Journal of Mathematical Research & Exposition
Vol.24, No.3, 421–429, August, 2004

Perturbation Analysis for the Drazin Inverses of Bounded Linear Operators on a Banach Space *

CAI Jing[1], *CHEN Guo-liang*[2]

(1. Faculty of Science, Huzhou Teachers' College, Zhejiang 313000, China;

2. Dept. of Math., East China Normal University, Shanghai 200062, China)

Abstract: Let X be a Banach space over the complex field C and let $T : X \to X$ be a bounded linear operator with $\mathrm{Ind}(T) = k$ and $R(T^k)$ closed. Denote the Drazin inverse of T by T^D. Let $\bar{T} = T + \delta T$, then $\bar{T}^D$ has the simple expression $\bar{T}^D = T^D(I + \delta T T^D)^{-1} = (I + T^D \delta T)^{-1} T^D$ under certain hypotheses. The upper bound for the relative error $\|\bar{T}^D - T^D\|/\|T^D\|$ and for the solution to the operator equation: $Tx = u$ $(u \in R(T^D))$ is also considered.

Key words: Drazin inverse; index; error bound.

Classification: AMS(2000) 47A05, 47A55, 65J10/CLC number: O151.21

Document code: A **Article ID**: 1000-341X(2004)03-0421-09

1. Introduction

The concept of the Drazin inverse of a bounded linear operator on a Banach space (to be defined in Section 2) appears in Caradus[1]. Its existence, uniqueness and basic properties were stated by Qiao[2]. Then Kuang[3] and Cai[4] studied its characterization and representation. Its applications in the infinite dimensional linear system were discussed by Campell[5] and Caradus[1]. Later Koliha and Vladimir[6,7] investigated its continuity.

Recently, there are a series of papers [8,9,10] concerning the expression of the generalized inverse of a perturbed linear operator on Hilbert spaces or Banach spaces. In the case of the Dazin inverse of a sqaure matrix, it was shown in [11] that if A and $B \in C^{n\times n}$ satisfy $R(E) \subseteq R(A^k), R(E^*) \subseteq R(A^{k*})$ and $\|A^D E\| < 1$, where $k = \mathrm{Ind}(A)$, then

$$B^D = A^D(I + EA^D)^{-1} = (I + A^D E)^{-1} A^D. \tag{1.1}$$

If , in addition, one has $\|A^D\|\|E\| < 1$, then

$$\frac{\|B^D - A^D\|}{\|A^D\|} \le \frac{K_D(A)\|E\|/\|A\|}{1 - K_D(A)\|E\|/\|A\|}\ , \tag{1.2}$$

*Received date: 2001-07-03
Foundation item: Supported by National Natural Science Foundation of China (19871029)

where $K_D(A) = \|A\|\|A^D\|$ is defined as the condition number with respect to the Dazin inverse. Later Wei and Wang[12] obtain (1.1) ((1.2)) under the assumption that $E = AA^DE = EAA^D$ and $\|A^DE\| < 1$ (in addition $\|A^D\|\|E\| < 1$). We should remark that the two sufficient conditions above are actually equivalent. Let L and M be complementary subspaces of C^n. $P_{L,M}$ denotes a projection of C^n onto L along M. Moreover, let $T \in C^{n\times n}$, then $P_{L,M}T = T$ if and only if $R(T) \subseteq L$ and $TP_{L,M} = T$ if and only if $N(T) \supseteq M$ (see Ben-Israel[13]). Since $AA^D = A^DA = P_{R(A^D),N(A^D)} = P_{R(A^k),N(A^k)}$ (see also Ben-Israel[13]). Then we have $E = AA^DE = EAA^D$ if and only if $R(E) \subseteq R(A^k)$ and $N(E) = R(E^*)^{\perp} \supseteq N(A^k) = R(A^{k*})^{\perp}$ which is equivalent to $R(E^*) \subseteq R(A^{k*})$.

