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Minimal Elements in the Poset of Graphic Sequences *

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Abstract: A nonincreasing sequence π of n nonnegative integers is said to be graphic if it is the degree sequence of a simple graph G of order n. The set of all graphic sequences of n terms with even sum $2m$ and trace f is a poset $G_{n,m,f}$ under majorization relation. The paper characterizes the minimal elements in the poset $G_{n,m,f}$ and determines the number of minimal elements in various posets of graphic sequences.

Key words: graph; graphic sequence; poset; minimal elements.

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1. Introduction

Let $\pi = (d_1, d_2, \cdots, d_n)$ be an intger sequence with $n-1 \geq d_1 \geq d_2 \geq \cdots \geq d_n \geq 0$. The set of all such sequences is denoted by NS_n. For a given $\pi \in NS_n$, denote $\sigma(\pi) = d_1 + d_2 + \cdots + d_n$, and $f(\pi) = \max\{i \in \langle n \rangle : d_i \geq i\}$, where $\langle n \rangle = \{1, 2, \cdots, n\}$. Then $\sigma(\pi)$ and $f(\pi)$ are the term sum and trace of π respectively. Define an $n \times n$ $(0,1)$-matrix $\overline{A(\pi)} = (a_{ij})$ as follows: for $1 \leq i \leq f(\pi)$,

$$a_{ij} = \begin{cases} 1 & \text{if } 1 \leq j \neq i \leq d_i + 1, \\ 0 & \text{otherwise.} \end{cases}$$

and for $f(\pi) + 1 \leq i \leq n$,

$$a_{ij} = \begin{cases} 1 & \text{if } 1 \leq j \leq d_i, \\ 0 & \text{otherwise.} \end{cases}$$

Then $\overline{A(\pi)}$ is called the off-diagonal leftmost matrix of π. Clearly, π is the row sum vector of $\overline{A(\pi)}$. The column sum vector of $\overline{A(\pi)}$ is called the corrected conjugate sequence of π, and denoted by $\overline{\pi} = (\overline{d_1}, \overline{d_2}, \cdots, \overline{d_n})$. It is clear that $\sigma(\pi) = \sigma(\overline{\pi})$.

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A sequence $\pi \in NS_n$ is graphic if it is the degree sequence of a simple graph G of order n. The set of all graphic sequences in NS_n is denoted by G_n. Berge [1] gave a criteria for a sequence in NS_n being graphic as follows:

Theorem 1.1 *Let $\pi = (d_1, d_2, \cdots, d_n)$ be a sequence in NS_n with even sum $\sigma(\pi)$. Then $\pi \in G_n$ if and only if for each $k \in \langle n \rangle$, $d_1 + d_2 + \cdots + d_k \leq \overline{d_1} + \overline{d_2} + \cdots + \overline{d_k}$, or in equivalent words, if and only if (1) holds for each $k \in \langle f(\pi) \rangle$.*

It is clear that, if $\pi \in G_n$ with $\sigma(\pi) = 2m$, then m is the number of edges of any realization G of π. Denote $G_{n,m,f} = \{\pi \in G_n : \sigma(\pi) = 2m$ and $f(\pi) = f\}$, $G_{n,m,*} = \{\pi \in G_n : \sigma(\pi) = 2m\}$, $G_{n,*,f} = \{\pi \in G_n : f(\pi) = f\}$. Moreover, the set of graphic sequences π of positive integers with $\sigma(\pi) = 2m$ is denoted by $G_{*,m,*}$.

Let $\pi = (d_1, d_2, \cdots, d_n)$ and $\rho = (r_1, r_2, \cdots, r_n)$ be sequences in NS_n. We say that π majorizes ρ, denoted by $\pi \succ \rho$, if $d_1 + d_2 + \cdots + d_k \geq r_1 + r_2 + \cdots + r_k$ for each $k \in \langle n \rangle$ and equality holds when $k = n$ (see Marshall and Olkin [2]). It is easy to see that each of G_n, $G_{n,m,f}$, $G_{n,m,*}$, $G_{n,*,f}$ and $G_{*,m,*}$ is a poset under majorization relation.

