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Homomorphisms between Two Sets of Fuzzy Subsemigroups *

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Abstract: Let S and T be semigroups. $F(S)$ and $F_s(S)$ denote the sets of all fuzzy subsets and all fuzzy subsemigroups of S, respectively. In this paper, we discuss the homomorphisms between $F(S)(F_s(S))$ and $F(T)(F_s(T))$. We introduce the concept of fuzzy quotient subsemigroup and generalize the fundamental theorems of homomorphism of semigroups to fuzzy subsemigroups.

Key words: fuzzy subsemigroup; fuzzy quotient semigroup; fuzzy congruence; homomorphism.

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1. Introduction

In this paper, S, T always denote semigroups. If S is homomorphic (isomorphic) to T, it is denoted by $S \sim T(S \cong T)$.

Let X be a non-empty set. We call a mapping $\mu : X \to [0,1]$ a fuzzy subset of X. The collection of all fuzzy subset of X is denoted by $F(X)$. A mapping $\mu : S \to [0,1]$ is called a fuzzy subsemigroup of S, if

$$\mu(a,b) \geq \mu(a) \wedge \mu(b)$$

for all $a, b \in S$. The set of all fuzzy subsemigroup of S is denoted by $F_s(S)$.

In [2],[3] and [4], the fuzzy congruences on a semigroup were studied with many results. In this paper, it is proved that $F(S)$ is a semigroup; $F_s(S)$ is a subsemigroup of $F(S)$; if $S \sim T(S \cong T)$, then

$$F(S) \sim F(T)(F(S) \cong F(T)), F_s(S) \sim F_s(T)(F_s(S) \cong F_s(T)), F(S) \sim F(S/\rho);$$

and if $\rho \leq \sigma$, then $F((S/\rho)/(\sigma/\rho)) \cong F(S/\sigma)$, where ρ, σ are fuzzy congruences on S. We introduce the concept of fuzzy quotient subsemigroup, and generalize the fundamental

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theorems of homomorphism of semigroup to fuzzy subsemigroup (see Theorem 4.9).

2. Preliminaries

A mapping $\rho : X \times X \to [0,1]$ is called a fuzzy relation on X. Let ρ and σ be two fuzzy relations on $X, \rho \leq \sigma$ means $\rho(a,b) \leq \sigma(a,b)$ for all $a, b \in X$.

Definition 2.1 *A fuzzy relation ρ on a semigroup S is called a fuzzy equivalence relation on S, if, for all $a, b, c \in S$,*

(1) $\rho(a,a) = 1$,

(2) $\rho(a,b) = \rho(b,a)$,

(3) $\rho(a,c) \geq \min(\rho(a,b), \rho(b,c)) = \rho(a,b) \wedge \rho(b,c)$.

Furthermore, if ρ also satisfies the following condition:

(4) $\rho(ab,cd) \geq \rho(a,c) \wedge \rho(b,d)$ *(or* $\rho(ac,bc) \geq \rho(a,b)$ *and* $\rho(ca,cb) \geq \rho(a,b)$*),*

then ρ is said to be a fuzzy congruence on S.

The notation $\mathrm{Con}_f(S)$ denotes the set of all fuzzy congruences on S.

It is not difficult to prove that a fuzzy relation ρ on S is a fuzzy congruence if and only if $\rho^t = \{(a,b) \in S \times S : \rho(a,b) \geq t\}$ is a congruence on S for all $t \in [0,1]$.

Let ρ be a fuzzy equivalence relation on S. For each $a \in S$ a fuzzy subset ρ_a of S is defined as $\rho_a(x) = \rho(a,x)$ for all $x \in S$[3].

Lemma 2.2[3] *Let ρ be a fuzzy equivalence (congruence) relation on S, and $a, b \in S$. Then*

$$\rho_a = \rho_b \textit{ if and only if } \rho(a,b) = 1.$$

Let $A, B \in F(S)$, then the product $A \circ B$ is defined by, for all $a \in S$,

$$(A \circ B)(a) = \begin{cases} \sup\limits_{a=xy} \min\{A(x), B(y)\}, \\ 0, \text{ if } a \text{ is not expressible as } a = xy. \end{cases} \tag{A}$$

For $\rho \in \mathrm{Con}_f(S)$, we write $S/\rho = \{\rho_a : a \in S\}$.

