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Structural Theorem for Completely Simple Γ-Semigroups and Γ-Semigroups with a Completely Simple Γ-Kernel *

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Abstract: We prove structural theorem for completely simple Γ-semigroups and semigroups with a completely simple Γ-kernel.

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1. Introduction

Let M and Γ be two nonempty sets. M is called a Γ-semigroup if the following conditions are satisfied (1)$(a\alpha b)\in M$ and $\alpha\in\Gamma$,(2)$a\alpha b\beta c = a\alpha(b\beta c)$ for any $a,b,c\in M,\alpha,\beta\in\Gamma$. A right(left) Γ-ideal of a Γ-semigroup is a nonempty subset I of M such that $I\Gamma M\subseteq I(M\Gamma I \subseteq I)$. If I is both right and left Γ-ideal, then we call I is a Γ-ideal of M. A Γ-semigroup M is called a Γ-semigroup with a completely simple Γ-kernel I if M has a completely simple Γ-ideal I. An element of a Γ-semigroup M is called a α-idempotent if $e\alpha e = e$ for $\alpha\in\Gamma$. Let E be the set of all idempotents of Γ-semigroup M. We define the partial order relation ω on E by $e\omega f$ if and only if $(\alpha,\beta\in\Gamma)(e\alpha e = e, f\beta f = f, e = e\alpha f = f\beta e)$. Define $a.b$ in M by $a.b = a\alpha b$ for $a,b\in M$, then M is a semigroup. Denote this semigroup by M_α and call it the interrelated semigroup of M. Let M_1 be a Γ_1-semigroup and M_2 be a Γ_2-semigroup. A pair of mappings $f_1{:}M_1\to M_2$ and $f_2{:}\Gamma_1\to\Gamma_2$ is said to be a homomorphism from (M_1,Γ_1) to (M_2,Γ_2), if $(a\alpha b)f_1 = (af_1)(\alpha f_2)(bf_1)$ for all $a,b\in M_1$, and $\alpha\in\Gamma_1$.If f_1 and f_2 are both bijections then (f_1,f_2) is said to be an isomorphism of (M_1,Γ_1) onto (M_1,Γ_2).

Let G be a group and I,$\bigwedge$ be index sets and Γ be the collection of some $\bigwedge\times I$ matrices over G^0,the group with zero. Let μ^0 be the set of elements $(a)_{i\lambda}$ where $i\in I,\lambda\in\bigwedge$ and $(a)_{i\lambda}$ is the $I\times\bigwedge$ matrix over G^0 having a in the i-th row and λ-th column, its remaining entries being zero. The expression$(o)_{i\lambda}$ will be used to denote $I\times\bigwedge$ zero matrix. for any $(a)_{i\lambda},(b)_{j\mu},(c)_{k\gamma}\in\mu^0$. Then it is easy to verify that

$$[(a)_{i\lambda}\alpha(b)_{jk}]\beta(c)_{k\gamma} = (a)_{i\lambda}\alpha[(b)_{j\mu}\beta(c)_{k\gamma}].$$

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Thus μ^0 is a Γ-semigroup.

We shall call Γ the sandwich matrix set and μ^0 Rees $I \times \wedge$ matrix Γ-semigroups over G^0 with sandwich matrix set Γ and denote it by $\mu^0(G : I, \wedge; \Gamma)$. Sandwich matrix set Γ is called regular if for each row $i \in I$ there exists a matrix $\alpha = (p^{\alpha}_{\mu i}) \in \Gamma$ and for each column $\lambda \in \wedge$ there exists a martix $\beta = (q^{\beta}_{\lambda i}) \in \Gamma$ such that $\lambda = (p^{\alpha}_{\mu i})$ has at least one nonzero entry in i-th row and $\beta = (q^{\alpha}_{\mu j})$ has at least non-zero entry in λ-th column, then $\mu^0(G : I, \wedge; \Gamma)$ is called a regular Rees $I \times \wedge$ matrix Γ-semigroup and it is denoted by $\mu^0[G : I, \wedge; \Gamma]$. Let $\mu[G : I, \wedge; \Gamma]$ be the set of all elements $(a)_{i\lambda}$ where $i \in I, \lambda \in \wedge$ and $(a)_{i\lambda}$ is the $I \times \wedge$ matrix over G and Γ be the collection of some $\wedge \times I$ matrix over G, then $\mu[G : I, \wedge; \Gamma]$ is a completely simple Γ-semigroup. It is clear that the inter-related semigroup $\mu^0[G : I \wedge; \Gamma]_{\alpha}$ of $\mu^0[G : I, \wedge; \Gamma]$ may be not a completely 0-simple semigroup. In 1989, Seth.A. shown in [1] that a Γ-semigroup is a completely 0-simple Γ-semigroup if and only if it is isomorphic with a $\mu^0[G : I, \wedge; \Gamma]$. In this paper we first prove that a Γ-semigroup is compeletely simple if only if it is isomorphic whith a $\mu[G : I, \wedge; \Gamma]$, then we give a structural theorem for semigroups with a completely simple Γ-kernel. Unless otherwise defined our notations will follow that of [1-4].