Theorem 1.2[3] *A graphic sequence $\pi = (d_1, d_2, \cdots, d_n)$ is a maximal element in the poset G_n if and only if $\overline{\pi} = \pi$, or in equivalent words, $\overline{d_k} = d_k$ for each $k \in \langle f(\pi) \rangle$.*

Theorem 1.3[4] *Let $G_{n,m,f} \neq \emptyset$. Then $\pi \in G_{n,m,f}$ is a maximal element in the poset $G_{n,m,f}$ if and only if π is a maximal element in the poset G_n.*

The purpose of this paper is to determine all minimal elements in above various poset of graphic sequences. Furthermore, we also count the number of minimal elements in above various posets of graphic sequences.

2. Main Results

Lemma 2.1 *Let $\pi = (d_1, d_2, \cdots, d_n)$ and $\rho = (r_1, r_2, \cdots, r_n)$ be sequences in NS_n. If $\sigma(\pi) = \sigma(\rho)$ and there exists an integer $k \in \langle n \rangle$ such that $d_i \geq r_i$ for each $i \in \langle k \rangle$ and $d_i \leq r_i$ for each $i \in \langle n \rangle - \langle k \rangle$, then $\pi \succ \rho$.*

Proof This is a clear consequence of the definition of majorization. □

Theorem 2.2 *Let $G_{n,m,f} \neq \emptyset$. Then*

$$f(f+1) \leq 2m \leq 2nf - f(f+1). \tag{1}$$

Proof By Theorem 1.3, $G_{n,m,f}$ has a maximal element $\pi = (d_1, d_2, \cdots, d_n)$, and π is also maximal in G_n. By Theorem 1.2, $\overline{\pi} = (\overline{d_1}, \overline{d_2}, \cdots, \overline{d_n}) = \pi$. Hence the row sum vector and column sum vector of $\overline{A(\pi)}$ are the same. Since $\overline{A(\pi)}$ is a (0,1)-matrix with trace zero, $\overline{A(\pi)} = (a_{ij})$ is symmetric. Hence, $\sum_{1 \leq i < j \leq n} a_{ij} = d_1 + (d_2 - 1) + \cdots + (d_f - f + 1) = m$. Observe that $n - 1 \geq d_1 \geq d_2 \geq \cdots \geq d_f \geq f$. Hence $n - 1 \geq d_1 > d_2 - 1 > \cdots > d_f - f + 1 > 0$. Therefore $f - k + 1 \leq d_k - k + 1 \leq n - k$ for each $k \in \langle f \rangle$. Thus

$$\begin{aligned} \frac{1}{2} f(f+1) = f + (f-1) + \cdots + 1 \leq d_1 + (d_2 - 1) + \cdots + (d_f - f + 1) &= m \\ \leq (n-1) + (n-2) + \cdots + (n-f) & \\ = nf - \frac{1}{2} f(f+1). & \end{aligned}$$

In other words, (1) holds. □

Remark It is easy to see that $\pi_1 = (f^{f+1}, 0^{n-f-1}) \in G_{n,m,f}$, where f^{f+1} means that f occurs $f+1$ times in π_1, and $\sigma(\pi_1) = 2m = f(f+1)$. Hence first equality in (1) holds if $\pi = \pi_1$. Moreover, $\pi_2 = \left((n-1)^f, f^{n-f}\right) \in G_{n,m,f}$, where $\sigma(\pi_2) = 2m = 2nf - f(f+1)$. So second equality in (1) holds if $\pi = \pi_2$.

Theorem 2.3 *If $G_{n,m,f} \neq \emptyset$ and $2m = nf_1$, then $\pi_1 = (f_1^n)$ is the least element in $G_{n,m,f}$.*

Proof It follows by the matrix $\overline{A(\pi_1)}$ that $\overline{\pi_1} = \left((n-1)^{f_1}, f_1^1, 0^{n-f_1-1}\right)$. Denote $\pi_1 = (d_1^{(1)}, d_2^{(1)}, \cdots, d_n^{(1)})$. Then $\sum_{i=1}^k \overline{d_i^{(1)}} - \sum_{i=1}^k d_i^{(1)} = k(n-1-f_1) \geq 0$ for each $k \in \langle f_1 \rangle$. By Theorem 1.1, $\pi_1 \in G_{n,m,f_1}$.

