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Optimal Controls of Nonlinear Evolution Inclusions with Pseudomonotone Operator *

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Abstract: In this paper, we study the optimal control problem of nonlinear differential inclusions with principle operator being pseudomonotone. First, we give some properties of solutions of certain evolution equations. Further, we prove the existence of admissible trajectories for evolution inclusions. Then, we extend the Fillipov's selection theorem and discuss a general Lagrange type optimal control problem. Finally, we present an example that demonstrates the applicability of our results.

Key words: evolution inclusion; set-valued analysis; pseudomonotone operator; upper semicontinuity; support function; parabolic system.

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1. Introduction

In recent years, the optimal control problem of nonlinear evolution inclusions has attracted the interest of several mathematicians, such as Papageorgiou([3]-[4]), Ahmed([5]), and Cesari. They studied the existence of optimal control for semilinear and quasi-linear evolution inclusions. More recently, Papageorgiou([3]) considered nonlinear evolution inclusions with principle operator being monotone and sequentially weakly continuous by utilizing the compactness of the set of admissible trajectories and the Cesari-Rockafellar reduction technique, but he made the two restrictive assumptions in the sense of weak topology. However, by replacing the upper semicontinuity with upper hemicontinuity on the orientor field in the sense of strong topology, we may aslo solve the optimal control problem.

The purpose of this paper is to modify some assumptions of [3] in the sense of strong topology and investigate the existence of optimal control for nonlinear evolution inclusions where the principle operator is pseudomonotone with the admissible set being the set of strong measurable functions, which develops the case in which principle operator

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is monotone. Moreover, our results extend that of [6] governed by evolution equations with principle operator being pseudomonotone. Our method is different from that of [3] and [6]. Namely, generalizing Fillipov's selection theorem, using some properties of convex functions and pseudomonotone operators, we overcome the difficulty by applying the compactness to deal with the existence of optimal control of evolution inclusions.

2. Preliminaries

Let T be a fixed positive number, $I = [0, T]$ the time horizon, H a separable Hilbert space, X a subspace of H carrying the structure of a separable reflexive Banach space, and X^* the dual space of X. Indentifying H with its dual, we have $X \hookrightarrow H \hookrightarrow X^*$, with all embeddings being continuous ([2]). In the literature, triple (X, H, X^*) is usually called evolution triple. By $L^p(X)$, we denote a Banach space consisting of all strongly measurable functions from I to X with L^p-norm. We denote by $\langle .,. \rangle, \langle\langle .,. \rangle\rangle$, and $(.,.)$ the duality brackets for the pairs (X, X^*), $(L^p(X), L^q(X^*))$, and the inner product of H, respectively; here, $p > 1$, and $\frac{1}{p} + \frac{1}{q} = 1$. By $||.||$ (resp. $|.|$, $||.||_*$), we denote the norm of X (resp.H, X^*). Let $\longrightarrow^s$ and $\longrightarrow^w$ stand for the strong and weak convergence, respectively. We set

$$W^{1,p}(I,H) = \{x \in L^p(X) | \dot{x} \in L^q(X^*), \\ ||x||_{W^{1,p}(I,H)} = (||x||_{L^p(X)} + ||\dot{x}||_{L^q(X^*)})^{\frac{1}{2}}\}$$

where the above derivative should be understood in the sense of vector-valued distributions. Hence, $W^{1,p}(I, H)$ is a separable reflexive Banach space, and $W^{1,p}(I, H) \hookrightarrow C(I, H)$ continuously ([2]).

Let Ω be a topology space, (Ω, Θ) a measurable space, (Ω, Θ, μ) a complete $\sigma-$ finite measure space, and $F : \Omega \longrightarrow 2^X$ a set-valued map. By S_F^p we denote the set of all selectors of $F(.)$ that belong to the Lebesgue-Bochner space $L^p(X)$, $(1 \le p \le \infty)$, i.e.,

$$S_F^p = \{u \in L^p(X) : u(\omega) \in F(\varpi), \text{ a.e. } \Omega\},$$

which is nonempty if

$$\varpi \longrightarrow \sup\{||y||, y \in F(\varpi)\} \in L_+^p,$$

in which case the multifunction is called L^p-bounded. In addition, by

$$\sigma_A(x^*) = \sup\{\langle x, x^* \rangle : x \in A\},$$

we denote the support function of the set A. By S_F we denote the set of all strong measurable selectors.