Recently, Wei and Wu[14] presented a sufficient and necessary condition such that B^D has the simple form (1.1) as follows:

Let $B = A + E \in C^{n\times n}$ with $\text{Ind}(A) = k$ and $\text{Ind}(B) = j$. Let $l = \max\{k, j\}$ and $E(A^l) = B^l - A^l$. If $\|A^DE\| < 1$, then B^D has the expression (1.1) if and only if

$$\text{rank}(B^j) = \text{rank}(A^k) \text{ and } AA^DE(A^l) = E(A^l) = E(A^l)AA^D. \tag{1.3}$$

Let X be a Banach space over the complex field C. $L(X)$ denotes the vector space of all linear operators: $T : X \to X$. Let $B(X)$ denote the Banach space of all bounded linear operators $T : X \to X$ with the norm $\|T\| = \sup\ \{\ \|Tx\| : \|x\| = 1, x \in X\ \}$. For T and $\bar{T} = T + \delta T \in B(X)$, $R(T)$ and $N(T)$ denote the range and the null space of T. Let dim(A) denote the dimension of the subspace A of X. In this paper, we aim to extend the above results in [11,12,14] to the case of Drazin inverses of bounded linear operators on a Banach space. However, things are somewhat complicated in this case. As for T and $\bar{T} = T + \delta T \in B(X)$, sufficient and necessary condition such that $\bar{T}^D$ has the simple expression $\bar{T}^D = T^D(I + \delta TT^D)^{-1} = (I + T^D\delta T)^{-1}T^D$ can't be replaced by the following condition:

$$\dim R(\bar{T}^j) = \dim R(T^k) \text{ and } TT^DE(T^l) = E(T^l) = E(T^l)TT^D. \tag{1.4}$$

which reduces to the known result (1.3) for the square matrix case. Since if X is an finite dimensional Banach space, then $\dim R(\bar{T}^l) = \dim R(T^l)$ and $R(\bar{T}^l) \subseteq R(T^l)$ following from (1.4) imply $R(\bar{T}^l) = R(T^l)$. While it's not correct for the case that X is an infinite dimensional Banach space.

In the next section, we give some necessary concept and preliminary results. We present our main results in section 3. In section 4, we give an error bound for the solution for operator equation:$Tx = u$ ($u \in R(T^D)$). Conclusions will be put in section 5.

2. Preliminaries

First, let us recall that $\alpha(T)$ ($\delta(T)$), the ascent (descent) of $T \in B(X)$ is the smallest non-negative n such that $N(T^n) = N(T^{n+1})$ ($R(T^n) = R(T^{n+1})$). If no such n exists, then $\alpha(T) = \infty$ ($\delta(T) = \infty$). If T has finite ascent and descent, then *the index* of T is equal to the ascent of T (see [15]).

Definition 2.1[2] *Let $T \in L(X)$. If for some non-negative integer k, there exists $S \in L(X)$ such that*

$$TST^k = T^k, STS = S, TS = ST, \tag{2.1}$$

then S *is called* the Dazin inverse *of* T *and is denoted by* $S = T^D$. *In particular, when* k*=1, the operator* S *satisfying (2.1) is called* the group inverse *of* T *and is denoted by* $S = T^{\#}$.

We list several useful properties of T^D in the following lemma. For more details, see [2].

Lemma 2.1[2] *Let* $T \in B(X)$ *with Ind*$(T) = k$ *and* $R(T^k)$ *closed,* $l \geq k$*, then*

(1) $R(T^D) = R(T^l), N(T^D) = N(T^l)$,

(2) $X = R(T^D) \oplus N(T^D)$ *(topological direct sum decomposition),*

(3) $TT^D = T^DT = P_{R(T^D),N(T^D)} = P_{R(T^l),N(T^l)}$,

where $P_{R(T^D),N(T^D)}$ *denotes the projection of* X *onto* $R(T^D)$ *along* $N(T^D)$.

Definition 2.2[10] *Let* X_1, X_2 *be two Banach spaces over the complex field* C *and let* $B(X_1, X_2)$ *be the Banach space of all bounded linear operators:*$T : X_1 \to X_2$. *Let* $T \in B(X_1, X_2)$ *with* $R(T)$ *closed. If there exist two projections (idempotents) of* $P : X_1 \to N(T), Q : X_2 \to R(T)$*, then* T *has uniquely the generalized inverse* T^+ *(with respect to P,Q) such that*

$$TT^+T = T, T^+TT^+ = T^+, T^+T = I - P, TT^+ = Q \tag{2.2}$$

If X_1, X_2 *are Hilbert space, we require* $(T^+T)^* = T^+T, (TT^+)^* = TT^+$.