Now suppose $\pi = (d_1, d_2, \cdots, d_n) \in G_{n,m,f_1}$. Then $f(\pi) = f_1$ and $d_1 \geq d_2 \geq \cdots \geq d_{f_1} \geq f_1 \geq d_{f_1+1} \geq \cdots \geq d_n$. In other words, $d_k \geq d_k^{(1)}$ for each $k \in \langle f_1 \rangle$ and $d_k \leq d_k^{(1)}$ for each $k \in \langle n \rangle - \langle f_1 \rangle$. By Lemma 2.1, $\pi \succ \pi_1$. It shows that π_1 is the least element in G_{n,m,f_1}. □

Theorem 2.4 *Let f_2 be an integer which satisfies $f_2(f_2+1) \leq 2m < nf_2$. Denote $2m - f_2^2 = (n-f_2)q_2 + r_2$, where q_2 and r_2 are nonnegative integers and $r_2 < n - f_2$. Then $\pi_2 = \left(f_2^{f_2}, (q_2+1)^{r_2}, q_2^{n-f_2-r_2}\right) \in G_{n,m,f_2}$ and π_2 is the least element of G_{n,m,f_2}.*

Proof Denote $\pi_2 = (d_1^{(2)}, d_2^{(2)}, \cdots, d_n^{(2)})$. Then $\sigma(\pi_2) = f_2^2 + (n-f_2)q_2 + r_2 = 2m$. Since $2m < nf_2$, we have $q_2 + 1 \leq f_2$. Thus $f(\pi_2) = f_2$. We consider the following cases:

Case 1. $q_2 = 0$. In this case, $\pi_2 = (f_2^{f_2}, 1^{r_2}, 0^{n-f_2-r_2})$. It follows by the matrix $\overline{A(\pi_2)}$ that $\overline{\pi_2} = \left((f_2+r_2-1)^1, (f_2-1)^{f_2-1}, f_2^1, 0^{n-f_2-1}\right)$. It is easy to check that $\overline{\pi_2} \succ \pi_2$. By Theorem 1.2, $\pi_2 \in G_{n,m,f_2}$.

Now suppose $\pi = (d_1, d_2, \cdots, d_n) \in G_{n,m,f_2}$, where $d_1 \geq d_2 \geq \cdots \geq d_{f_2} \geq f_2 \geq d_{f_2+1} \geq \cdots \geq d_l > d_{l+1} = \cdots = d_n = 0$, and $f_2 + 1 \leq l \leq n$. Then $2m = f_2^2 + r_2 = (d_1 + d_2 + \cdots + d_{f_2}) + (d_{f_2+1} + \cdots + d_l) \geq f_2^2 + (l - f_2)$. Hence, $l \leq f_2 + r_2$. Therefore, $d_k \geq d_k^{(2)}$ for each $k \in \langle l \rangle$ and $d_k \leq d_k^{(2)}$ for each $k \in \langle n \rangle - \langle l \rangle$. By Lemma 2.1, $\pi \succ \pi_2$.

Case 2. $q_2 \geq 1$.

Subcase 2.1: $r_2 = 0$. In this case, $\pi_2 = (f_2^{f_2}, q_2^{n-f_2})$. Clearly, $q_2 < f_2$. By the matrix $\overline{A(\pi_2)}$, we have $\overline{\pi_2} = \left((n-1)^{q_2}, (f_2-1)^{f_2-q_2}, f_2^1, 0^{n-f_2-1}\right)$. Since $f_2(f_2+1) \leq 2m < nf_2$, we have $f_2 \leq n-1$. Hence, for each $k \in \langle q_2 \rangle$,

$$\sum_{i=1}^k \overline{d_i^{(2)}} = \sum_{i=1}^k d_i^{(2)} + k(n-1-f_2) \geq \sum_{i=1}^k d_i^{(2)}.$$

In addition, for each $k \in \langle f_2 \rangle - \langle q_2 \rangle$,

$$\begin{aligned}\sum_{i=1}^{k} \overline{d_i^{(2)}} &= \sum_{i=1}^{k} d_i^{(2)} + q_2(n-1-f_2) - (k-q_2) \\ &\geq \sum_{i=1}^{k} d_i^{(2)} + (n-f_2)q_2 - f_2 \\ &= \sum_{i=1}^{k} d_i^{(2)} + 2m - f_2(f_2+1) \geq \sum_{i=1}^{k} d_i^{(2)}.\end{aligned}$$

By Theorem 1.1, $\pi_2 \in G_{n,m,f_2}$.

