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Matrix Approach in Wavelet Analysis *

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Abstract: In this paper, we give a survey of matrix approach in wavelet theory , and describe some related results which were obtained by ourselves.

Key words: wavelet; matrix; biorthogonality; symmetry; matrix extension.

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1. Introduction

Wavelet analysis is a rapidly developing area in the mathematical sciences which is emerging as a brisk and important field of investigation. Moreover, it has already created a tight link between mathematicans and electrical engineers, and has even drawn a great deal of attention from scientists and engineers in other disciplines.

Historically, the study of wavelet analysis was based on the standard approach of functional analysis (see [1], [2], [3], [4] and [5]). However, the basic aspects of wavelet analysis can be derived using fairly elementary means of matrix algebra, and matrix method plays an important role in the study of wavelets (see [6] and [7]).

In this paper, we will give a survey about matrix approach in wavelet analysis. Some related results, which were deduced by ourselves, also presented.

2. Wavelet matrices

Let us consider an integer dilation parameter $m \geq 2$. The main idea in the constructions of wavelet bases is the multiresolution analysis.

Definition 1 *A sequence of closed subspaces $\{V_j\}_{j\in Z}$ in $L^2(R)$ is a multiresolution analysis of $L^2(R)$ (abbr. MRA)if it satisfies the following conditions:*

- $V_k \subset V_{k+1}$, *for all* $k \in Z$;

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- *$f(x) \in V_k$ if and only if $f(mx) \in V_{k+1}$, for all $k \in Z$;*
- $\bigcap\limits_{k\in Z} V_k = \{0\}$ and $\overline{\bigcup\limits_{k\in Z} V_k} = L^2(R)$;
- *there is an element $\varphi \in V_0$ such that $\{\varphi(\cdot - l)\}_{l\in Z}$ is an orthonormal basis of V_0, where R denotes the set of all real numbers and Z the set of all integers.*

By the definition of multiresolution analysis above, φ satisfies a dilation equation (or sometimes, we call it refinable equation) of the form

$$\varphi(x) = m^{\frac{1}{2}} \sum_{k\in Z} h_k \varphi(mx - k) \tag{1}$$

and there are $m-1$ wavelet functions $\psi^{(s)}$, $s = 1, 2, \cdots, m-1$, which are defined as some special linear combinations of the scaling function

$$\psi^{(s)}(x) = m^{\frac{1}{2}} \sum_{k\in Z} g_k^{(s)} \varphi(mx - k), \tag{2}$$

where $\{h_k\}_{k\in Z}$ and $\{g_k^{(s)}\}_{k\in Z}$ are sequences of numbers.

Definition 2 *Let $\{A_n\}_{n\in Z}$ be a matrix sequence,*

$$A_n = \begin{pmatrix} h_{nm} & h_{nm+1} & \cdots & h_{nm+m-1} \\ g_{nm}^{(1)} & g_{nm+1}^{(1)} & \cdots & g_{nm+m-1}^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ g_{nm}^{(m-1)} & g_{nm+1}^{(m-1)} & \cdots & g_{nm+m-1}^{(m-1)} \end{pmatrix},$$

each A_n is a square matrix of size $m \times m$, then, matrix $A = (\cdots \quad A_0 \quad A_1 \quad A_2 \quad \cdots)$ is called the wavelet matrix.

As to the definition of general wavelet matrix, one may refer to [6].

The orthogonality of the integer translates of scaling function φ and wavelet functions $\psi^{(s)}$ $(s = 1, 2, \cdots, m-1)$ implies that

$$\sum_{l\in Z} A_l A_{k+l}^T = \delta_{0,k} I, \quad k \in Z, \tag{3}$$

where $\delta_{0,k}$ is the kronecker delta.

Now we give two simple examples of wavelet matrices ($m = 2$).