2. Main result

Lemma 2.1[3] *Let M be a Γ-semigroup. Then the following conditions are equivalent:*

(1) *M is a completely simple Γ-semigroup;*
(2) *M_{α} is completely simple for any $\alpha \in \Gamma$;*
(3) *M_{α} is completely simple for some $\alpha \in \Gamma$;*
(4) *M_{α} is regular and every idempotent of M_{α} is minimal for some $\alpha \in \Gamma$.*

Theorem 2.2 *A Γ-semigroup is completely simple if and only if it is isomorphic to a $\mu[G : I, \wedge; \Gamma]$.*

Let M be a completely simple Γ-semigroup. Then $M \cup 0$ is a completely 0-simple Γ-semigroup where $0 \alpha M = 0$ for any $\alpha \in \Gamma$. By the Rees theorem for Γ-semigroups in [1] and for semigroup in [5] and the Lemma 2.1, it is not hard to obtain the results of theorem 2.2.

Let M be a nonempty set and Γ be a set of operations defined on M. M is called a partial Γ-semigroup if $x\alpha(y\beta z) = (x\alpha y)\beta z$ for any $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Let $\mu[G : I, \wedge; \Gamma]$ be a Rees matrix Γ-semigroups over a group G, and M be a partial Γ'-semigroyup such that $G \times I \times \wedge \cap M = \varphi$, where there exists a bijection $\psi : \alpha \to \alpha'$ from Γ onto Γ'.

Let $\xi^{\alpha} : p \to \xi^{\alpha}_p$ be a mapping from M into the semigroup T(I) of all the mappings of I into itself, and $\eta^{\alpha} : p \to \eta^{\alpha}_p$ be a mapping from M into the semigroup T($\wedge$) of all the mappings of $\wedge$ into itself where $\alpha \in \Gamma$, $p, q \in M$.

(i) If $p\alpha' q \in M$, then $\xi^{\beta}_{p\alpha' q} = \xi^{\beta}_q \xi^{\alpha}_p$ and $\eta^{\beta}_{p\alpha' q} = \eta^{\beta}_p \eta^{\alpha}_q$ for ang $\beta \in \Gamma$.

(ii) If $p\alpha' q \in M$, then $\xi^{\alpha}_q \xi^{\alpha}_p = \xi^{\beta}_q \xi^{\alpha}_p = const$ and $\eta^{\beta}_p \eta^{\alpha}_q = \eta^{\alpha}_p \eta^{\alpha}_q = const$ for any $\beta \in \Gamma$.

Again, let $\varphi_{\alpha} : M \times I \to G$ be a mapping for $\alpha \in \Gamma$ such that

(iii) If $p\alpha' q \in M$, then $\varphi_{\beta}(p\alpha' q, i) = \varphi_{\alpha}(p, i\xi^{\beta}_q)\varphi_{\beta}(q, i)$ for any $\beta \in \Gamma$.

(iv) $p^{\alpha}_{\lambda i \xi^{\beta}_p} \varphi_{\beta}(p, i)(p^{\beta}_{\lambda \eta^{\alpha}_p i})^{-1}$ does not depend on i and β.

The term from (iv) is denote by $\Psi_\alpha(p,\lambda)$.

Let us define a mulitiplication set Γ'' on $\sum = G\times I\times\bigwedge\cup M$ with

(1) There exists a bijiection $\psi_2:\alpha''\to\alpha$ from Γ'' onto Γ(there exists a bijiection $\psi_2':\alpha''\to\alpha'$ from Γ'' onto Γ', too)

(2) $(a;i,\lambda)\alpha''(b;j,\mu)=(a;i,\lambda)\alpha(b;j,\mu)=(ap^\alpha_{\lambda j}b;i,\mu)$ for $(a;i,\lambda),(b;j,\mu)\in G\times I\times\bigwedge$.