Lemma 2.3[3] *Let $\rho \in \mathrm{Con}_f(S)$ and $a, b \in S$. Define the multiplication in S/ρ by $\rho_a\rho_b = \rho_{ab}$ for all $\rho_a, \rho_b \in S/\rho$. Then S/ρ is a semigroup.*

Lemma 2.4 *For all $\rho \in \mathrm{Con}_f(S)$, S/ρ is isomorphic to S/ρ^1.*

Let f be a homomorphism from S onto T, $A \in F(S)$ and $B \in F(T)$. Define mappings

$$\theta_f : F(S) \to F(T), \theta_f(A)(t) = \sup_{f(s)=t} A(s) \text{ for each } t \in T, \tag{B}$$

$$\theta_f^{-1} : F(T) \to F(S), \theta_f^{-1}(B)(s) = B(f(s)) \text{ for each } s \in S \tag{C}$$

and fuzzy point $x_\lambda \in F(S) (\lambda \in (0,1])$

$$x_\lambda(a) = \begin{cases} \lambda, & a = x \\ 0, & a \neq x \end{cases} \quad \text{for all } a \in S.$$

3. The homomorphisms between $F(S)$ and $F(T)$

Lemma 3.1 *$F(S)$ is a semigroup with respect to the operation $\circ$.*

Proof It is obvious that $A \circ B \in F(S)$ for all $A, B \in F(S)$. Now, let $A, B, C \in F(S)$. Then, for $a \in S$, we suppose there exist $i, j, k \in S$ such that $a = ijk$, and $A(i) > 0, B(j) > 0$ and $C(k) > 0$.

$$\begin{aligned}
(A \circ (B \circ C))(a) &= \sup_{a=st} \min\{A(s), (B \circ C)(t)\} = \sup_{a=st} \min\{A(s), \sup_{t=uv} \min\{B(u), C(v)\}\} \\
&= \sup_{a=st} \min\{s_{A(s)}(s), \sup_{t=uv} \min\{u_{B(u)}(u), u_{C(v)}(v)\}\} \\
&= \sup_{a=st} \min\{s_{A(s)}(s), \sup_{t=uv} (uv)_{\min\{B(u), C(v)\}}\} \\
&= \sup_{a=st} \sup_{t=uv} \min\{s_{A(s)}(s), (uv)_{\min\{B(u), C(v)\}}\} \\
&= \sup_{a=st} \sup_{t=uv} \{(s(uv))_{\min\{A(s), B(u), C(v)\}}\} \\
&= \sup_{a=s(uv)} \{(s(uv))_{\min\{A(s), B(u), C(v)\}}\} \\
&= \sup_{a=xv} \sup_{x=su} \{((su)v)_{\min\{A(s), B(u), C(v)\}}\} \quad \text{(since } S \text{ is a semigroup)} \\
&= \sup_{a=xv} \sup_{x=su} \min\{(su)_{\min\{A(s), B(u)\}}, v_{C(v)}\} \\
&= \sup_{a=xv} \sup_{x=su} \min\{\min\{A(s), B(u)\}, v_{C(v)}\} \\
&= \sup_{a=xv} \min\{\sup_{x=su} \min\{A(s), B(u)\}, C(v)\} \\
&= \sup_{a=xv} \min\{(A \circ B)(x), C(v)\} = ((A \circ B) \circ C)(a).
\end{aligned}$$

It means that $F(S)$ is a semigroup. $\square$

Theorem 3.2 *If there is a homomorphism from S onto T, then $F(S)$ is homomorphic to $F(T)$.*

Proof According to the condition, there exists a homomorphism f from S onto T. By (B), we know that $\theta_f(A) \in F(T)$ for all $A \in F(S)$. Hence θ_f is a mapping from $F(S)$ to $F(T)$. Notice that, for all $B \in F(T), \theta_f^{-1}(B) \in F(S)$ and

$$\theta_f(\theta_f^{-1}(B))(t) = \sup_{f(s)=t} \theta_f^{-1}(B)(s) = \sup_{f(s)=t} B(f(s)) = B(t).$$