Example 1 Haar wavelet matrix

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} \cdots & 0 & 1 & 1 & 0 & \cdots \\ \cdots & 0 & 1 & -1 & 0 & \cdots \end{pmatrix}.$$

Example 2 Daubechies wavelet's two scaling equation is

$$\varphi_N(x) = \sum_{k=0}^{2N-1} c_k \varphi_N(2x - k),$$

where N is a positive integer. When $N = 2$, we can obtain Daubechies wavelet matrix:

$$A = \frac{\sqrt{2}}{4}\begin{pmatrix} \cdots & 0 & 1+\sqrt{3} & 3+\sqrt{3} & 3-\sqrt{3} & 1-\sqrt{3} & 0 & \cdots \\ \cdots & 0 & -1+\sqrt{3} & 3-\sqrt{3} & -3-\sqrt{3} & 1+\sqrt{3} & 0 & \cdots \end{pmatrix}.$$

On the other hand, if a wavelet matrix A is given such that it contains only a finite number of nonzero blocks, satisfiies the conditon (3) and the sum of the elements in the first row is $m^{\frac{1}{2}}$, then, equations (1) and (2) can be solved. The translates and dilates of the wavelet functions form at least a tight frame; in almost all cases this frame turns out to be an orthonormal basis (see [8] and [9]). When $m = 2$, Lawton ([9]) obtained the following Theorem.

Theorem 1 *Assume that $m_0(\xi)$ is a trigonometric polynomial of the form $2^{-\frac{1}{2}}\sum_{n=0}^{N} h_n e^{in\xi}$, satisfing*

$$|m_0(\xi)|^2 + |m_0(\xi+\pi)|^2 = 1$$

and $m_0(0) = 1$, define φ and $(2N-1)\times(2N-1)$ matrix A respectively as follows.

$$\hat{\varphi}(\xi) = (2\pi)^{-\frac{1}{2}}\prod_{j=1}^{+\infty} m_0(2^{-j}\xi),$$

$$A = (A_{lk})_{l,k}$$

and

$$A_{lk} = \sum_{n=0}^{N} h_n \overline{h_{k-2l+n}}, \quad -N+1 \le l,k \le N-1.$$

If the eigenvalue 1 of A is a single root of eigenpolynomial of A, then $\{\varphi(\cdot - l)\}_{l\in Z}$ are orthonormal.

Remark 1 As for the convergence of the infinite product in Theorem 1, one may refer to [1] (or [9]).

Thus the wavelet matrices can be used to study compactly supported of higher multiplicity (i.e. the dilation parameter m) wavelets by means of linear algebra.

3. Factorizations of compactly supported orthogonal wavelet matrix

In this section, we describe all matrices which can give rise to orthogonal wavelets with compact support. The wavelet matrix A must then contain only a finite number of nonzero blocks. Without loss of generality, we suppose that $A_k = 0$ whenever $k < 0$ and $k \ge p$ for some p and write $A = (A_0 \quad A_1 \quad \cdots \quad A_{p-1})$. The out of the range blocks will be automaticality considered to be zero. We will also identify the matrix A and its extension constructed from A by adding one or more $m\times m$ zero blocks to either side. We can thus add or discard zero blocks at either side and renumber blocks when requested.

Let us focus on the condition (3). We can impose the group structure to the set of the matrices conforming to equations (3) by using, for example, the following product.

Definition 3 *Let* $A = (A_0 \quad A_1 \quad \cdots \quad A_{p-1})$, $B = (B_0 \quad B_1 \quad \cdots \quad B_{q-1})$. *We define*

$$A * B = (D_0 \quad D_1 \quad \cdots \quad D_{p+q-2}),$$

where $D_j = \sum_k A_k B_{j-k}$.

Under this product, the matrices comforming to the equation (3) form a group, the unit element being the identity matrix of order m and the element inverse to A having the form

$$A^{inv} = (A_{p-1}^T \quad A_{p-2}^T \quad \cdots A_0^T), \tag{4}$$

where matrix A^T means transpose of A .

