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Global Attractor of a Spatially Discretized Reaction Diffusion System with Hamiltonian Structure *

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In recent years there has been a growing interest on discrete models, see e.g.[1].[2]. We consider a reaction diffusion equation which space-independent system is a Hamilton system with one degree of freedom:

$$\begin{cases} u_t = u_{xx} + v, & 0 < x < 1, t > 0, \\ v_t = v_{xx} - u + u^2, & 0 < x < 1, t > 0, \\ u(0,t) = u(1,t) = 0, v(0,t) = v(1,t) = 0, & t > 0, \\ u(0,x) = u_0, v(0,x) = v_0, & 0 < x < 1. \end{cases} \tag{1}$$

For Hamilton structured reaction diffusion systems, the local dynamics is structurally unstable while most conditions imposed on gradient-structured system are not gradient-structured equation: $u_t = \gamma\Delta u - f(u)$ with five restrictions on function ([1]), Huang and Lu studied the existence of global attractor of Henon-Heiles hamilton system,([2])but functions $f(v) = v$ and $g(u) = -u + u^2$ in system (1) do not satisfy the five restrictions on the functions.

Let us discretize spatial variable of (1). The discretized negative Laplacian operator $-\Delta$ with Dirichlet boundary condition by using the finite difference is set to be A,

$$A = \begin{pmatrix} 2 & -1 & 0 & \cdots & \cdots & 0 \\ -1 & 2 & -1 & \cdots & \cdots & 0 \\ 0 & -1 & 2 & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \vdots \\ & & & -1 & 2 & -1 \\ 0 & \cdots & \cdots & 0 & -1 & 2 \end{pmatrix}, \quad \begin{aligned} u(t) &= (u_1, u_2, \cdots, u_{m-1})^{\mathrm{T}}, \\ v(t) &= (v_1, v_2, \cdots, v_{m-1})^{\mathrm{T}}, \\ u^2(t) &= (u_1^2, u_2^2, \cdots, u_{m-1}^2)^{\mathrm{T}}, \\ u_0 &= (u_1^0, u_2^0, \cdots, u_{m-1}^0)^{\mathrm{T}}, \\ v_0 &\doteq (v_1^0, v_2^0, \cdots, v_{m-1}^0)^{\mathrm{T}}, \end{aligned}$$

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where $u_i(t) = u(\frac{i}{m}, t)$, $v_i(t) = v(\frac{i}{m}, t), i = 1, 2, \cdots, m-1$, and A is an $(m-1) \times (m-1)$ sysmetric and positive definite matrix. And its eigenvalues satisfying: $\frac{1}{c_p^2} \le \lambda_i \le \frac{c_0}{h^2}$, $(mh = 1)$

With the above nations ,the spatially finite difference discretized version of (1) can be written by:

$$\begin{cases} u_t = -m^2 Au + v, \\ v_t = -m^2 Av - u + u^2, \\ u(0) = u_0, v(0) = v_0, \end{cases} \tag{2}$$

and we further introduce $P(t) = (u,v)^T$, $P_i(t) = (u_i, v_i)^T$, $u_i, v_i \in R^{m-1}$, $B = \mathrm{diag}(m^2 A, m^2 A)$. Then, the matrix B is positive define, and we define inner products and norms respectively:

$(u,v) = u^T v$, $\mid u \mid^2 = (u,u)$; $(u,v)_A = u^T Av$,
$\|u\|_A^2 = (u,u)_A, (P_1, P_2) = P_1^T P_2$, $\mid P \mid^2 = (P,P)$,
$(P_1, P_2)_B = P_1^T B P_2$, $\|P\|_B^2 = (P,P)_B$.

It is easy to obtain the following lemma:

Lemma *1)* $\mid P \mid^2 = \mid u \mid^2 + \mid v \mid^2$; *2)* $\|P\|_B^2 = m^2(\|u\|_A^2 + \|v\|_A^2)$;
3) $\mid u \mid \le c_p \|u\|_A$, $\|u\|_A^2 \le \frac{c_0}{h^2} \mid u \mid^2$;
4) $\|u\|_A \le c_p \mid Au \mid$, $\mid Au \mid^2 \le \frac{c_0}{h^2} \|u\|_A^2$.

Theorem 1 $\Omega = \{(u,v)^T \in R^{2m-2}, \mid u \mid^2 + \mid v \mid^2 \le 2c$, $0 < c < \frac{1}{2}\frac{m^4}{c_p^4}\}$ *is an invariant region for (2).*

We denote $E_0 = L^2(\Omega, \mid \cdot \mid)$, $E_1 = L^2(\Omega, \| \cdot \|_B)$. *Similarly, we can obtain the theorems:*

Theorem 2 *For any* $P_0 = (u_0, v_0)^T \in E_0$ *, there is a unique global solution* $P(t) = (u,v)^T$ *for (2),the semigroup* $\{S(t)\}_{t \ge 0}$ *defined by* $S(t)P_0 = P(t), (t \ge 0)$ *is continuous in* E_0.

Theorem 3 *There exist constants* ρ_1 *and* ρ_2 *such that* $B_1 = \{P \in E_0, \mid P \mid \le \rho_1\}$ *and* $B_2 = \{P \in E_1, \|P\|_B \le \rho_2\}$ *are absorbing sets for the semigroup* $\{S(t)\}_{t \ge 0}$ *in* E_0 *and* E_1 *respectively.*

Theorem 4 *There exist maximal attractors* G_0 *and* G_1 *in* E_0 *and* E_1 *respectively,* $G_0 = \omega(B_0), G_1 = \omega(B_1)$ *where* B_0, B_1 *are the bounded set in* E_0 *and* E_1 *respectively.* G_0 *and* G_1 *are connected.*

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