(3) $p\alpha''(a;i,\lambda)=(\varphi_\alpha(p,i)a;i\xi^\alpha_p,\lambda)$ for $p\in M$ and $(a;i,\lambda)\in G\times I\times\bigwedge$.

(4) $(a;i,\lambda)\alpha'p=(a\Psi_\alpha(p,\lambda);i,\lambda\eta_p)$ for $p\in M$ and $(a;i,\lambda)\in G\times I\times\bigwedge$.

(5) If $p\alpha'q\in M$, then $p\alpha'q=p\alpha'q\in\sum=G\times I\times\bigwedge\cup M$ for $p,q\in M$.

(6) If $p\alpha'q\in M$, then $p\alpha'q=(\varphi_\alpha(p,i\xi^\alpha_q)\varphi_\alpha(q,i)(p^\alpha_{\lambda\eta^\alpha_p\eta^\alpha_q i})^{-1};i\xi^\alpha_q\xi^\alpha_p,\lambda\eta^\alpha_p\eta^\alpha_q)$, for $p,q\in M$ and $i\in M$ and $i\in I,\lambda\in\bigwedge$.

We will denote$\sum$ with a multiplication set Γ'' by $\mu[G:I,\wedge;\Gamma']$.

Theorem 2.3 *$\mu[G:I,\bigwedge;\Gamma'';M,\varphi,\Psi,\xi,\eta]$ is a Γ''-semigroup with a completely simple Γ''-kernel $\mu[G:I,\bigwedge;\Gamma'']$.*

Proof It is obvious that $a\alpha''b\in\mu[G:I,\wedge;\Gamma'',M,\varphi,\Psi,\eta]$ for any $a,b\in\mu[G:I,\bigwedge;\Gamma'',M,\varphi,\Psi,\eta]$ and $\alpha''\in\Gamma$. By (2) and (5) we can get $(a\alpha''b)\beta''c=a\alpha''(b\beta''c)$ on M, or $G\times I\times\bigwedge$.

Let $p,q\in M,(a;i,\lambda),(b;j,\mu)\in G\times T\times\wedge$, and $\alpha'',\beta''\in\Gamma''$. Then by (3) and (2) we have

$$[p\alpha''(a;i,\lambda)]\beta''(b;j,\mu)=\cdots=(\varphi_\alpha(p,i)ap^\beta_{\lambda j}b;i\xi^\alpha_p,\mu).$$

By (4) and (2) we can obtain

$$[(a;i,\lambda)\alpha''(b;j,\mu)]\beta''p=(ap^\alpha_{\lambda j}b;i,\mu)\beta''p=\cdots=(ap^\alpha_{\lambda j}b\Psi_\beta(p,\mu);i,\mu\eta^\beta_p).$$

i.e., $[(a;i,\lambda)\alpha''(b;j,\mu)]\beta''p=(a;i,\lambda)\alpha''[(b;j,\mu)\beta''p]$. By (3),(4) and (2) we have that

$$[(a;i,\lambda)\alpha'']\beta''(b;j,\mu)=(a\Psi_\alpha(p,i);i,\lambda\eta^\alpha_p)\beta''(b;j,\mu)=\cdots=(a\Psi_{p\lambda})p^\beta_{\lambda\eta^\alpha_p j}b;i,\mu)$$

i.e., $[(a;i,\lambda)\alpha''p]\beta''(b;j,\mu)=(a;i,\lambda)\alpha''[p\beta''(b;j,\mu)]$. Similarly, we have

$$[p\alpha''(a;i,\lambda)]\beta''q=p\alpha''[(a;i,\lambda)\beta''q].$$

Let $p,q\in M,p\alpha''q\in M,\alpha'',\beta''\in\Gamma$ and $x=(a;i,j)$. Then by $(i),(iii)$ and (3) we get

$$[p\alpha''q]\beta''x=(p\alpha'q)\beta(a;i,j)=\cdots=(\varphi_\alpha(p,i\xi^\beta_q)\varphi_\beta(q,i)a;i\xi^\beta_p\xi^\alpha_p,j)$$

i.e., $[p\alpha''q]\beta''x=p\alpha''[q\beta''x]$. It follows from (ii), (iii), (iv) and (4) that