In [6], J. Kautsky and R. Turcajova obtained that

Theorem 2 *Matrix* $A = (A_0 \quad A_1 \quad \cdots \quad A_{p-1})$ *satisfies (3) if and only if there exists Hermitian matrices* P_j, $j = 1, 2, \cdots, p-1$, *and an orthogonal matrix* H *such that*

$$A = H * (P_1 \quad I - P_1) * (P_2 \quad I - P_2) * \cdots * (P_{p-1} \quad I - P_{p-1}). \tag{5}$$

Then the problem arises, whether the factorization in Theorem 2 is unique. We have the following result.

Theorem 3 *If rank* $A_0 +$ *rank* $A_{p-1} = m$, *then the* P_j *in (5) and consequently also the linear factor (i.e. the factors in (5)) are determined uniquely.*

When rank A_0+rank $A_{p-1} < m$, one has to impose some additional conditions. For example, one can choose in the process of factorizing always the Hermitian martices having the lowest possible rank (which is unique).

There are several factorizations similar to the one described here. Their particular features make them more or less suitable for different applications (see [5]).

4. Biorthogonal Wavelet matrix

The shifted biorthogonal conditions analogous to equation (3) are as follows.

$$\sum_{l \in Z} A_l \tilde{A}_{l+k}^T = \delta_{0,k} I, \quad k \in Z, \tag{6}$$

where $\tilde{A} = (\cdots \quad \tilde{A}_0 \quad \tilde{A}_1 \quad \tilde{A}_2 \quad \cdots)$ denotes some matrix. We will focus on here the case where all the basis functions are compactly supported, that is, both A and $\tilde{A}$ contain only a finite number of nonzero blocks. Here, we will study these problems: What must A satisfy so as $\tilde{A}$ satisfying conditions (6) and $\tilde{A}$ contains only a finite number of nonzero blocks? How do we construct such pairs? Can they be factorized in some way?

When canceling or adding zero square blocks or renumbering blocks, we do the same with both A and $\tilde{A}$, because the mutual position of the blocks of A and $\tilde{A}$ is important. Without loss of generality, we will suppose that $A_j = \tilde{A}_j = 0$, for $j < 0$, and describe the exact extent of nonzero blocks in the following way.

Definition 4 *We say that*

$$\Theta = \left\{ \begin{array}{c} A \\ \tilde{A} \end{array} \right\}$$

is an (l,p,k,q)-biorthogonal pair (or a biorthogonal pair of type (l,p,k,q)) if

- $A_j = 0$ *for* $0 \le j \le l-1$, $j \ge l+p$, $A_l \ne 0$ *and* $A_{l+p-1} \ne 0$;
- $\tilde{A}_j = 0$ *for* $0 \le j \le k-1$, $j \ge k+q$, $\tilde{A}_k \ne 0$ *and* $\tilde{A}_{k+q-1} \ne 0$.

We can extend the definition of the $*$ product (see Definition 3) to biorthogonal pairs in the following way:

$$\left\{ \begin{array}{c} A \\ \tilde{A} \end{array} \right\} * \left\{ \begin{array}{c} B \\ \tilde{B} \end{array} \right\} = \left\{ \begin{array}{c} A * B \\ \tilde{A} * \tilde{B} \end{array} \right\}.$$

Similarly as in the case of orthogonal wavelets, the biorthogonal pairs form a group under this product. The unit element is the (0,1,0,1)–biorthogonal pair formed by two identity matrices; the element inverse to

$$\Theta = \left\{ \begin{array}{cccc} (A_0 & A_1 & \cdots & A_r) \\ (\tilde{A}_0 & \tilde{A}_1 & \cdots & \tilde{A}_r) \end{array} \right\}$$

is then

$$\Theta^{inv} = \left\{ \begin{array}{cccc} (\tilde{A}_r^T & \tilde{A}_{r-1}^T & \cdots & \tilde{A}_0^T) \\ (A_r^T & A_{r-1}^T & \cdots & A_0^T) \end{array} \right\}.$$

We can generalize Theorem 2 to biorthogonal case.