It is deduced that θ_f is surjective. Next, we only show $\theta_f(A \circ B) = \theta_f(A) \circ \theta_f(B)$ for all $A, B \in F(S)$. Let $A, B \in F(S)$. For $t \in T$, we suppose there exists $j, k \in S$ such that $f(jk) = t$, $A(j) > 0$ and $B(k) > 0$. Then,

$$\theta_f(A \circ B)(t) = \sup_{f(s)=t} (A \circ B)(s) = \sup_{f(s)=t} \sup_{s=jk} \min\{A(j), B(k)\},$$

and

$$(\theta_f(A) \circ \theta_f(B))(t) = \sup_{t=xy} \min\{\theta_f(A)(x), \theta_f(B)(y)\} = \sup_{t=xy} \min\{\sup_{f(u)=x} A(u), \sup_{f(v)=y} B(v)\}.$$

It is clear that

$$\sup_{f(s)=t}\sup_{s=jk}\min\{A(j),B(k)\}\le\sup_{t=xy}\min\{\sup_{f(u)=x}A(u),\sup_{f(v)=y}B(v)\}.$$

If $\theta_f(A\circ B)(t)<(\theta_f(A)\circ\theta_f(B))(t)$, then

$$\sup_{t=xy}\min\{\sup_{f(u)=x}A(u),\sup_{f(v)=y}B(v)\}-\sup_{f(s)=t}\sup_{s=jk}\min\{A(j),B(k)\}>0.$$

Hence there exist $u,v\in S$ and $\varepsilon>0$ satisfying $t=f(u)f(v)$ and

$$\min\{A(u),B(v)\}>\sup_{f(s)=t}\sup_{s=jk}\min\{A(j),B(k)\}+\varepsilon.$$

If we take $s=uv$, then

$$\sup_{s=jk}\min\{A(j),B(k)\}\ge\min\{A(u),B(v)\}>\sup_{f(s)=t}\sup_{s=jk}\min\{A(j),B(k)\}.$$

Therefore,

$$\sup_{f(s)=t}\sup_{s=jk}\min\{A(j),B(k)\}>\sup_{f(s)=t}\sup_{s=jk}\min\{A(j),B(k)\}.$$

It contradicts with the hypothesis. Hence $\theta_f(A\circ B)(t)=(\theta_f(A)\circ\theta_f(B))(t)$. Consequently, $\theta_f(A\circ B)=\theta_f(A)\circ\theta_f(B)$. □

Theorem 3.3 *$S\cong T$ implies $F(S)\cong F(T)$.*

Proof Let f be an isomorphic mapping from S onto T. By the proof of Theorem 3.2, θ_f is a homomorphic mapping from $F(S)$ onto $F(T)$. We only show θ_f is injective. If $\theta_f(A)=\theta_f(B)$ for $A,B\in F(S)$, then, for all $t\in T,\theta_f(A)(t)=\theta_f(B)(t)$, this is, $\sup_{f(s)=t}A(s)=\sup_{f(s)=t}B(s)$. Since f is an isomorphic mapping from S to T, there exists uniquely $s\in S$ such that $f(s)=t$ for each $t\in T$. Therefore, $A(s)=B(s)$ for all $s\in S$. So $A=B$. It shows that θ_f is injective. □

4. Fundamental theorems of homomorphisms

Lemma 4.1 *$F_s(S)$ is a subsemigroup of $F(S)$.*

Proof Since $F(S)$ is a semigroup, we only show that $F_s(S)$ is closed with respect to multiplication $\circ$. To do it, let $A,B\in F_s(S)$. Suppose there exist $x,y\in S$ such that $ab=xy$ and $A(x),B(y)>0$ for $a,b\in S$. Then

$$\begin{aligned}(A\circ B)(ab)&=\sup_{ab=xy}\min\{A(x),B(y)\}\\&\ge\sup_{ab=xy,x=ij,y=uv}\min\{\min\{A(i),A(j)\},\min\{B(u),B(v)\}\}\\&\ge\sup_{a=ij,b=uv}\min\{\min\{A(i),A(j)\},\min\{B(u),B(v)\}\}\\&\ge\min\{\sup_{a=ij}\min\{A(i),B(j)\},\sup_{b=uv}\min\{A(u),B(v)\}\}\\&=(A\circ B)(a)\wedge(A\circ B)(b).\end{aligned}$$