Lemma 2.2[4] *Let* $T \in B(X)$ *with Ind*$(T) = k$*, then there exists* $T^{k+} \in L(X)$ *such that*

$$\begin{aligned} T^kT^{k+}T^k &= T^k \\ T^{k+}T^kT^{k+} &= T^{k+} \\ T^{k+}T^k &= T^kT^{k+} = P_{R(T^k),N(T^k)} = I - P_{N(T^k),R(T^k)}. \end{aligned}$$

Remark 2.1 Noting the fact that $R(T^k) = R(T^l), N(T^k) = N(T^l) (l \geq k)$, thus if we replace k by l in lemma 2.2, the result still holds.

Lemma 2.3[16,p.185] *Let* $T \in B(X)$ *with* $\|T\| < 1$*, then* $I + T$ *is invertible and* $(I + T)^{-1} : X \to X$ *is bounded. Moreover*

$$\|(I + T)^{-1}\| \leq \frac{1}{1 - \|T\|}. \tag{2.3}$$

The proof of the following lemmas is easy, thus omitted here.

Lemma 2.4 *Let* L *and* M *be complementary subspaces of* X*, and let* $P_{L,M}$ *be a projection of* X *onto* L *along* M*,* $T \in B(X)$*, then*

(1) $P_{L,M}T = T$ *if and only if* $R(T) \subseteq L$,

(2) $TP_{L,M} = T$ *if and only if* $N(T) \supseteq M$.

Lemma 2.5 *Let* T, U *and* $V \in B(X)$*, if* U *and* V *are invertible, then*

(1) $U^{-1}R(UT) = R(T) = R(TV)$,

(2) $N(UT) = N(T) = VN(TV)$.

3. Main results

Now we establish the sufficient and necessary condition such that $\bar{T}^D = T^D(I + \delta TT^D)^{-1} = (I + T^D\delta T)^{-1}T^D$.

Theorem 3.1 *Let $T \in B(X)$ with $\mathrm{Ind}(T) = k$ and $R(T^k)$ closed. Let $\bar{T} = T + \delta T \in B(X)$ with $\mathrm{Ind}(\bar{T}) = j$ and $\|T^D\|\|\delta T\| < 1$. Denote $l = \max\{k, j\}$ and $E(T^l) = \bar{T}^l - T^l$, then*

$$\bar{T}^D = T^D(I + \delta TT^D)^{-1} = (I + T^D\delta T)^{-1}T^D \tag{3.1}$$

if and only if

$$\bar{T}\bar{T}^D E(T^l) = E(T^l) = E(T^l)\bar{T}\bar{T}^D \text{ and } TT^D E(T^l) = E(T^l) = E(T^l)TT^D. \tag{3.2}$$

Proof From the assumption that $\|T^D\|\|\delta T\| < 1$, we get both $I + T^D\delta T$ and $I + \delta TT^D$ are invertible. Since $(I + \delta TT^D)^{-1} = \sum_{n=0}^{\infty}(-1)^n(\delta TT^D)^n$, we have

$$T^D(I + \delta TT^D)^{-1} = (I + T^D\delta T)^{-1}T^D. \tag{3.3}$$

Necessarity: It follows from (3.1) and lemma 2.5 that

$$R(\bar{T}^D) = R(T^D), N(\bar{T}^D) = N(T^D). \tag{3.4}$$

Hence, by Lemma 2.1 (1), we have

$$R(\bar{T}^l) = R(T^l), N(\bar{T}^l) = N(T^l). \tag{3.5}$$

Thus we have

$$R(E(T^l)) \subseteq R(T^l), N(E(T^l)) \supseteq N(T^l)$$

and

$$R(E(T^l)) \subseteq R(\bar{T}^l), N(E(T^l)) \supseteq N(\bar{T}^l).$$

According to Lemma 2.1 (3) and Lemma 2.4, we get

$$\bar{T}\bar{T}^D E(T^l) = E(T^l) = E(T^l)\bar{T}\bar{T}^D \text{ and } TT^D E(T^l) = E(T^l) = E(T^l)TT^D.$$