Now suppose that $\pi = (d_1, d_2, \cdots, d_n) \in G_{n,m,f_2}$. Then there exists an interger $l \geq f_2+1$ such that $d_l \geq q_2 > d_l + 1$. Hence $d_k \geq d_k^{(2)}$ for each $k \in \langle l \rangle$ and $d_k \leq d_k^{(2)}$ for each $k \in \langle n \rangle - \langle l \rangle$,. By Lemma 2.1, $\pi \succ \pi_2$.

Subcase 2.2: $r_2 > 0$. In this case, $\overline{\pi}_2 = \left((n-1)^{q_2}, (f_2+r_2-1)^1, (f_2-1)^{f_2-q_2-1}, f_2^1, 0^{n-f_2-1}\right)$. It is easy to prove that $\pi_2 \in G_{n,m,f_2}$ and $\pi \succ \pi_2$ for any $\pi \in G_{n,m,f_2}$. □

Theorem 2.5 *Let f_3 be an interger which satisfies $nf_3 < 2m \leq 2nf_3 - f_3(f_3+1)$ and $2m - nf_3 = f_3 q_3 + r_3$, where q_3 and r_3 are nonnegative integers and $r_3 < f_3$. Then $\pi_3 = \left((f_3+q_3+1)^{r_3}, (f_3+q_3)^{f_3-r_3}, f_3^{n-f_3}\right)$ is the least element of G_{n,m,f_2}.*

Proof Clearly $f_3 > 0$. Since $nf_3 < 2m \leq 2nf_3 - f_3(f_3+1)$, we have $0 < 2m - nf_3 = f_3q_3 + r_3 \leq nf_3 - f_3(f_3+1)$. Hence, $nf_3 \geq f_3(f_3+q_3+1) + r_3$. Therefore, $f_3 + q_3 + 1 \leq n-1$ if $r_3 > 0$ and $f_3 + q_3 \leq n-1$ if $r_3 = 0$. Thus $\pi_3 \in NS_n$, and $f(\pi_3) = f_3$. We distinguish the following cases:

Case 1. $q_3 = n - f_3 - 1$. In this case, we have $2nf_3 - f_3(f_3+1) \geq 2m = nf_3 + f_3(n - f_3 + 1) + r_3 = 2nf_3 - f_3(f_3+1) + r_3$. Hence, $r_3 = 0$ and $\pi_3 = \left((n-1)^{f_3}, f_3^{n-f_3}\right)$. By the matrix $\overline{A(\pi_3)}$, we obtain $\overline{\pi}_3 = \pi_3$. By Theorem 1.1, $\pi_3 \in G_{n,m,f_3}$. By Theorems 1.2 and 1.3, π_3 is a maximal element in G_{n,m,f_3}.

Now suppose $\pi = (d_1, d_2, \cdots, d_n) \in G_{n,m,f_3}$. Then $f(\pi_3) = f_3$ and $d_k^{(3)} \geq d_k$ for each $k \in \langle n \rangle$, where $\pi_3 = (d_1^{(3)}, d_2^{(3)}, \cdots, d_n^{(3)})$. Hence, it follows by $\sigma(\pi_3) = \sigma(\pi)$ that $\pi = \pi_3$. So $G_{n,m,f_3} = \{\pi_3\}$. Hence π_3 is also the least element of G_{n,m,f_3}.

Case 2. $q_3 \leq n - f_3 - 2$. In this case, $f_3 + q_3 + 1 \leq n-1$ and $r_3 \geq 0$. By the matrix $\overline{A(\pi_3)}$, we have $\overline{\pi}_3 = \left((n-1)^{f_3}, f_3^{q_3}, r_3^1, 0^{n-f_3-q_3-1}\right)$. Hence, $\sum_{i=1}^{k} \overline{d_i^{(3)}} \geq \sum_{i=1}^{k} d_i^{(3)}$ for each $k \in \langle f_3 \rangle$. By Theorem 1.1, $\pi_3 \in G_{n,m,f_3}$. Moreover, it is easy to prove that π_3 is the least element of G_{n,m,f_3}. □

Combining Theorems 2.3, 2.4 and 2.5, we have the following

Theorem 2.6 *If $G_{n,m,f} \neq \emptyset$, then $G_{n,m,f}$ has only one minimal element and the minimal element is the least element in $G_{n,m,f}$.* □

3. Enumeration

In this section we consider the enumerations concerning the minimal elements in various posets of graphic sequences.