Therefore, $A \circ B \in F_s(S)$. $\square$

For $\rho \in \mathrm{Con}_f(S)$ and $A \in F_s(S)$, a fuzzy set $A/\rho \in F_s(S/\rho)$ is defined as follows: for all $\rho_a \in S/\rho$

$$(A/\rho)(\rho_a) = \sup_{\rho(x,a)=1} A(x).$$

We say A/ρ is a fuzzy quotient subsemigroup of S with respective to the fuzzy congruence ρ.

Lemma 4.2 *Let $\rho \in \mathrm{Con}_f(S)$, then A/ρ is a fuzzy subsemigroup of S/ρ.*

Proof First, it is clear that A/ρ is a mapping from S/ρ to [0,1]. Next, for all $\rho_a, \rho_b \in S/\rho$,

$$\begin{aligned}(A/\rho)(\rho_a\rho_b) = (A/\rho)(\rho_{ab}) &= \sup_{\rho(x,ab)=1} A(x) \\ &= \sup_{\rho(y,a)=1,\rho(z,b)=1} A(yz) \geq \sup_{\rho(y,a)=1,\rho(z,b)=1} \min\{A(y), A(z)\} \\ &= \min\{\sup_{\rho(y,a)=1} A(y), \sup_{\rho(z,b)=1} A(z)\} = (A/\rho)(\rho_a) \wedge (A/\rho)(\rho_b).\end{aligned}$$

Hence, A/ρ is a fuzzy subsemigroup of S/ρ. $\square$

Theorem 4.3 *For $\rho \in \mathrm{Con}_f(S)$, there exists a homomorphism θ_ρ from $F_s(S)$ onto $F_s(S/\rho)$ satisfying $\theta_\rho(A) = A/\rho$ for all $A \in F_s(S)$.*

Proof For $\rho \in \mathrm{Con}_f(S)$, by Lemma 2.4, we know there exists an isomorphism $g : S/\rho^1 \to S/\rho$ satisfying $g(a\rho^1) = \rho_a$. According to Theorem 5.3 in page 22 of [1], there exists a homomorphism h from S onto S/ρ^1 such that, for all $a \in S, h(a) = a\rho^1$. Hence $f = gh$ is a homomorphism from S onto S/ρ such that $f(a) = \rho_a$ for all $a \in S$. Furthermore, let $A \in F_s(S)$. Then, for all $\rho_a \in S/\rho$,

$$\begin{aligned}\theta_f(A)(\rho_a) &= \sup_{f(x)=\rho_a} A(x) = \sup_{\rho_x=\rho_a} A(x) \\ &= \sup_{\rho(x,a)=1} A(x) \quad \text{(by Lemma 2.2)} \\ &= (A/\rho)(\rho_a).\end{aligned}$$

Therefore, $\theta_f(A) = A/\rho$ for all $A \in F_s(S)$. To show θ_f is surjective, let $B \in F_s(S/\rho)$. We define a fuzzy subsemigroup A of S given by, for all $s \in S$,

$$A(s) = B(f(s)).$$

Then, for all $\rho_a \in S/\rho$,

$$(A/\rho)(\rho_a) = \sup_{\rho(x,a)=1} A(x) = \sup_{\rho_x=\rho_a} A(x) = \sup_{f(x)=\rho_a} B(f(x)) = B(\rho_a).$$

$A/\rho = B$ shows that θ_f is surjection satisfying $\theta_f(A) = B$. According to proof of Theorem 3.2, θ_f is a homomorphism. Take $\theta_\rho = \theta_f$, as required. $\square$

We called θ_ρ a homomorphism induced by ρ from $F_s(S)$ onto $F_s(S/\rho)$.

Corollary 4.4 *Let $\rho \in \mathrm{Con}_f(S)$, then $F_s(S/\rho) = \{A/\rho : A \in S\}$.*

The next theorem is concerned with a more general situation.