Theorem 4 *Let Θ be a biorthogonal pair of type $(l,p,k,q) \ne (0,p,p-1,q)$. Then there exists an biorthogonal pair Γ such that $\Lambda = \Theta * \Gamma$ is of the type $(\tilde{l},\tilde{p},\tilde{k},\tilde{q})$, where $\tilde{q} \le q$ and exactly one of the following three possibilities occurs:*

$$\begin{array}{ccc} \tilde{l} = l, & \tilde{p} = p, & \tilde{k} = k+1 \quad or \\ \tilde{l} = l-1, & \tilde{p} = p, & \tilde{k} = k \quad or \\ \tilde{l} = l, & \tilde{p} = p-1, & \tilde{k} = k. \end{array} \tag{7}$$

Definition 5 *If $A_{p-1}\tilde{A}_{p-1} = I$, then we call Θ defined in Definition 4 a biorthogonal atom. When $A_{p-1} = \tilde{A}_{p-1} = I$, we say that the atom is normalized.*

By repeated application of Theorem 4, we obtain the next Theorem.

Theorem 5 *Every (l,p,k,q)-biorthogonal pair can be factorized into the product of a (0,1,0,1)-biorthogonal pair, a normalized atom and $l+p-k-1$ biorthogonal pairs.*

Suppose that $A = (A_0 \quad A_1 \quad \cdots \quad A_{p-2} \quad I)$ is given. There is a question: when does a biorthogonal counterpart of the form $\tilde{A} = (I \quad \tilde{A}_p \quad \cdots \quad \tilde{A}_{p+q-2})$ exist such that

$$\Theta = \left\{ \begin{array}{c} A \\ \tilde{A} \end{array} \right\}$$

is a normalized (p,q)-atom? The following Theorem replies this question.

Theorem 6 *Let* $A = (A_0 \quad A_1 \quad \cdots \quad A_{p-2} \quad I)$. *Then there exists* $\tilde{A}$ *such that the biorthogonal pair* Θ *is a normalized* (p,q)*- atom if and only if the matrix* G

$$G = \begin{pmatrix} -A_{p-2} & -A_{p-3} & -A_{p-4} & \cdots & -A_1 & -A_0 \\ I & 0 & 0 & \cdots & 0 & 0 \\ 0 & I & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & I & 0 \end{pmatrix} \tag{8}$$

is nilpotent with index $p+q-2$.

5. Symmetric orthogonal wavelet matrices

In some applications, symmetry of wavelets is of importance. More precisely, one requires each of the sequences $\{h_k\}_{k\in Z}$ and $\{g_k^{(s)}\}_{k\in Z}$, $s = 1, 2, \cdots, m-1$, in equations (1) and (2), respectively, to be symmetric or antisymmetric about some point. It is well known that except for the trivial case of Haar basis, there are no classical (multiplicity $m = 2$) compactly supported symmetric wavelets. Usually, when symmetry is required, biorthogonal wavelets are used, instead. Another possibility, however, is to use higher multiplicity wavelets.

We consider here only the case when all the rows of the wavelet matrix

$$A = (A_0 \quad A_1 \quad \cdots \quad A_{p-1})$$

are symmetric about the middle of the length of A.

Definition 6 *If* $A_j = A_{p-1-j}$ $(j = 0, 1, \cdots, p-1)$ *in Definition 2, then, matrix* $A = (A_0 \quad A_1 \quad \cdots \quad A_{p-1})$ *is called centrosymmetric.*

Theorem 7 *A centrosymmetric matrix* $A = (A_0 \quad A_1 \quad \cdots \quad A_{p-1})$ *with an even number of rows satisfies the shifted orthogonality conditions (3) if and only if there exist centrosymmetric linear factors* $(P_j \quad I - P_j)$, $j = 1, 2, \cdots, p-1$, $(P_j$ *are Harmitian matrices) and a centrosymmetric orthogonal matrix* H *such that*

$$A = H * (P_1 \quad I - P_1) * (P_2 \quad I - P_2) * \cdots * (P_{p-1} \quad I - P_{p-1}).$$

When m, the number of rows, is odd, the situation is more complicated. The simple fact that $\frac{m}{2}$ is not an integer, causes a series of problems. But, a complete characterization similar to the one for m even is still possible. The following Theorem holds.

