, 0): Z \to Z$ is a linear homeomorphism, $T: Z \to Z^*$ is a symmetric linear homeomorphism. By means of implicit function theorem: There exists a neighborhood U_2 of O in V and a unique C^∞ mapping $\gamma_2: U_2 \to Z$ such that $\gamma_2(0) = 0$ and $h(v, \gamma_2(v)) = 0$. This is equivalent to $P_{Z^*} T \Psi(v, \gamma_2(v)) = 0$. Namely $< T \Psi(v, \gamma_2(v)), \bar{z} > = < G_1' \circ \Phi(v, \gamma_2(v)), \bar{z} > = 0, \forall \bar{z} \in Z$.

By differentiating the equation $P_{Z^*} T \Psi(v, \gamma_2(v)) = 0$ at $v = 0$, giving

$$P_{Z^*} T \left(\frac{\partial \Psi}{\partial v}(0, 0) + \frac{\partial \Psi}{\partial z}(0, 0) \gamma_2'(0) \right) v = 0 \text{ for all } v.$$

Namely $P_{Z^*} T \frac{\partial \Psi}{\partial z}(0, 0) \gamma_2'(0) v = 0 (\forall v \in V)$. Therefore $\gamma_2'(0) = 0$.

Similarly, for $G(v, z)$, there exists a neighborhood U_1 of O in V and a unique C^∞ mapping $\gamma_1: U_1 \to Z$ such that $\gamma_1(0) = 0$, $< G'(v, \gamma_1(v)), \bar{z} > = 0, \forall \bar{z} \in Z$, and $\gamma_1'(0) = 0$.

Take $U = U_1 \cap U_2$, then $U, \gamma_1: U \to Z$ and $\gamma_2: U \to Z$ are desired.

Proof of theorem 1 For $G(v, z)$: In $U, z = \gamma_1(v)$, thereby $G(v, z) = G(v, \gamma_1(v)) = \frac{1}{2} < T \gamma_1(v), \gamma_1(v) > + g(v)$. From Lemma 4, $< G'(v, \gamma_1(v)), z > = 0, \forall z \in Z$. In particular, take $z = \gamma_1(v)$, we have $< G'(v, \gamma_1(v)), \gamma_1(v) > = 0$. Namely $< T \gamma_1(v), \gamma_1(v) > = 0$. Therefore $G(v, \gamma_1(v)) = g(v)$ for all $v \in U$.

Similarly, for $G_1(v, z)$, in $U: z = \gamma_2(v)$. Thereby

$$G_1 \circ \Phi(v, z) = G_1 \circ \Phi(v, \gamma_2(v)) = \frac{1}{2} < T \Psi(v, \gamma_2(v)), \Psi(v, \gamma_2(v)) > + g_1 \circ \varnothing(v, \gamma_2(v)).$$

However, $< G_1' \circ \Phi(v, \gamma_2(v)), z > = 0, \forall z \in Z$. In particular, take $z = \Psi(v, \gamma_2(v))$, we get

$$< G_1' \circ \Phi(v, \gamma_2(v)), \Psi(v, \gamma_2(v)) > = 0.$$

Namely $< T \Psi(v, \gamma_2(v)), \Psi(v, \gamma_2(v)) > = 0$.

Therefore $G_1 \circ \Phi(v, \gamma_2(v)) = g_1 \circ \varnothing(v, \gamma_2(v))$ *in* U. Thus, in $U \subset V$: $G(v, \gamma_1(v)) = G_1 \circ \Phi(v, \gamma_2(v))$. Namely $g(v) = g_1 \circ \varnothing(v, \gamma_2(v))$.

Now, only need to prove: $\varnothing: (V, 0) \to (V, 0), v \mapsto \varnothing(v, \gamma_2(v))$ is a local diffeomorphism. Differentiating $\varnothing(v, \gamma_2(v))$ at $v = 0$:

$$\varnothing'(0) = \frac{\partial \varnothing}{\partial v}(0, 0) + \frac{\partial \varnothing}{\partial z}(0, 0) \gamma_2'(0) = \frac{\partial \varnothing}{\partial v}(0, 0).$$

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By Lemma 3, we know that $\frac{\partial \varnothing}{\partial v}(0,0): V \to V$ is a linear homeomorphism. Therefore, $\varnothing: (V, 0) \to (V, 0), v \mapsto \varnothing(v, \gamma_2(v))$ is a local diffeomorphism. Which ends the proof.

The proof of Theorem 2 Similar to Theorem 1. Omit.

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一类实奇芽等价的充要条件

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摘 要

给定一自反的Banach 空间X. 在c^∞函数芽环$\epsilon(X)$中, 任何在原点$O \in X$的实奇芽f, 若其二阶Frechet 导数$f''(0) = T$是一个Fredholm 算子, 则它同构于形如: $\frac{1}{2} < Tz, z > + g(v)$的一个芽. 如果代替$g$以一个与之同构的芽$g_1$, 显然得到一个与原来的芽同构的芽. 然而, 其逆是否真呢? 本文将证明其逆命题亦真

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