Theorem 3.1 *Let $g(n,*,f)$ be the number of minimal elements in $G_{n,*,f}$. Then*

$$g(n,*,f) = nf - f(f+1) + 1. \tag{1}$$

Proof By Theorem 2.2, we have $G_{n,*,f} = \bigcup_{k\le m\le l} G_{n,m,f}$, where $k = \frac{1}{2}f(f+1)$ and $l = nf - \frac{1}{2}f(f+1)$. From Theorem 2.6, for each $m \in \langle l\rangle - \langle k-1\rangle$, there exists an unique minimal element in $G_{n,m,f}$. Clearly, for any $m_1, m_2 \in \langle l\rangle - \langle k-1\rangle, m_1 \neq m_2$, the minimal elements of $G_{n,m_1,f}$ and $G_{n,m_2,f}$ are not comparable. Thus (1) holds. □

Theorem 3.2 *Let $g(n,m,*)$ be the number of minimal elements in $G_{n,m,*}$. Then $g(n,m,*) = 1$.*

Proof Denote $F = \{f : f(f+1) \le 2m \le 2nf - f(f+1)\}, F_1 = \{f : f(f+1) \le 2m < nf\}$ and $F_2 = \{f : nf < 2m \le 2nf - f(f+1)\}$. Then by Theorem 2.2, $G_{n,m,*} = \bigcup_{f\in F} G_{n,m,f}$. Suppose $f_1, f_2 \in F_1$ and $f_1 > f_2$. Then by Theorem 2.4, $\pi_1 = \left(f_1^{f_1}, (q_1+1)^{r_1}, q_1^{n-f_1-r_1}\right)$ and $\pi_2 = \left(f_2^{f_2}, (q_2+1)^{r_2}, q_2^{n-f_2-r_2}\right)$ are the least elements in G_{n,m,f_1} and G_{n,m,f_2} respectively. Since $\sigma(\pi_1) = \sigma(\pi_2) = 2m$, we have

$$(f_1 - f_2)(f_1 + f_2 - q_2 - 1) + (n - f_1)(q_1 - q_2 - 1) + r_1 + (n - f_2 - r_2) = 0. \tag{i}$$

Observe that $f_1 - f_2 > 0, f_2 > q_2, n - f_2 > r_2$ and $r_1 \ge 0$. Hence, $q_1 \le q_2$. If $q_1 = q_2$, then by $(i), r_2 = (f_1-f_2)(f_1+f_2-q_2)+r_1 > f_1+f_2-q_2+r_1$. Hence $(f_2+r_2)-(f_1+r_1) > 2f_2-q_2 > 0$. Denote $\pi_1 = (d_1^{(1)}, d_2^{(1)}, \cdots, d_n^{(1)})$ and $\pi_2 = (d_1^{(2)}, d_2^{(2)}, \cdots, d_n^{(2)})$. Then $d_k^{(1)} \ge d_k^{(2)}$ for each $k \in \langle f_1+r_1\rangle$ and $d_k^{(1)} \le d_k^{(2)}$ for each $k \in \langle n\rangle - \langle f_1+r_1\rangle$. By Lemma 2.1, $\pi_1 \succ \pi_2$. If $q_1 < q_2$, then $d_k^{(1)} \ge d_k^{(2)}$ for each $k \in \langle f_1\rangle$ and $d_k^{(1)} \le d_k^{(2)}$ for each $k \in \langle n\rangle - \langle f_1\rangle$. Consequently, $\pi_1 \succ \pi_2$. From now on we assume that $f_2 = min\{f \in F_1\}$.

Next suppose that $f_3, f_4 \in F_2$ and $f_3 > f_4$. By Theorem 2.5, $\pi_3 = \left((f_3+q_3+1)^{r_3}, (f_3+q_3)^{f_3-r_3}, f_3^{n-f_3}\right)$ and $\pi_4 = \left((f_4+q_4+1)^{r_4}, (f_4+q_4)^{f_4-r_4}, f_4^{n-f_4}\right)$ are the least elements in G_{n,m,f_3} and G_{n,m,f_4} respectively. Since $f_3 > f_4, r_4 < f_4 < f_3$ and