Sufficiency: From the assumption $TT^D E(T^l) = E(T^l) = E(T^l)TT^D$ and Lemma 2.4, we have

$$R(E(T^l)) \subseteq R(T^l), N(E(T^l)) \supseteq N(T^l),$$

which implies

$$R(\bar{T}^l) \subseteq R(T^l), N(\bar{T}^l) \supseteq N(T^l). \tag{3.6}$$

On the other hand, it follows from the assumption $\bar{T}\bar{T}^D E(T^l) = E(T^l) = E(T^l)\bar{T}\bar{T}^D$ and Lemma 2.4 that

$$R(E(T^l)) \subseteq R(\bar{T}^l), N(E(T^l)) \supseteq N(\bar{T}^l).$$

Thus

$$R(T^l) \subseteq R(\bar{T}^l), N(T^l) \supseteq N(\bar{T}^l). \tag{3.7}$$

Hence (3.6) and (3.7) give

$$R(\bar{T}^D) = R(\bar{T}^l) = R(T^l) = R(T^D) \tag{3.8}$$

and

$$N(\bar{T}^D) = N(\bar{T}^l) = N(T^l) = N(T^D). \tag{3.9}$$

Thus by (3.8) (3.9) and Lemma 2.1 (3), we obtain

$$\bar{T}\bar{T}^D = TT^D. \tag{3.10}$$

It follows that

$$\begin{aligned}\bar{T}^D - T^D &= -\bar{T}^D\delta TT^D + \bar{T}^D - T^D + \bar{T}^D(\bar{T} - T)T^D \\ &= -\bar{T}^D\delta TT^D + \bar{T}^D - \bar{T}^D TT^D - T^D + \bar{T}\bar{T}^D T^D \\ &= -\bar{T}^D\delta TT^D.\end{aligned}$$

i.e., $\bar{T}^D(I + \delta TT^D) = T^D$. Because of the invertibility of $I + \delta TT^D$ and (3.3), we have

$$\bar{T}^D = T^D(I + \delta TT^D)^{-1} = (I + T^D\delta T)^{-1}T^D.$$

□

Note that the condition in Theorem 3.1 is difficult to verify in practice. Next we give a sufficient condition which is easy to check.

For convenience, we denote the following condition by (W) condition:

Let $T \in B(X)$ with $\mathrm{Ind}(T) = k$ and $R(T^k)$ colsed. Let $\bar{T} = T + \delta T \in B(X)$ with $\mathrm{Ind}(\bar{T}) = j$. Denote $l = max\{k, j\}$ and $E(T^l) = \bar{T}^l - T^l$. Denote $\varepsilon(T^l) = \sum_{i=0}^{l-1} C_l^i \|T\|^i \|\delta T\|^{l-i} \geq \|E(T^l)\|$, where C_l^i is the binomial coefficient. Suppose $R(\delta T) \subseteq R(T^l)$ and $N(\delta T) \supseteq N(T^l)$ and $\|T^{l+}\|\varepsilon(T^l) < 1$.

Then we can prove the following lemma.

Lemma 3.1 *Suppose (W) condition holds, then*

$$R(\bar{T}^l) = R(T^l), N(\bar{T}^l) = N(T^l). \tag{3.11}$$

Proof Since

$$\begin{aligned}E(T^l) =& \bar{T}^l - T^l \\ =& (T^{l-1}\delta T + T^{l-2}\delta T \cdot T + \cdots + \delta T \cdot T^{l-1}) + \cdots + \\ & [T(\delta T)^{l-1} + \delta T \cdot T(\delta T)^{l-2} + \cdots + (\delta T)^{l-1}T] + (\delta T)^l,\end{aligned} \tag{3.12}$$