Finally, let Y be the control space with the structure of a separable Banach space, Σ a parameter set. Recall that an operator $A : X \longrightarrow X^*$ is called pseudomonotone if, for each $x \in X$, and a sequence (x_n) in X,

$$x_n \longrightarrow^w x \text{ in } X \text{ and } \overline{\lim}_{n\to\infty} \langle Ax_n, x_n - x \rangle \le 0$$

imply

$$\langle Ax, x - w \rangle \le \underline{\lim}_{n\to\infty} \langle Ax_n, x_n - w \rangle, \text{ with all } \omega \in X.$$

In addition, an operator $A : \Omega \longrightarrow X$ is called affine if,

$$A(\lambda\varpi_1 + (1-\lambda)\varpi_2) = \lambda A(\varpi_1) + (1-\lambda)A(\varpi_2), \lambda \in [0,1], \varpi_1, \varpi_2 \in \Omega.$$

For brevity we introduce the following notation:

$$P_{f(c)}(X) = \{A \subseteq X : \text{ nonempty, closed (convex)}\},$$
$$P_{kc}(X) = \{A \subseteq X : \text{ nonempty, compact, convex}\}.$$

3. Existence of admissible trajectories

Consider the following nonlinear, controlled evolution incusion

$$\begin{cases} \dot{x}(t) \in -A(t, x(t)) + F(t, x(t), u(t)), \text{ a.e. } I, \\ x(0) = x_0, \quad u \in S_U \end{cases} \tag{3.1}$$

where $A : I \times X \longrightarrow X^*$ is pseudomonotone and $F : I \times X \times Y \longrightarrow 2^{X^*}$ a set-valued map.

A function $x \in W^{1,p}(I, H)$ is said to be a solution of (3.1) if, there exists a control $u \in S_U$ and $f \in S^q_{F(\cdot, x(\cdot), u(\cdot))}$ s.t.

$$\begin{cases} \dot{x}(t) \in -A(t, x(t)) + f(t), \text{ a.e. } I, \\ x(0) = x_0. \end{cases} \tag{3.2}$$

We next give some basic hypotheses on the data of the system (3.1).

Hypothesis (H$_1$) $A : I \times X \longrightarrow X^*$ is an operator s.t.

(1) $t \longrightarrow A(t, x)$ is measurable;

(2) $x \longrightarrow A(t, x)$ is pseudomonotone;

(3) $||A(t, x)||_* \leq a_1(t) + b_1||x||^{p-1}$, with $a_1 \in L^q_+(0, T), b_1 > 0, 2 \leq p < \infty, \frac{1}{p} + \frac{1}{q} = 1$;

(4) $\langle A(t, x) - A(t, y), x - y\rangle \geq c||x - y||^p - d|x - y|^2$, with $x, y \in X, c > 0, d \geq 0$;

(5) $x \longrightarrow A(t, x)$ is affine.

Hypothesis (H$_2$) $F : I \times X \times Y \longrightarrow P_{fc}(X^*)$ satisfies

(1) $(t, x, u) \longrightarrow F(t, x, u)$ is graph measurable;

(2) $x \longrightarrow F(t, x, u)$ is *u.h.c.*, i.e., $x \longrightarrow \sigma_{F(t,x,u)}(y)$ is *u.s.c.*, with each given $y \in X$;

(3) $F(t, x, u) \subset G(t)$, a.e. I, with $G : I \longrightarrow P_f(X^*)$ being L^q-bounded measurable;

(4) $x \longrightarrow F(t, x, u)$ is convex.

Hypothesis (H$_3$) $x_0 \in H$.

Hypothesis (H$_4$) $U : I \longrightarrow P_f(Y)$ is measurable.