Theorem 4.5 *Let f be a homomorphism from S onto T and $\rho \in \mathrm{Con}_f(S)$. If $\rho^1 \subsetneq \ker f(\rho^1 = \ker f)$, where*

$$\ker f = \{(a, b) \in S \times S : f(a) = f(b)\}.$$

Then there exists a homomorphism (isomorphism) α from $F_s(S/\rho)$ onto $F_s(T)$ such that $\alpha(A/\rho) = \theta_f(A)$ for all $A/\rho \in F_s(S/\rho)$ and the diagram

$$\begin{tikzcd}
F_s(S) \arrow[r, "\theta_f"] & F_s(T) \\
F_s(S/\rho) \arrow[u, "\theta_\rho"] \arrow[ur, "\alpha"'] &
\end{tikzcd}$$

commutes.

Proof We define $g : S/\rho \to T, g(\rho_a) = f(a)$ for all $\rho_a \in S/\rho$. If $\rho_a = \rho_b$ for $\rho_a, \rho_b \in S$, by Lemma 2.2, $\rho(a, b) = 1$. Hence $(a, b) \in \ker f$ and so $f(a) = f(b)$. That is to say g is a mapping.

For all $\rho_a, \rho_b \in S/\rho$,

$$g(\rho_a \rho_b) = g(\rho_{ab}) = f(ab) = f(a)f(b) = g(\rho_a)g(\rho_b).$$

Hence g is a homomorphism from S/ρ to T. Furthermore, for all $t \in T$,

$$\begin{aligned}\theta_g(A/\rho)(t) &= \sup_{g(\rho_s)=t} (A/\rho)(\rho_s) = \sup_{f(s)=t} (A/\rho)(\rho_s) = \sup_{f(s)=t} \sup_{\rho(x,s)=1} A(x) \\ &= \sup_{f(s)=t} A(s) = \theta_f(A)(t).\end{aligned}$$

Therefore, $\theta_g(A/\rho) = \theta_f(A)$. If $\rho^1 = \ker f$, then $f(a) = f(b)$ for $a, b \in S$ implies $g(\rho_a) = g(\rho_b)$. That is, θ_g is an isomorphism. Take $\alpha = \theta_g$, as required. □

Lemma 4.6 *Let $\rho \leq \sigma$ for $\rho, \sigma \in \mathrm{Con}_f(S)$. Then the fuzzy relation σ/ρ on S/ρ, defined by, for all $a, b \in S$,*

$$(\sigma/\rho)(\rho_a, \rho_b) = \sigma(a, b),$$

is a fuzzy congruence on S/ρ.

Proof Let $\rho \leq \sigma$ for $\rho, \sigma \in \mathrm{Con}_f(S)$. Suppose $\sigma(a, b) \neq \sigma(c, d)$ for $(a, b), (c, d) \in S \times S$. Then $(\rho_a, \rho_b) \neq (\rho_c, \rho_d)$. Otherwise, $\rho_a = \rho_c$ imply $\rho(a, c) = 1$ and $\rho(b, d) = 1$. Since $\rho \leq \sigma, \rho(a, c) \leq \sigma(a, c) = 1$ and $\rho(b, d) \leq \sigma(b, d) = 1$. We deduce $\sigma(a, b) = \sigma(c, d)$. It contradicts with the hypothesis. So σ/ρ is a mapping. Then, for all $\rho_a, \rho_b, \rho_c \in S/\rho$,

(1) $(\sigma/\rho)(\rho_a, \rho_a) = \sigma(a, a) = 1$;

(2) $(\sigma/\rho)(\rho_a, \rho_b) = \sigma(a, b) = \sigma(b, a) = (\sigma/\rho)(\rho_b, \rho_a)$;

(3) $(\sigma/\rho)(\rho_a, \rho_c) = \sigma(a,c) \geq \sigma(a,b) \wedge \sigma(b,c) = (\sigma/\rho)(\rho_a, \rho_b) \wedge (\sigma/\rho)(\rho_b, \rho_c)$;

(4) $(\sigma/\rho)(\rho_a\rho_c, \rho_b\rho_d) = (\sigma/\rho)(\rho_{ac}, \rho_{bc}) = \sigma(ac, bd) \geq \sigma(a,b) \wedge \sigma(c,d) = (\sigma/\rho)(\rho_a, \rho_b) \wedge (\sigma/\rho)(\rho_c, \rho_d)$.