by assumption that $R(\delta T) \subseteq R(T^l)$, we can show that

$$R(E(T^l)) \subseteq R(T^l). \tag{3.13}$$

On the other hand, $\forall x \in N(T^l)$, for any $0 \le m \le l$, i.e.,

$$T^{l-m}x \in N(T^m) \subseteq N(T^l) \subseteq N(\delta T). \tag{3.14}$$

Then it follows from (3.12) that $E(T^l) \cdot x = 0$, i.e.,

$$N(E(T^l)) \supseteq N(T^l). \tag{3.15}$$

According to Lemma 2.2, Remark 2.1 and Lemma 2.4, we obtain

$$\begin{aligned}\bar{T}^l &= T^l + E(T^l)\\ &= T^l + P_{R(T^l),N(T^l)}E(T^l) = T^l + E(T^l)P_{R(T^l),N(T^l)}\\ &= T^l + T^lT^{l+}E(T^l) = T^l + E(T^l)T^{l+}T^l\\ &= T^l[I + T^{l+}E(T^l)] = [I + E(T^l)T^{l+}]T^l\end{aligned}$$

in which, by virtue of $\|T^{l+}E(T^l)\| \le \|T^{l+}\|\varepsilon(T^l) < 1$, $I + T^{l+}E(T^l)$ is invertible. Similarly $I + E(T^l)T^{l+}$ is also invertible.

Hence it follows from Lemma 2.5 that

$$R(\bar{T}^l) = R(T^l), N(\bar{T}^l) = N(T^l).$$

□

Remark 2.2 The (W) condition in Lemma 3.1 is easy to verify.

For example, let $T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\delta T = \begin{pmatrix} 0 & \varepsilon & 0 \\ \varepsilon & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, where $0 < \varepsilon < 1$, and $\bar{T} = T + \delta T$.

Since $\text{rank}(T) = \text{rank}(T^2) = 2$ and $\text{rank}(\bar{T}) = \text{rank}(\bar{T}^2) = 2$, thus $\text{Ind}(T) = \text{Ind}\bar{T} = 1$ and so T has a group inverse $T^{\#}$. Moreover, $l = \max\{\text{Ind}(T), \text{Ind}(\bar{T})\} = 1$, $E(T^l) = \bar{T}^l - T^l = \bar{T} - T = \delta T$ and T^{l+} in (W) condition becomes T^+. Let $\|\cdot\|_2$ denote the spectral norm. Then $\varepsilon(T^l) = \|\delta T\|_2 = \varepsilon$.

On the other hand, it is easily verified that $R(\delta T) \subseteq R(T)$ and $N(\delta T) \supseteq N(T)$. Noting that T^{k+} given by Lemma 2.2 where $k = \text{Ind}(T)$ reduces to $(T^k)^{\#}$ for the square matrix case. Then by direct computation, we have $\|T^{l+}\|_2\varepsilon(T^l) = \|T^+\|_2\|\delta T\|_2 = \|T^{\#}\|_2\|\delta T\|_2 = \varepsilon < 1$. Therefore (W) condition holds in this case.

Next we present a theorem bounding $\frac{\|\bar{T}^D - T^D\|}{\|T^D\|}$.

Theorem 3.2 *Let $T, \bar{T}$ be as in Lemma 3.1. Denote $K_D(T) = \|T\|\|T^D\|$, then*

$$\bar{T}^D = T^D(I + \delta T T^D)^{-1} = (I + T^D\delta T)^{-1}T^D,$$

$$\|\bar{T}^D\| \le \frac{\|T^D\|}{1 - \|T^D\delta T\|},$$

$$\frac{\|\bar{T}^D - T^D\|}{\|T^D\|} \le \frac{K_D(T)\|\delta T\|/\|T\|}{1 - K_D(T)\|\delta T\|/\|T\|}.$$

Proof It follows from Lemma 3.1 and Lemma 2.1 (3) that

$$\bar{T}\bar{T}^D = TT^D.$$

Then from the proof of Theorem 3.1, we get

$$\bar{T}^D = T^D(I + \delta TT^D)^{-1} = (I + T^D\delta T)^{-1}T^D.$$

By taking $\|\cdot\|$ of both sides and using Lemma 2.3, we get

$$\|\bar{T}^D\| \le \frac{\|T^D\|}{1 - \|T^D\delta T\|}.$$

Since $\bar{T}^D - T^D = -\bar{T}^D\delta TT^D$, we have

$$\|\bar{T}^D - T^D\| \le \|\bar{T}^D\|\|T^D\|\|\delta T\| \le \frac{\|T^D\|^2\|\delta T\|}{1 - \|T^D\|\|\delta T\|}.$$