From [6], we know that the Cauchy problem (3.2) has a unique solution with each $f \in L^q(X)$. Write the solution map as $f \longrightarrow x$. Next we introduce the Nemyckii operator $\hat{A} : L^p(X) \longrightarrow L^q(X^*)$, given by $(\hat{A}x)(t) = A(t, x(t))$, with $x \in L^p(X)$.

Lemma 3.1 *Under hypothsis (H_1), the solution map $f \longrightarrow x$ is affine and continuous from S^q_G into $L^p(I,X)$; moreover,*

$$\max_{t\in I}|x(t)| + ||x||_{W^{1,p}(I,H)} + ||\hat{A}x||_{L^q(X^*)} \le \rho(|x_0|, ||f||_{L^q(X^*)}), \tag{3.3}$$

where $\rho : R_+ \times R_+ \longrightarrow R$ is a continuous function.

Proof It follows from hypothesis $(H_1)(5)$ and uniqueness of solution for (3.2) that the solution map $f \longrightarrow x$ is affine. Using hypothesis $(H_1)(3)$-(4), generalized integration by parts formula, Cauchy's inequality, Gronwall's inequality, one can show that (3.3) is true. Next, let $f_n \longrightarrow^s f$ in S^q_G. Then, there exists a sequence (x_n), satisfying

$$\begin{cases} \dot{x}_n(t) + A(t, x_n(t)) = f_n(t), \text{ a.e., } I, \\ x_n(0) = x_0. \end{cases} \tag{3.4}$$

Hence we have that $x_n(T) \longrightarrow^w z$ in H, $x_n \longrightarrow^w x$ in $W^{1,p}(I,H)$, and $\hat{A}x_n \longrightarrow^w b$ in $L^q(X^*)$. Further, (x,z,b) satisfies the following operator equation (see Lemma 30.5([2], pp.776) or Theorem1 ([6]))

$$\begin{cases} \dot{x} + b = f(.), \\ x(0) = x_0, x(T) = z. \end{cases} \tag{3.5}$$

In addition, (3.4)-(3.5) and hypothesis$(H_1)(1)$-(4) imply the operator $\hat{A}$ satisfies the Condition (M) ([2], pp.583 or Lemma1 of [6]). Therefore, x is a solution of (3.2) ([2], pp.768-770). From hypothesis $(H_1)(4)$ and (3.2) and (3.4), we obtain that $x_n \longrightarrow^s x$ in $L^p(I,X)$. □

Lemma 3.2 *Let X be a Banach space, (Ω, Θ, μ) a complete σ-finite measure space. Let $F : \Omega \longrightarrow P_f(X)$ be L^1-bounded measurable. Then, for any $y \in L^p(X)$, $p > 1, \frac{1}{p} + \frac{1}{q} = 1$, we have*

(1) $\sigma_{S^q_F}(y) = \int_\Omega \sigma_{F(\varpi)}(y(\varpi))\mu(d\varpi)$;

(2) there exists a measurable selection $f \in S^q_F$ s.t.

$$\sigma_{S^q_F}(y) = \langle\langle f, y\rangle\rangle.$$

Proof It is obvious that $(\varpi, z) \longrightarrow \sigma_{F(\varpi)}(z)$ is a Carathéordory function. Hence,

$$\langle\langle h, y\rangle\rangle \le \int_\Omega \sigma_{F(\varpi)}(y(\varpi))\mu(\mathrm{d}\varpi), \text{ for } h \in S^q_F;$$

on the other hand, pick out $k \in L^1_+(\Omega, R)$ and $\varepsilon > 0$. Define $G : \Omega \longrightarrow 2^X$, given by

$$G(\varpi) = [\sigma_{F(\varpi)}(y(\varpi)) - \varepsilon k(\varpi), \sigma_{F(\varpi)}(y(\varpi)) + \varepsilon k(\varpi)].$$

By means of Corrollary 8.2.13([1]), $G(\cdot)$ and the set-valued map

$$\varpi \longrightarrow H(\varpi) = \{x \in F(\varpi) | \ (x, y(\varpi)) \in G(\varpi)\}$$

are measurable. So, using measurable selection theorem ([1], pp.308), $H(\cdot)$ admits a measurable selection $f : \Omega \longrightarrow X$ such that $f(\varpi) \in H(\varpi)$, a.e. Ω. Again since ε is arbitrary, the conclusion is true. □

Theorem 3.1 *Assume that hypotheses* (H_1)-(H_4) *hold. Then, for each* $u \in S_U$, *the system (3.1) admits a solution* $x(.) \in W^{1,p}(I,H) \subset C(I,H)$.