$$x\beta''(p\alpha''q)=(a;i,j)\beta''(p\alpha'q)=\cdots=(a\Psi_\beta(p,j)\Psi_\alpha(q,j\eta^\beta_p;i,j\eta^\beta_p\eta^\alpha_q),$$

$$[x\beta''p]\alpha''q=[(a;i,j)\beta''p]\alpha''q=(a\Psi_\beta(p,j)\Psi_\alpha(q,j\eta^\beta_p;i,j\eta^\beta_p\eta^\alpha_q),$$

i.e., $x\beta''(p\alpha''q)=\Psi_\beta(p,j)\Psi_\alpha(q,j\eta^\beta_p)$. Let $p,q\in M,p\alpha''q\in M$ and $(x;k,l)\in G\times I\times\bigwedge$. In this case

$$\xi^\alpha_q\xi^\alpha_p=\xi^\alpha_q\xi^\alpha_p=\text{const},\quad\eta^\beta_p\eta^\alpha_q=\eta^\alpha_p\eta^\alpha_q=\text{const}.$$

for any $\beta \in \Gamma$. It remains to prove that $\varphi_\alpha(p, k\xi_q^\beta)\varphi_\beta(q,k)(p^\beta_{\lambda\eta_p^\alpha\eta_q^\alpha k})^{-1}$ does not dependent on k and $\beta \in \Gamma$, and

$$\varphi_\alpha(p, k\xi_q^\beta)\varphi_\beta(q,k)(p^\beta_{\lambda\eta_p^\alpha\eta_q^\alpha k})^{-1} = \cdots = (p^\alpha_{\lambda k\xi_q^\beta\xi_p^\alpha})^{-1}\Psi_\alpha(p,\lambda)\Psi_\alpha(q,\lambda\eta_p^\alpha).$$

Since $k\xi_q^\beta\xi_p^\alpha(\xi_q^\beta\xi_p^\alpha = \text{const})$ and $\Psi_\alpha(p,\alpha)\Psi_\alpha(q,\alpha\eta_p^\alpha)$ not depend on k and β, it follows that $\varphi_\alpha(p,k\xi_q^\beta)\varphi_\beta(q,k)(p^\beta_{\lambda\eta_p^\alpha\eta_q^\alpha})^{-1}$ does not depend on k and β, too. Hence

$$\varphi_\alpha(p, k\xi_q^\beta)\varphi_\beta(q,k)(p^\beta_{\lambda\eta_p^\alpha\eta_q^\alpha k})^{-1} = \varphi_\alpha(p, i\xi_q^\alpha)\varphi_\alpha(q,k)(p^\alpha_{\lambda\eta_p^\alpha\eta_q^\alpha i})^{-1}.$$

By (6) and the above equivality we have

$$p[\alpha'' q]\beta''(x;k,l) = \cdots = (\varphi_\alpha(p,k\xi_q^\beta)\varphi_\beta(q,k)x; i\xi_p^\alpha\xi_q^\alpha, l).$$

On the other hand

$$p\alpha''[q\beta''(x;k,l)]p\alpha''(\varphi_\beta(q,k)x; k\xi_q^\beta, l) = (\varphi_\alpha(p,k\xi_p^\beta)\varphi_\beta(q,k)x; k\xi_q^\beta\xi_p^\alpha, l),$$

$$i\xi_q^\alpha\xi_p^\alpha = k\xi_q^\beta\xi_p^\alpha.$$

Thus $[p\alpha'' q]\beta''(x;k,l) = p\alpha''[q\beta''(x;k,l)]$. Again since

$$\begin{aligned}(x;k,l)\beta''[p\alpha'' q] &= (x;k,l)\beta''(\varphi_\alpha(p,i\xi_q^\alpha)\varphi_\alpha(q,i)(p^\alpha_{\lambda\eta_p^\alpha\eta_q^\alpha i})^{-1}; i\xi_q^\alpha\xi_p^\alpha, \lambda\eta_p^\alpha\eta_q^\alpha)\\ &= \cdots = (x\Psi_\beta(p,l)\Psi_\alpha(q,l\eta_p^\beta); k, \lambda\eta_p^\alpha\eta_q^\alpha),\end{aligned}$$

and

$$[(x;k,l)\beta'' p]\alpha'' q = (x\Psi_\beta(p,l); k, l\eta_p^\beta)\alpha'' q = (x\Psi_\beta(p,l)\Psi(q,l\eta_p^\beta); k, l\eta_p^\alpha\eta_q^\alpha).$$

It is clear that $\mu[G:I,\wedge;\Gamma''; M,\varphi,\Psi,\xi,\eta]$ is a Γ''-semigroups. But it is clear that

$$\mu[G:I,\wedge;\Gamma''; M,\varphi,\Psi,\xi,\eta]$$

has a completely simple Γ-kernel $\mu[G:I,\bigwedge;\Gamma]$.

