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Note on the Paper "A Note on the SVD of Matrices" *

HUANG Li-ping
(Inst. of Math., Xiangtan Polytechnic University, Hunan 411201, China)

Abstract: We correct two errors of the paper [1].

Key words: F-algebra with involution; singular value decomposition; quaternion field.

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Let F be a field, R be an F-algebra having SVD property[2], $R^+ = \{a \in R | \bar{a} = a\}$. In [2], we proved that

Lemma 1 *Let R be an F-algebra which having SVD property. Then R^+ is a subfield in F, R^+ is a formally real field, and R^+ is Pythagorean with no zero-divisor.*

In the proof of Lemma 1 of [2], thereis this step: "For any $a \in R^+$, clearly, there exists $\sigma \in R^+ \cap F$ such that $a^2 = a\bar{a} = \sigma^2$, thus $a = \pm\sigma \in F$, is follows that $R^+ \subseteq F$".

Recently, paper [1] shows that "The proof of this step is not right over logic. For example, in real quaternion division ring, if $i^2 = j^2 = -1$, then $j \neq \pm i, \cdots$".

We show that this criticism is wrong. This step in [2] is right over logic. In fact, for any $a \in R^+$, since R has the SVD property, thus there exist 1×1 unitary matrices $u, v \in R$ such that $a = u\sigma v$, where $\sigma \in F \cap R^+$. Note that σ in center, it is easy to see that $a = \sigma uv, a^2 = a\bar{a} = \sigma uv\bar{v}\bar{u}\sigma = \sigma^2$, $a^2 - \sigma^2 = (a-\sigma)(a+\sigma) = 0$, thus $a = \pm\sigma \in F$.

Paper [1] proposed a correction of Lemma 1 as follows:

Proposition[1] *Let R be an F-algebra which have SVD property. Then $R^+ = F$, and R^+ is a real Pythagorean field.*

In fact, let C be complex field with involution to be usual conjugate. Then C is a C-algebra with involution and having SVD property. Clearly, C^+ =real number field but $C \neq C^+$. Thus the Proposition is wrong. In fact, the first step of its proof in [1]: "By the definition of involution, it is clear that $F = F1 \subset R^+$" is wrong.

References:

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Biography: HUANG Li-ping (1954-), male, professor.