$$(n+q_3)f_3 \le 2m = (n+q_3)f_3 + r_3 = (n+q_4)f_4 + r_4 < (n+q_4+1)f_3,$$

we have $q_3 \le q_4$. Hence $f_4 + q_4 \ge f_3 + q_3 + 1$. Denote $\pi_3 = (d_1^{(3)}, d_2^{(3)}, \cdots, d_n^{(3)})$ and $\pi_4 = (d_1^{(4)}, d_2^{(4)}, \cdots, d_n^{(4)})$. Then $d_k^{(4)} \ge d_k^{(3)}$ for each $k \in \langle f_4\rangle$ and $d_k^{(4)} \le d_k^{(3)}$ for each $k \in \langle n\rangle - \langle f_4\rangle$. By Lemma 2.1, $\pi_4 \succ \pi_3$. In the following, we assume $f_3 = max\{f \in F_3\}$. Consider the cases as follows:

Case 1. There exists an integer $f_1 \in F$ such that $2m = nf_1$. In this case, $\pi_1 = (f_1^n)$ is the least element of G_{n,m,f_1}. Clearly, $f_2 > f_1$. Since $0 = nf_1 - (f_2^2 + (n-f_2)q_2 + r_2) = f_2(f_1 - f_2) + (n - f_2)(f_1 - q_2) - r_2$, we have $f_1 > q_2$. Hence $d_k^{(2)} \ge d_k^{(1)}$ for each $k \in \langle f_2\rangle$ and $d_k^{(2)} \le d_k^{(1)}$ for other k. Thus $\pi_2 \succ \pi_1$. Next, $f_3 < f_1$. If $f_3 + q_3 + 1 \le f_1$, then by a similar method, we can prove that $\pi_3 \succ \pi_1$.

Case 2. There is no integer f such that $2m = nf$. In this case, $f_3 < f_2$. By the choice of f_2 and f_3, we have $f_2 = f_3 + 1$, and $\pi_2 = ((f_3+1)^{r_3}, (f_3+1)^{f_3-r_3}, (f_3+1)^1, (q_2+1)^{r_2}, q_2^{n-f_3-r_2-1})$, $\pi_3 = \left((f_3+q_3+1)^{r_3}, (f_3+q_3)^{f_3-r_3}, f_3^1, f_3^{r_2}, f_3^{n-f_3-r_3-1}\right)$. If $q_2 \geq f_3$, then $\sum_{i=1}^{f_3} d_i^{(2)} - \sum_{i=1}^{f_3} d_i^{(3)} = \sum_{i=f_3+1}^{n} d_i^{(3)} - \sum_{i=f_3+1}^{n} d_i^{(2)} < 0$, i.e., $f_3(f_3+1) < f_3(f_3+q_3) + r_3 < (f_3+1)(f_3+q_3)$.

Hence $q_3 > 0$. Consequently, $d_k^{(3)} \geq d_k^{(2)}$ for each $k \in \langle f_3 \rangle$ and $d_k^{(3)} \leq d_k^{(2)}$ for other k. By Lemma 2.1, $\pi_3 \succ \pi_2$. Similarly, if $q_3 < f_3$, we can prove that $q_3 < 2$ and $\pi_2 \succ \pi_3$. □

Theorem 3.3 *Let $g(n)$ be the number of minimal elements in G_n. Then $g(n) = \binom{n}{2} + 1$.*

Proof *Clearly, $G_n = \bigcup_{0 \leq m \leq \binom{n}{2}} G_{n,m,*}$. By Theorem 3.2,*

$$g(n) = \sum_{m=0}^{\binom{n}{2}} g(n,m,*) = \sum_{m=0}^{\binom{n}{2}} 1 = \binom{n}{2} + 1.$$ □

Theorem 3.4 *For a given positive integer m, let $g(*,m,*)$ be the number of minimal elements in the pose $G_{*,m,*}$. Then $g(*,m,*) = 1$.*

Proof *Clearly, $\pi_0 = (1^{2m}) \in G_{*,m,*}$ is the least element in $G_{*,m,*}$.* □

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可图序列偏序集的极小元

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摘 要: n 项非增非负整数序列 π 是可图的，若 π 是某个 n 阶简单图的度序列. 所有项和为 $2m$ 、迹为 f 的 n 项可图序列的集合 $G_{n,m,f}$ 在优超关系下是一个偏序集. 本文刻划了偏序集 $G_{n,m,f}$ 的极小元，并确定各种可图序列偏序集中极小元的个数.