Theorem 2.4 *A Γ-semigroup S has a Γ-ideal which is a completely simple Γ subsemigroup of S if and only if S is isomorphic to some $\mu[G:I,\bigwedge;\Gamma''; M,\varphi,\Psi,\xi,\eta]$.*

Proof Let a Γ-semigroup S have a Γ-ideal k which is a completely simple semigroup. Then $M = S\backslash K$ is a partial Γ-semigroup such that $S = K \cup M \cong \mu[G:I,\bigwedge;\Gamma]\cup M$ (K is isomorphic to $\mu[G:I,\bigwedge;\Gamma]$ by theorem 2.4).
Let $p\in M, (1;i,l)\in K$ and $\alpha,\beta\in\Gamma$. We have

$$p\alpha(1;i,l) = (g;k,s)\in K$$

where $g = \varphi_\alpha(p;i,l), k = i\xi^\alpha_{p.l}, s = l\eta^\alpha_{p.i}$. So

$$\begin{aligned}(\varphi_\alpha(p;i,l); i\xi_{p.l}, l\eta^\alpha_{p.i}) &= p\alpha[(1;i,l)\alpha((p^\alpha_{lk})^{-1};k,l)] = \cdots\\ &= (\varphi_\alpha(p;i,l)p^\alpha_{l\eta_{p.i}}(p^\alpha_{lk})^{-1}; i\xi^\alpha_{p.l}, l).\end{aligned}$$

Hence $l\eta_{p.i} = l$, so that $p\alpha(1;i,l) = (\varphi_\alpha(p;i,l); i\xi^\alpha_{p.l}, l)$.

Furthermore $(\varphi_\alpha(p;i,\lambda); i\xi_{p.\lambda}, \lambda) = \cdots = (\varphi_\alpha(p;i,l); i\xi^\alpha_{p.l}, \lambda)$. Thus $\varphi_\alpha(p;i,\lambda) = \varphi_\alpha(p;i,l)$, i.e., φ does not depend on $\lambda \in \bigwedge$ and $i\xi^\alpha_{p.\lambda} = i\xi^\alpha_{p.l}$, i.e., ξ does not depend on $\lambda \in \bigwedge$. Hence $p\alpha(1;i,\lambda) = (\varphi_\alpha; i\xi^\alpha_p, \lambda)$, where $\varphi : M \times I \to G$ and $\xi^\alpha_p : I \to I$. Similarly

$$(1;i,\lambda)\alpha p = (\Psi_\alpha(p,\lambda); i, \lambda\eta^\alpha_p),$$

where $\Psi_\alpha : M \times \wedge \to G$, and $\eta^\alpha_p : \wedge \to \wedge$. Since $[(1;i,\lambda)\alpha p]\beta(1;i,\lambda) = \cdots = (p^\alpha_{\lambda i\xi^\alpha_p}\varphi_\beta(p,i); i, \lambda)$, so $\Psi_\alpha(p,\lambda)p^\beta_{\lambda\eta^\alpha_p i} = p^\alpha_{\lambda i\xi^\beta_p}\psi_\beta(p,i)$, i.e., $\Psi_\alpha(p,\lambda) = p^\alpha_{\lambda i\xi^\beta_p}\varphi_\alpha(p,i)(p^\beta_{\lambda\eta^\alpha_p i})^{-1}$. And the term $p^\alpha_{\lambda i\xi^\beta_p}\varphi_\beta(p,i)(p^\beta_{\lambda\eta^\alpha_p i})^{-1}$ does not depend on i and β. For $p \in M, (g;i,\lambda) \in K$ and $\alpha \in \Gamma$, we obain that

$$p\alpha(g;i,\lambda) = \cdots = (\varphi_\alpha(p,i)p^\alpha_{li}(p^\alpha_{li})^{-1}g; i\xi^\alpha_p, \lambda)$$