Theorem 8 *A centrosymmetric matric* $A = (A_0 \quad A_1 \quad \cdots \quad A_{p-1})$ *with an odd number of rows satisfies the orthogonality conditions (3) if and only if*

$$A = H * W_1 * W_2 * \cdots * W_{p-1},$$

where, for $j = 1, 2, \cdots, \frac{p-1}{2}$,

$$W_{2j-1} = (U_{2j-1}U_{2j-1}^T + V_jV_j^T \quad JU_{2j-1}U_{2j-1}^T J),$$
$$W_{2j} = (U_{2j}U_{2j}^T \quad JU_{2j-1}U_{2j-1}^T J + V_jV_j^T),$$

both $(U_{2j-1} \quad V_j \quad JU_{2j-1}J)$ *and* $(U_{2j} \quad V_j \quad JU_{2j}J)$ *are centrosymmetric orthogonal matrices,* J *stands for a permutation matrix of the appropriate size.*

6. Matrix extension

Because constructions of many wavelets are attributed to matrix extension, matrix extension is an important and difficult subject in wavelet theory.

In frequency domain, (1) can be rewritten as follows.

$$\hat{\varphi}(\xi) = m^{-\frac{1}{2}} \sum_{k\in Z} h_k e^{-i\frac{k}{m}\xi} \hat{\varphi}(\frac{\xi}{m}) = m_0(\frac{\xi}{m})\hat{\varphi}(\frac{\xi}{m}),$$

where $m_0(\xi) = m^{-\frac{1}{2}} \sum_{k\in Z} h_k e^{-ik\xi}$ is called symbol function (or low pass filter) associated with the scaling function φ. The so–called matrix extension question is: assume that

$$\sum_{k=0}^{m-1} |m_0(\xi + \frac{2k\pi}{m})|^2 = 1.$$

How do we find functions $m_j(\xi) \in C(R)$, $j = 1, 2, \cdots, m-1$, such that the following matrix

$$\begin{pmatrix} m_0(\xi) & m_0(\xi+\frac{2\pi}{m}) & m_0(\xi+\frac{4\pi}{m}) & \cdots & m_0(\xi+\frac{2(m-1)\pi}{m}) \\ m_1(\xi) & m_1(\xi+\frac{2\pi}{m}) & m_1(\xi+\frac{4\pi}{m}) & \cdots & m_1(\xi+\frac{2(m-1)\pi}{m}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m_{m-1}(\xi) & m_{m-1}(\xi+\frac{2\pi}{m}) & m_{m-1}(\xi+\frac{4\pi}{m}) & \cdots & m_{m-1}(\xi+\frac{2(m-1)\pi}{m}) \end{pmatrix}$$

is unitary? This matrix extension is called B–extension.

If $m = 2$, matrix extension question has been well solved (see [1] and [10]). As for $m > 2$, this problem is still open except for several special cases.

In multidimensional wavelet analysis and vector wavelet analysis, there are also analogous matrix extension problems. But, this problem is more difficult and complicated. In [11], Chen and Xiao gave a matrix extension by using a new kind of Householder type matrix, then apply it to the construction of multivariate wavelet vectors and the construction of compactly supported multivariate wavelets. In [12], Lawton, Lee and Shen gave an algorithm for matrix extension to construct orthogonal wavelets, and S. S. Goh et al ([13]) generalized this algorithm to biorthogonal case. In vector case, Jiang ([14] and [15]) did a series of works. His results provided symmetric paraunitary matrix extension and parametrization of correcponding symmetric orthogonal multifilter banks. These matrix extension problems were considered under certain conditions, but in the general case, the problems have not been solved.

In order to show problem of matrix extension,we list here two results. In the following process, we consider the case which $m = 2$ and multidimension. Suppose dimensionality is d .