Consequently,

$$\frac{\|\bar{T}^D - T^D\|}{\|T^D\|} \le \frac{\|T^D\|\|\delta T\|}{1 - \|T^D\|\|\delta T\|} = \frac{K_D(T)\|\delta T\|/\|T\|}{1 - K_D(T)\|\delta T\|/\|T\|}.$$

4. Applications

In this section, we give an error bound for the solution for the operator equation:

$$Tx = u \quad (u \in R(T^D)) \ . \tag{4.1}$$

It is well known that we actually compute the perturbed system:

$$\bar{T}y = \bar{u} \quad (\bar{u} \in R(\bar{T}^D)), \tag{4.2}$$

where $\bar{T} = T + \delta T, \bar{u} = u + \delta u$.

We simply estimate the distance between the exact solution $T^D u$ and $\bar{T}^D\bar{u}$ for (4.1) and (4.2).

Theorem 4.1 *Let T and $\bar{T}$ be as in Lemma 3.1, and denote $x = T^D u, y = \bar{T}^D\bar{u}, K_D(T) = \|T\|\|T^D\|$, then*

$$\frac{\|y - x\|}{\|x\|} \le \frac{K_D(T)}{1 - K_D(T)\|\delta T\|/\|T\|}(\|\delta T\|\|T\| + \frac{\|\delta u\|}{\|u\|}). \tag{4.3}$$

Proof From the assumption it follows that

$$\begin{aligned} y-x &= \bar{T}^D\bar{u} - T^D u \\ &= \bar{T}^D(u+\delta u) - T^D u = (\bar{T}^D - T^D)u + \bar{T}^D\delta u \\ &= -\bar{T}^D\delta T T^D u + \bar{T}^D\delta u. \end{aligned}$$

By taking $\|\cdot\|$ of both sides and noting that $\|u\| \le \|T\|\|x\|$, we get

$$\begin{aligned} \|y-x\| &\le \|\bar{T^D}\|\|\delta T\|\|x\| + \|\bar{T^D}\|\|\delta u\| \\ &\le \frac{\|T^D\|}{1-\|T^D\|\|\delta T\|}(\|\delta T\|\|x\| + \frac{\|\delta u\|}{\|u\|}\|T\|\|x\|) \\ &= \frac{\|T^D\|\|T\|\|x\|}{1-\|T^D\|\|\delta T\|}(\frac{\|\delta T\|}{\|T\|} + \frac{\|\delta u\|}{\|u\|}) \\ &= \frac{K_D(T)\|x\|}{1-K_D(T)\|\delta T\|/\|T\|}(\frac{\|\delta T\|}{\|T\|} + \frac{\|\delta u\|}{\|u\|}). \end{aligned}$$

Then (4.3) holds. □

5. Conclusions

In this paper, we generalized some results in the perturbation analysis for the Drazin inverse of a square matrix to a more general situation. Our research makes progress in the perturbation analysis for the Drazin inverse of a bounded linear operator on a Banach space. However, the case discussed here is only the special case that $\bar{T}^D$ has the simple form (3.1). The more general situation will be investigated in the future.

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Banach 空间上有界线性算子的 Drazin 逆的扰动分析

蔡 静[1], 陈 果 良[2]

(1. 湖州师范学院理学院, 浙江 湖州 313000; 2. 华东师范大学数学系, 上海 200062)

摘 要: 设 X 为一复域 C 上的 Banach 空间, 设 $T: X \to X$ 为一有界线性算子, 其指标为 k 且 $R(T^k)$ 闭. 记 T 的 Drazin 逆为 T^D. 设 $\bar{T} = T + \delta T$, 则在一定条件下, $\bar{T}^D$ 有简明分解式 $\bar{T}^D = T^D(I + \delta T T^D)^{-1} = (I + T^D \delta T)^{-1} T^D$, 从而导出了相对误差 $\|\bar{T}^D - T^D\|/\|T^D\|$ 的上界和算子方程: $Tx = u (u \in R(T^D))$ 的解的扰动界.

关键词: Drazin 逆; 指标; 误差界.