Proof Let $u \in S_U$ be a given element and $P(f)(.)$ denote the solution of (3.2) corresponding to $f \in S_G^q$. It is easy to see that $S^q_{F(\cdot,P(f)(\cdot),u(\cdot))}$ is closed and convex in S_G^q. Let $p_n(.)$ be L^q-bounded simple functons s.t. $p_n(\cdot) \longrightarrow^s P(f)(\cdot)$ in $L^q(X^*)$, and then $t \longrightarrow F(t,p_n(t),u(t))$ is measurable and L^q-bounded. Hence, we can find that

$$g_n(\cdot) \in S^q_{F(\cdot,p_n(\cdot)(\cdot),u(\cdot))} \subset S_G^q \subset L^q(X^*).$$

Owing to boundedness of S_G^q, we may assume that $g_n(\cdot) \longrightarrow^w g(\cdot)$ in $L^q(X^*)$. Consequently, it follows from hypothesis (H_2)(2) and Lemma 3.2 that

$$\begin{aligned}\langle\langle g,v\rangle\rangle &\leq \overline{\lim}_{n\to\infty}\int_0^T \sigma_{F(t,p_n(t),u(t))}(v(t))\mathrm{d}t\\ &\leq \int_0^T \sigma_{F(t,p(f)(t),u(t))}(v(t))\mathrm{d}t, \quad v \in L^p(X).\end{aligned}$$

Using separation theorem, one can know $g \in S^q_{F(\cdot,P(f)(\cdot),u(\cdot))}$, which shows that $S^q_{F(\cdot,P(f)(\cdot),u(\cdot))} \neq \emptyset$. Next, define a set-valued map $R(\cdot): L^q(X^*) \longrightarrow 2^{S_G^q}$, given by

$$R(f) = S^q_{F(\cdot,P(f)(\cdot),u(\cdot))}, \text{ with } f \in L^q(X^*).$$

It is well-known that upper semicontinuity of $\sigma_{R(\cdot)}(y)$ on $L^q(X^*)$ with $y \in L^p(X)$ is equivalent to that of $\sigma_{R(\cdot)}(y)$ on $L^q(X^*)_w$, provided that $\sigma_{R(\cdot)}(y)$ is convex function; here $L^q(X^*)_w$ is the space of $L^q(X^*)$ with its weak topology. Therefore, let $f_1, f_2 \in L^q(X^*), y \in L^p(X), \lambda \in [0,1], f_n \longrightarrow^s f$ in $L^q(X^*)$. By Lemma 3.1-3.2 and hypotheses (H_1)(5) and (H_2)(2), we have

$$\begin{aligned}\sigma_{R(\lambda f_1+(1-\lambda)f_2)}(y) &= \int_0^T \sigma_{F(t,\lambda P(f_1)(t)+(1-\lambda)P(f_2)(t),u(t))}(y(t))\mathrm{d}t\\ &\leq \lambda\sigma_{R(f_1)(y)+(1-\lambda)\sigma_{R(f_2)}}(y),\end{aligned}$$

and

$$\overline{\lim}_{n\to\infty}\sigma_{R(f_n)}(y) \leq \int_0^T \sigma_{F(t,P(f)(t),u(t))}(y(t))\mathrm{d}t$$

which means that $\sigma_{R(\cdot)}(y)$ is $u.s.c.$ on $L^q(X^*)$, and hence on $L^q(X^*)_w$. Thus, $R(\cdot)$ is $u.s.c.$ on $L^q(X^*)_w$. Applying the Kakutani fixed-point theorem ([1], pp.87), we get $g \in R(g)$. So, $P(g)(\cdot) \in W^{1,p}(I,H)$ is a solution of (3.1) corresponding to $u \in S_U$.

