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P_n^3 is a Graceful Graph *

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Abstract: Let $G(V,E)$ be a simple graph and G^k be a k-power graph defined by $V(G^k)=V(G), E(G^k)=E(G)\cup\{uv|d(u,v)=k\}$ for natural number k. In this paper, it is proved that P_n^3 is a graceful graph.

Key words: graph P_n^3; graceful labelling; graceful graph.

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1. Introduction

The graceful of graph has been widely studied since G .Kingd proposed the conjecture "all trees are graceful" in 1960's. A series of results on graceful of special graphs were obtained. In this paper, the graceful of a class graph is given.

Definition 1 *Let $G(V,E)$ be a simple graph with n vertices and m edges. A mapping $f{:}V(G)\to\{0,1,2,\cdots,m\}$, f is called a graceful labelling of graph G with a graceful graph G, if is satisfies*

1 f *be an injection;*

2 $\max\{f(v)|v\in V\}=m$;

3 $f(uv)=|f(u)-f(v)|, e=uv\in E$;

4 $\{f(uv)|uv\in E\}=\{1,2,\cdots,m\}$.

Definition 2 *Let $G(V,E)$ be a simple graph and k be a natural number, $V(G^k)=V(G), E(G^k)=E(G)\cup\{uv|d(u,v)=k\}$, that is iff the distance is k between two vertices of G, an edge is added. G^k is callled k-power graph of G.*

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Let P_n be a path of order n.

Theorem 1 *When $n = 4, 5$, P_n^3 is graceful.*
The theorem is obviously true.

Theorem 2 *For $n \geq 6$, P_n^3 is graceful.*

Proof Now to prove the theorem by six cases.

Case 1. When $n = 6k$.
A labelling mapping f of P_n^3 is given as follows.

$$f(v_{6i}) = 7i, f(v_{6i+}) = 7i + 2, f(v_{6i+4}) = 7i + 5,$$

$$f(v_{6i+1}) = 12k - 5i - 4, f(v_{6i+3}) = 12k - 5i - 5, f(v_{6i+5}) = 12k - 5i - 6,$$

where $i = 0, 1, 2, \cdots, k - 1$. Now f is going to be proved a graceful labelling.

(1) According to the labelling above, labels of even vertices arrange from small to large, the maximum is $7k - 2$; labels of odd vertices arrange from large to small, the minimum is $7k - 1$; then f is an injection.

(2) $\max\{f(v)|v \in v\} = f(v_1) = 12k - 4 = |E(P_n^3)|$.

(3) For $i \neq j$,

$$|f(v_i) - f(v_{i+1})| \neq |f(v_j) - f(v_{j+1})|; |f(v_i) - f(v_{i+3})| \neq |f(v_j) - f(v_{j+3})|;$$
$$|f(v_i) - f(v_{i+1})| \neq |f(v_i) - f(v_{i+3})|.$$

and moreover

$$\max\{f(uv)|uv \in E\} = \max\{|f(v_i) - f(v_{i+1})|, |f(v_i) - f(v_{i+3})|\} = f(v_{6k-1}) - f(v_{6k-2}) = 1$$

As the labels of all edges are $\{1, 2, \cdots, 12k - 4\}$, f is a graceful labelling of P_n^3.

Case 2. When $n = 6k + 1$.
A labelling mapping f of P_n^3 is given as follows

$$f(v_{6i}) = 7i, f(v_{6i+2}) = 7i + 2, f(v_{6i+4}) = 7i + 5,$$

$$f(v_{6i+1}) = 12k - 5i - 2, f(v_{6i+3}) = 12k - 5i - 3, f(v_{6i+5}) = 12k - 5i - 4,$$

where $i = 0, 1, 2, \cdots, k - 1$. It is similar to lemma 2 to be proved f is a graceful labelling.

Case 3. When $n = 6k + 2$.
A labelling mapping f of P_n^3 is given as follows

$$f(v_0) = 0, f(v_1) = 12k, f(v_3) = 12k - 1,$$

$$f(v_{6i+2}) = 7i + 2, f(v_{6i+4}) = 7i + 4, f(v_{6i+6}) = 7i + 6,$$

$$f(v_{6i+5}) = 12k - 5i - 4, f(v_{6i+7}) = 12k - 5i - 5,$$

where $i = 0, 1, 2, \cdots, k-1$; $f(v_{6i+9}) = 12k - 5i - 6$, where $i = 0, 1, 2, \cdots, k-2$. It is similar to lemma 2 to be prove f is a graceful labelling.

Case 4. When $n = 6k + 3$.

A labelling mapping f of P_n^3 is given as follows

$$f(v_0) = 0, f(v_1) = 12k + 2, f(v_2) = 2,$$

$$f(v_{6i+4}) = 7i + 5, f(v_{6i+6}) = 7i + 8, f(v_{6i+8}) = 7i + 10,$$

$$f(v_{6i+3}) = 12k - 5i + 1, f(v_{6i+5}) = 12k - 5i, f(v_{6i+7}) = 12k - 5i - 1,$$

where $i = 0, 1, 2, \cdots, k-1$. It is similar to Lemma 2 to be proved f is a graceful labelling.

Case 5. When $n = 6k + 4$.

A labelling mapping f of P_n^3 is given as follows

$$f(v_0) = 0, f(v_1) = 12k + 4, f(v_3) = 12k + 3,$$

$f(v_{6i+2}) = 7i + 2$, where $i = 0, 1, 2, \cdots, k$; $f(v_{6i+4}) = 7i + 4, f(v_{6i+6}) = 7i + 6$, $f(v_{6i+5}) = 12k - 5i + 2, f(v_{6i+7}) = 12k - 5i + 1, f(v_{6i+9}) = 12k - 5i$, where $i = 0, 1, 2, \cdots, k-1$. It is similar to lemma 2 to be proved f is a graceful labelling.

Case 6. When $n = 6k + 4$.

A labelling mapping f of P_n^3 is given as follows

$$f(v_0) = 0, f(v_1) = 12k + 6, f(v_3) = 12k + 5,$$

$f(v_{6i+2}) = 7i + 2, f(v_{6i+4}) = 7i + 4$, where $i = 0, 1, 2, \cdots, k; f(v_{6i+6}) = 7i + 6, f(v_{6i+5}) = 12k - 5i + 4, f(v_{6i+7}) = 12k - 5i + 3, f(v_{6i+9}) = 12k - 5i + 2$, where $i = 0, 1, 2, \cdots, k-1$. It is similar to lemma 2 to be proved f is a graceful labelling.

This completes the proof.

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关于 P_n^3 的优美性

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摘 要: 设 $G(V, E)$ 是一个简单图, 对自然数 k, 当 $V(G^k) = V(G), E(G^k) = E(G) \cup \{uv | d(u, v) = k\}$, 则称图 G^k 为 k- 次方图. 本文证明了图 P_n^3 的优美性.

关键词: P_n^3 图; 优美标号; 优美图.