and

$$(g;i,\lambda)\alpha p = (g\Psi_\alpha(p,\lambda); i, \lambda\eta^\alpha_p).$$

Let $p, q \in M; p\alpha q \in M$ and $\lambda, \beta \in \Gamma$, we have that

$$(p\alpha q)\beta(1;i,\lambda) = \cdots = (\varphi_\alpha(p, i\xi^\alpha_q)\varphi_\beta(q,i); i\xi^\alpha_p\xi^\beta_q, \lambda)$$

i.e.,

$$\varphi_\beta(p\alpha q, i) = \varphi_\alpha(p, i\xi^\alpha_q)\varphi_\alpha(q,i), i\xi^\alpha_{p\alpha q} = i\xi^\beta_q\xi^\alpha_p.$$

Similarly

$$(1;i,\lambda)\beta(p\alpha q) = \cdots = (\Psi_\beta(p,\lambda)\Psi_\alpha(q,\lambda\eta^\beta_p); i, \lambda\eta^\beta_p\eta^\alpha_q).$$

Then

$$\Psi_\beta(p\alpha q,\lambda) = \Psi_\beta(p,\lambda)\Psi_\alpha(q,\lambda\eta^\beta_p)$$

and $\lambda\eta^\beta_{p\alpha q} = \lambda\eta^\beta_p\eta^\alpha_q$. Again, for $p, q \in M, p\alpha q \in M$ and $\alpha, \beta \in \Gamma$, we have

$$\begin{aligned} p\alpha q = (q;i,\lambda) &= (g;i,\lambda)\alpha((p^\alpha_{\lambda k})^{-1}; k, \lambda) = \cdots \\ &= (\varphi_\alpha(p, k\xi^\alpha_q)\varphi_\alpha(q,k)(p^\alpha_{\lambda k})^{-1}; k\xi^\alpha_q\xi^\alpha_p, \lambda). \end{aligned}$$

On the other hand

$$\begin{aligned} p\alpha q = (g,i,\lambda) &= (g;i,\lambda)\beta((p^\beta_{\lambda k})^{-1}; k, \lambda) = \cdots \\ &= (\varphi_\alpha(p, k\xi^\beta_q)\varphi_\beta(q,k)(p^\beta_{\lambda k})^{-1}; k\xi^\beta_q\xi^\alpha_p, \lambda), \end{aligned}$$

$$p\alpha q = (g;i,\lambda) = ((p^\beta_{li})^{-1}; i, l)\beta[p\alpha q] = ((p^\beta_{li})^{-1}\Psi_\beta(p,l)\Psi_\alpha(q, l\eta^\beta_p); i, l\eta^\beta_p\eta^\alpha_q).$$

Thus $\xi^\alpha_q\xi^\alpha_p = \xi^\beta_q\xi^\alpha_p, \eta^\alpha_p\eta^\alpha_q = \eta^\beta_p\eta^\alpha_q$. From this we conclude that

$$g = \varphi_\alpha(p, k\xi^\alpha_q)\varphi_\alpha(q,k)(p_{\lambda k})^{-1} = (p^\alpha_{li})^{-1}\Psi_\alpha(p,l)\Psi_\alpha(q, l\eta^\alpha_p)$$

$$i = k\xi^\alpha_q\xi^\alpha_p, \quad \lambda = \eta^\beta_p\eta^\alpha_q.$$

We have $\xi_q^\alpha \xi_p^\alpha = \text{const} = \xi_q^\beta \xi_p^\alpha, \eta_p^\alpha \eta_q^\alpha = \text{const} = \eta_p^\beta \eta_q^\alpha$ and g does not depend on k and l. Therefore

$$p\alpha q = (\varphi_\alpha(p, i\xi_p^\alpha)\varphi_\alpha(q, i)(p^\alpha_{\lambda\eta_p^\alpha\eta_q^\alpha i})^{-1}; i\xi_p^\alpha\xi_q^\alpha, \lambda\eta_p^\alpha\eta_q^\alpha).$$

So

$$S \cong \mu[G : I; \Gamma; M; \varphi, \Psi, \xi, \eta].$$

The other part of the result follows from Theorem 2.3.

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完全单 Γ- 半群的结构定理和具有完全单 Γ- 核的半群的结构定理

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摘 要: 本文给出了完全单 Γ- 半群的结构定理和具有完全单 Γ- 核的半群的结构定理.