It shows that σ/ρ is a fuzzy congruence on S/ρ. □

Theorem 4.7 *If $\rho \leq \sigma$ for $\rho, \sigma \in \mathrm{Con}_f(S)$, then there exists an isomorphism α from $F_s((S/\rho)/(\sigma/\rho))$ onto $F_s(S/\sigma)$ such that $\alpha((A/\rho)/(\sigma/\rho)) = A/\rho$ and the following diagram commutes.*

$$\begin{tikzcd}
F_s(S) \arrow[r, "\theta_\sigma"] & F_s(S/\sigma) \arrow[d, "\alpha"] \\
F_s(S/\rho) \arrow[u, "\theta_\rho"] \arrow[r, "\theta_{\sigma/\rho}"'] & F_s((S/\rho)/(\sigma/\rho))
\end{tikzcd}$$

Proof Since $\rho \leq \sigma, \rho^1 \subseteq \sigma^1$. Thus, we can define a mapping f from S/ρ to S/σ given by, for all $\rho_a \in S/\rho$,

$$f(\rho_a) = \sigma_a.$$

For $\rho_a, \rho_b \in S/\rho, f(\rho_a\rho_b) = f(\rho_{ab}) = \sigma_{ab} = \sigma_a\sigma_b = f(\rho_a)f(\rho_b)$. Hence f is a homomorphism. By Lemma 4.6, σ/ρ is a fuzzy congruence S/ρ. Notice

$$\begin{aligned}\ker f &= \{(\rho_a, \rho_b) \in S/\rho \times S/\rho : f(\rho_a) = f(\rho_b)\} = \{(\rho_a, \rho_b) \in S/\rho \times S/\rho : \sigma_a = \sigma_b\} \\ &= \{(\rho_a, \rho_b) \in S/\rho \times S/\rho : \sigma(a,b) = 1\} = (\sigma/\rho)^1.\end{aligned}$$

Since $F_s(S/\rho) = \{A/\rho : A \in F_s(S)\}$, we have

$$F_s((S/\rho)/(\sigma/\rho)) = \{(A/\rho)/(\sigma/\rho) : A/\rho \in F_s(S/\rho)\}.$$

By Theorem 4.5, there exists an isomorphism θ_f from $F_s((S/\rho)/(\sigma/\rho))$ to $F_s(S/\sigma)$ such that

$$\theta_f((A/\rho)/(\sigma/\rho)) = A/\sigma.$$

Hence $\alpha = \theta_f$. □

Definition 4.8 *Let f be a homomorphism (isomorphism) from S onto T. If $\theta_f(A) = B$ for $A \in F_s(S)$ and $B \in F_s(S)$. Then we say A is homomorphic (isomorphism) onto B, which is denoted as $A \sim B (A \cong B)$.*

The following results is from Theorem 4.5 and 4.7.

Theorem 4.9 *Let f be a homomorphism (isomorphism) from S onto T. The following results are true.*

(1) If $\rho^1 \subsetneq \ker f (\rho^1 = \ker f)$ for $\rho \in \mathrm{Con}_f(S)$, then $A/\rho \sim \theta_f(A) (A/\rho \cong \theta_f(A))$.

(2) If $\rho \leq \sigma$ for $\rho, \sigma \in \mathrm{Con}_f(S)$, then $(A/\rho)/(\sigma/\rho) \cong A/\sigma$.

Remark 4.10 In fact, Theorem 4.9 is a generalization of fundamental theorems of homomorphisms of semigroups in fuzzy subsemigroups.

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两个模糊子半群集合之间的同态

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摘 要: 设 S,T 是半群， $F(S)$ 和 $F_s(S)$ 分别表示 S 的所有模糊子集的集合和所有模糊子半群的集合. 文中，讨论了 $F(S)(F_s(S))$ 和 $F(T)(F_s(T))$ 之间的模糊同态，建立了模糊商子半群的概念，把分明半群的基本同态定理推广到模糊子半群.

关键词: 模糊子半群；模糊商半群；模糊同余；同态.