Suppose that $\{V_j\}$ is a MRA in $L^2(R^d)$, $\varphi(x)$ is the corresponding scaling function and symbol function is

$$m_0(\xi) = 2^{-\frac{d}{2}} \sum_k h_k e^{-ik\xi}.$$

Then the refinable equation is

$$\varphi(x) = 2^{\frac{d}{2}} \sum_k h_k \varphi(2x - k). \tag{9}$$

Let $d \le 3$, then, there exists a one-to-one mapping χ from E_d to E_d satisfying

$$\chi(0) = 0, \quad (\chi(\nu) + \chi(\mu))(\nu + \mu) \quad \text{is odd}, \quad \forall \nu, \mu \in E_d, \nu \neq \mu,$$

where E_d denotes the set of all vertices of unit cube $[0,1]^d$ in R^d (one refers to [16]). Then there is the following conclusion.

Theorem 9 *Assume that* $\varphi(x) = \bar{\varphi}(c - x)$, $c \in Z^d$, $m_0(\mu\pi) = \delta_{0,\mu}$, $m_0(\xi) \in C(R^d)$, $d \le 3$, *and*

$$\sum_{\mu \in E_d} |m_0(\xi + \mu\pi)|^2 = 1.$$

Define

$$m_\nu(\xi) = \begin{cases} e^{i\chi(\nu)\xi} m_0(\xi + \nu\pi), & \text{if} \quad \nu \cdot c \quad \text{is even and} \quad \nu \neq 0, \\ e^{i\chi(\nu)\xi} \bar{m}_0(\xi + \nu\pi), & \text{if} \quad \nu \cdot c \quad \text{is odd and} \quad \nu \neq 0. \end{cases}$$

Then matrix $M(\xi) = (m_\nu(\xi + \mu\pi))_{\nu,\mu}$ *is unitary.*

By (9),

$$\varphi(x) = 2^d \sum_k d_k \varphi(2x - k),$$

where $d_k = 2^{-\frac{d}{2}} h_k$.

Let

$$P_{0,\nu}(\xi) = \sum_l h_{2l+\nu} e^{-il\xi}.$$

Then we have another result of matrix extension.

Theorem 10 *Suppose that the scaling function* φ *in Definition 1 satisfies* $\varphi(x) = \bar{\varphi}(c - x)$, $c \in Z^d$ *and* $d > 1$. *Let* $(P_\nu(\xi))_{\nu \in E_d} = (P_{0,\nu}(\xi))_{\nu \in E_d}$. *Define*

$$P_{\nu,\mu}(\xi) = \begin{cases} \frac{\bar{P}_\nu(1+P_0)}{1+\bar{P}_0}, & \nu \in E_d, \quad \nu \neq 0, \quad \mu = 0, \\ -\delta_{\nu,\mu} + \frac{\bar{P}_\nu P_\mu}{1+P_0}, & \nu, \mu \in E_d - \{0\}. \end{cases}$$

Then $(P_{\nu,\mu}(\xi))_{\nu,\mu \in E_d}$ *is unitary.*

Matrix extension in Theorem 10 is called A–extension . In fact, A– extension is equivalent to B–extension (see [10]). Thus we think that there is no difference between the two extension.

7. Matrix extension in periodic case

It is well known that one must solve matrix extension problem in order to obtain the construction of wavelets. As for nonperiodic case, matrix extension problem is a very difficult one which was introduced in Section 6. However, the periodic case is different from nonperiodic one. In [17](also see [18]), we discussed the problem and completely solved matrix extension problem of periodic case by using matrix decomposition. Moreover, we presented the constructive process of multidimensional biorthogonal periodic multiwavelets with a dilation matrix A, where the dilation matrix A means that it is a integer matrix whose eigenvalues lie outside the closed unit disk. To the construction of multidimensional biorthogonal periodic multiwavelets, an algorithm was given and an example was also provided.

8. Others

There are many other matrix applications in wavelet analysis. For example, in multidimensional wavelet theory, vector wavelet analysis , multidimensional wavelet analysis with matrix dilation, etc., matrix approach was frequently used. As space is limited, it is impossible to list all development and results. One refers to the references [10]–[20].

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小波分析中的矩阵方法

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摘 要: 本文系统总结了小波分析中的矩阵方法，给出了我们得到的一些相关结果.

关键词: 小波；矩阵；双正交性；对称性；矩阵扩充.