4. Lagrange type optimal control problem

The cost functional is defined as follows:

$$J(x,u) = \int_0^T L(t,x(t),u(t))\mathrm{d}t.$$

Next, consider the existence of optimal control for the following problem with a parameter set:

(P) $\min J(x,u),$

subject to

$$\begin{cases} \dot{x}(t) \in -A(t,x(t)) + F(t,x(t),u(t),\rho), \text{ a.e. } I, \\ x(0) = x_0, u \in S_U, \rho \in \Sigma. \end{cases} \tag{4.1}$$

A pair (x,u,ρ) is called feasible if, x is a solution of (4.1) with $u \in S_U$ and $\rho \in \Sigma$. Now we impose some general assumptions.

Hypothesis (H$_5$) $F : I \times X \times Y \times \Sigma \to P_f(X^*)$ satisfies

(1) $(t,x,u) \longrightarrow F(t,x,u,\rho)$ is graph measurable;
(2) $(x,u,\rho) \longrightarrow F(t,x,u,\rho)$ is convex;
(3) $F(t,x,u,\rho) \subset G(t)$, a.e. I, with $G : I \longrightarrow P_f(X^*)$ being L^q-bounded measurable;
(4) $(x,u,\rho) \longrightarrow F(t,x,u,\rho)$ is u.h.c..

Hypothesis (H$_6$) $U : I \longrightarrow P_{kc}(Y)$ is measurable.

Hypothesis (H$_7$) Σ is a convex compact subset of a metric space.

Hypothesis (H$_8$) $L : I \times X \times Y \longrightarrow R$ is a function s.t.

(1) $(t,x,u) \longrightarrow L(t,x,u)$ is measurable;
(2) $(x,u) \longrightarrow L(t,x,u)$ is l.s.c.;
(3) $(x,u) \longrightarrow L(t,x,u)$ is convex;
(4) there exists a function $h \in L^1(I,R)$ s.t.

$$L(t,x,u) \geq h(t), \text{ a.e. } I.$$

Filliov's selection theortem plays an important role in the proof of the existence of optimal controls for systems governed by evolution equations. We shall generalize it into more general case, which can solve that of evolution inclusions.

Proposition 4.1 *Let X be a polish space, Y a separable Banach space, (Ω,Θ,μ) a complete σ-finite measure space. Assume that $F : \Omega \times X \longrightarrow P_{fc}(Y)$ is L^q-bounded measurable; $F(\cdot,x)$ is measurable; $F(\varpi,\cdot)$ is u.h.c.. Let $U : \Omega \longrightarrow P_f(X)$ be measurable, and $h : \Omega \longrightarrow X$ measurable such that $h(\varpi) \in F(\varpi,U(\varpi))$, a.e. Ω. Then, there exists a measurable selection $u : \Omega \longrightarrow X$ s.t.*

$$u(\varpi) \in U(\varpi), \text{ a.e. } \Omega,$$
$$h(\varpi) \in F(\varpi,u(\varpi)), \text{ a.e. } \Omega.$$

Proof Consider a multifunction $G : \Omega \longrightarrow 2^{X^*}$, given by

$$G(\varpi) = \{x \in U(\varpi) | h(\varpi) \in F(\varpi,x)\}.$$

Without loss of generality, we may assume that $G(\cdot)$ has nonempty images. By the definition of upper hemicontinuity([1]), pp.74), one can verify that $G(\cdot)$ has closed images. Define an operator $\varphi : \Omega \times X \longrightarrow \Omega \times X \times Y$, given by

$$\varphi(\varpi,x) = (\varpi,x,h(\varpi)).$$

We see easily that $\varphi(\cdot,\cdot)$ is a Carathéodory function. Meanwhile, we have

$$\text{Graph}(G) = \text{Graph}(U) \cap \varphi^{-1}(\text{Graph}(F)).$$

By Lemma 8.2.6 ([1]), we obtain that $G(.)$ is measurable. So, from measurable selection theorem ([1], pp.308), one can get that $u : \Omega \longrightarrow X$ s.t. $u(\varpi) \in G(\varpi)$, a.e. Ω, and hence the above conclusion is true. □

To obtain the existence of optimal control for (4.1), we introduce a set-valued map $\varepsilon(\cdot,\cdot,\cdot) : I \times X \times \Sigma \longrightarrow 2^{R\times X^*}$, given by

$$\varepsilon(t,x,\rho) = \{(z^*,z) \in R \times X^* | z^* \geq L(t,x,u), z \in F(t,x,u,\rho), u \in U(t)\}.$$

Theorem 4.1 *Under hypotheses (H_1), (H_3), and (H_5)-(H_8), the problem (P) has at least an optimal pair* $(x,u) \in W^{1,p}(I,H) \times S_U$.

Proof Let (x_n, u_n) be a minimizing sequence. Then, there exist corresponding sequences (ρ_n) and (f_n) such that (x_n, u_n, ρ_n) is feasible. Using hypothesis (H_5)(3), we may assume that $f_n \longrightarrow^w f$ in $L^q(X^*)$. Utilizing Mazur's Lemma, we acquire that

$$f_j(\cdot) = \Sigma_{i\geq 1}\alpha_{ij} f_{i+j}(\cdot) \longrightarrow^s f(\cdot) \text{ in } L^q(X^*), \quad \Sigma_{i\geq 1}\alpha_{ij} = 1, \alpha_{ij} \geq 0.$$

Write simply,

$$x_j(\cdot) = \Sigma_{i\geq 1}\alpha_{ij}x_{i+j}(\cdot), \quad u_j(\cdot) = \Sigma_{i\geq 1}\alpha_{ij}u_{i+j}(\cdot),$$
$$\rho_j = \Sigma_{i\geq 1}\alpha_{ij}\rho_{i+j}, \psi_j(\cdot) = L(\cdot, x_j(\cdot), u_j(\cdot)),$$
$$\psi_0(t) = \underline{\lim}_{j\to\infty}\psi_j(t), \text{ a.e. } I.$$

Therefore, by virtue of Lemma 3.1, we know that $x_j(\cdot) \longrightarrow^s x(\cdot)$ in $L^p(I,X)$; hence x is a solution of (3.2) with $f \in S_G^q$. Further, we may assume that $f_j(t) \longrightarrow^s f(t)$ in X^* a.e. I, $x_j(t) \longrightarrow^s x(t)$ in X, a.e.I, and $\rho_n \longrightarrow \rho$ in Σ. Observe that hypothesis (H_5)(2) yields $f_j(t) \in F(t, x_j(t), u_j(t), \rho_j)$, a.e. I, $j = 1, 2, \cdots$. Next we show that $(f(t), \psi_0(t)) \in \varepsilon(t, x(t), \rho)$, a.e. I. Fix $t_0 \in I$ and $z \in X$. Due to hypothesis (H_6), we can assume that $u_j(t) \longrightarrow^s u$ in Y. According to hypotheses (H_5)(4)and (H_8)(2), we have that, for any $\varepsilon' > 0$, as j large enough,

$$(f_j(t_0), z\rangle \leq \sigma_{F(t_0,x_j(t_0),u_j(t_0),\rho_j)}(z) < \sigma_{F(t_0,x(t_0),u,\rho)}(z) + \varepsilon',$$
$$\psi_j(t_0) = L(t_0, x_j(t_0), u_j(t_0)) \geq L(t_0, x_0(t), u) - \varepsilon'.$$

By the separation theorem, as $j \to \infty$, we can get that

$$(f(t_0), \psi_0(t_0)) \in \varepsilon(t_0, , x(t_0), \rho).$$

Since t_0 is arbitrary in I, (4.2) is true. Again using Proposition 4.1, we get a measurable selection $u \in S_U$ s.t.

$$f(t) \in F(t, x(t), u(t), \rho), \quad \psi_0(t) \geq L(t, x(t), u(t)) \geq h(t), \text{ a.e. } I.$$

Besides this, hypothesis (H_8) and Fatou's Lemma yield that

$$\int_0^T \psi_0(t)\mathrm{d}t = \int_0^T \underline{\lim}_{j\to\infty}\psi_j(t)\mathrm{d}t \leq \underline{\lim}_{j\to\infty}\int_0^T \psi_j(t)\mathrm{d}t \leq \lim_{j\to\infty} J(x_j, u_j).$$

So, $(x,u) \in W^{1,p}(I,H) \times S_U$ is the desired optimal pair.

Example

Let be Ω bounded open domain in R^n with smooth boundary $\Gamma=\partial\Omega$, and T a fixed positive number with $I = [0,T]$. Consider a control system governed by the following parabolic boundary-initial-value problem with a parameter set:

$$J(x,u) = \int_0^T \int_\Omega \ell(t,z,x(t,z),u(t,z))\mu(\mathrm{d}z)\mathrm{d}t \longrightarrow \inf = m, \quad (Q)$$

subject to

$$\begin{aligned}
&\frac{\partial}{\partial t}x(t,z) \in -\Sigma_{|\alpha|\le m}(-1)^{|\alpha|}A_\alpha(t,z,\theta(x(t,z))) + F_0(t,z,x(t,z),u(t,z),\rho), \text{ on } I\times\Omega,\\
&D^\beta x(t,z) = 0, \text{ on } I\times\Gamma, \quad |\beta| \le m-1,\\
&x(0,z) = x_0(z), \text{ on } \Omega,\\
&|u(t,z)| \le K(t), \text{ a.e. } I, K(\cdot) \in L^1(I,R).
\end{aligned}$$

where $\theta\ (x) = \{D^\beta x, |\beta| \le m\}$. We choose spaces

$$X = W_0^{m,p}(\Omega), H = L^2(\Omega), X^* = W^{-m,q}(\Omega), p > 1, \frac{1}{p} + \frac{1}{q} = 1.$$

Note that the hypothesis of A_α is the same as that of [6].

Let $F_0 : I \times \Omega \times R \times R \times R \longrightarrow P_f(R)$ be a set-valued map s.t.

(1) $(t,z) \longrightarrow F_0(t,z,x,u,\rho)$ is graph measurable;

(2) $(x,u,\rho) \longrightarrow F_0(t,z,x,u,\rho)$ is convex and $u.h.c.$;

(3) $F_0(t,z,x,u,\rho) \subset G(t,z)$, a.e. I, with $G(.,.) : I \times \Omega \longrightarrow P_f(R)$ being L^q-bounded measurable;

Let $\ell : I \times R \times R \longrightarrow \overline{R} = R \cup \{+\infty\}$ be an integrand s.t.

(1) $(t,z,x,u) \longrightarrow \ell(t,z,x,u)$ is measurable;

(2) $(x,u) \longrightarrow \ell(t,z,x,u)$ is $l.s.c.$ and convex;

(3) $h(t,z) \le \ell(t,z,x,u)$, a.e. I, with $h \in L^1(I \times \Omega)$.

One can check that hypotheses (H_1)-(H_8) hold. Rewrite the problem (Q) as the abstract form equivalent to the problem (P). Then, Theorem 4.1 shows that the problem (Q) has at least an optimal pair $(x,u) \in W^{1,p}(0,T;L^2(\Omega)) \cap L^p(0,T;W_0^{m,p}(\Omega)) \times S_U$.

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具有伪单调算子的非线性微分包含约束的最优控制

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摘 要: 本文研究一类受主算子为伪单调算子的非线性微分包含约束的最优控制问题. 首先，探讨抛物型发展方程的柯西问题其解的性质及微分包含问题的容许轨线的存在性；然后，利用一个新的可测选择定理解决了受非线性微分包含约束的最优控制的存在性. 最后，给一例子加以说明所获结果的应用性.