Journal of Mathematical Research & Exposition
Vol.24, No.4, 571–576, November, 2004

Browder's Theorem and Weyl's Theorem *

CAO Xiao-hong[1], *GUO Mao-zheng*[2], *MENG Bin*[2]

(1. College of Math. & Info. Sci., Shaanxi Normal University, Xi'an 710062, China;
2. School of Math. Sci., Peking University, Beijing 100871, China)

Abstract: In this paper, by defining two new spectral sets, we give the necessary and sufficient conditions for Browder's theorem and Weyl's theorem for bounded linear operator T and $f(T)$, where $f \in \mathcal{H}(\sigma(T))$ and $\mathcal{H}(\sigma(T))$ denotes the set of all analytic functions on an open neighborhood of $\sigma(T)$.

Key words: Browder's theorem; Weyl's theorem; spectrum.

Classification: AMS(2000) 47A10, 47A53, 47A55/CLC number: O177.2

Document code: A **Article ID**: 1000-341X(2004)04-0511-06

1. Introduction

H.Weyl[11] examined the spectra of all compact perturbations $T+K$ of a hermitian operator T and discovered that $\lambda \in \sigma(T+K)$ for every compact operator K if and only if λ is not an isolated eigenvalue of finite multiplicity in $\sigma(T)$. Today this result is known as Weyl's theorem, and it has been extended from hermitian operators to hyponormal operators and to Toeplitz operators by L.Coburn[3], to several classes of operators including seminormal operators by S.Berberian[1],[2], and to a few classes of Banach space operators[7],[8]. Similar to the Weyl's theorem, there is a-Browder theorem and a-Weyl's theorem[4],[9]. The aim of this paper is to give the necessary and sufficient conditions for Browder's theorem and Weyl's theorem.

Throughout this paper, X denotes a complex infinite Banach space, and $B(X)$ and $K(X)$ denote respectively the algebra of bounded linear operators and the ideal of compact operators on X. For $T \in B(X)$, $N(T), R(T)$ respectively denote the null and the range space. Let $\sigma(T)$ be the spectrum of T and $\rho(T) = \mathbf{C}\backslash\sigma(T)$. It is well known that the following sets form semi-groups of semi-Fredholm operators on X:

$$\Phi_+(X) = \{T \in B(X) : R(T) \text{ is closed and } \dim N(T) < \infty\}$$

and

$$\Phi_-(X) = \{T \in B(X) : R(T) \text{ is closed and } \dim X/R(T) < \infty\}.$$

*Received date: 2003-01-13

If $T \in \Phi_+(X) \cap \Phi_-(X)$, we call T is a Fredholm operator. Let $\Phi(X)=\Phi_+(X)\cap\Phi_-(X)$ be the set of all Fredholm operators on X. If T is semi-Fredholm operator and $n(T) = \dim N(T)$ and $d(T) = \dim X/R(T)$, then we define the index of T by $\mathrm{ind}(T) = n(T) - d(T)$. We also consider the sets: $\Phi_0(X) = \{T \in \Phi(X) : \mathrm{ind}(T) = 0\}$ (Weyl operators). An operator T is called Browder operator if T is Fredholm operator "of finite ascent and descent": equivalently ([5], Theorem 7.9.3) if T is Fredholm operator and $T - \lambda I$ is invertible for sufficiently small $\lambda \neq 0$ in $\mathbf{C}$.

The following definitions are well known: the essential spectrum of T is $\sigma_e(T) = \{\lambda \in \mathbf{C}, T - \lambda I$ is not in $\Phi(X)\}$, the Weyl spectrum of T is $\sigma_w(T) = \{\lambda \in \mathbf{C} : T - \lambda I$ is not in $\Phi_0(X)\}$ and the Browder spectrum of T is $\sigma_b(T) = \bigcap\{\sigma(T+K) : TK = KT, K \in K(X)\}=\{\lambda \in \mathbf{C} : T - \lambda I$ is not Browder operator $\}$. Let $P_{00}(T) = \sigma(T)\backslash\sigma_b(T)$, $\rho_w(T) = \mathbf{C}\backslash\sigma_w(T)$, $\rho_b(T) = \mathbf{C}\backslash\sigma_b(T)$ and Let $\pi_{00}(T)$ be the set of all $\lambda \in \mathbf{C}$ such that λ is an isolated point of $\sigma(T)$ and $0 < n(\lambda I - T) < \infty$. We write $isoK$ (∂K) for the isolated (boundary) points set of $K \subseteq \mathbf{C}$.

We say that T obeys Weyl's theorem if $\sigma(T)\backslash\sigma_w(T) = \pi_{00}(T)$, and T obeys Browder's theorem if $\sigma(T)\backslash\sigma_w(T) = P_{00}(T)$ or $\sigma_w(T) = \sigma_b(T)$.

Clearly, if T obeys Weyl's theorem, then it obeys Browder's theorem. $T \in B(X)$ is called isoloid if for any $\lambda \in \mathrm{iso}\sigma(T)$, then $\dim N(\lambda I - T) > 0$.

Let $\mathcal{H}(T)$ $(\mathcal{H}(\sigma(T)))$ denote the set of all analytic functions in some neighbourhood (region) of $\sigma(T)$. Clearly, $\mathcal{H}(\sigma(T)) \subseteq \mathcal{H}(T)$.

2. Browder's theorem and Weyl's theorem I

Let

$$\rho_1(T) = \{\lambda \in \mathbf{C} : T - \lambda I \text{ is Weyl operator and } N(T-\lambda I) \subseteq \bigcap_{n=1}^{\infty} R[(T-\lambda I)^n]\}$$

and $\sigma_1(T) = \mathbf{C}\backslash\sigma_1(T)$.

Clearly, $\rho(T) \subseteq \rho_1(T) \subseteq \rho_w(T)$, $\sigma_w(T) \subseteq \sigma_1(T) \subseteq \sigma(T)$ and $\mathrm{iso}\sigma(T) \subseteq \sigma_1(T)$. In fact, if $\lambda_0 \in \mathrm{iso}\sigma(T)$ but $\lambda_0 \in \rho_1(T)$, then $T - \lambda_0 I$ is Browder operator, so by Lemma 3.4 of [10], $n(T-\lambda_0 I) = \dim[N(T-\lambda_0 I) \cap \bigcap_{n=1}^{\infty} R[(T-\lambda_0 I)^n]] = 0$, thus $T - \lambda_0 I$ is invertible. It contradicts the fact that $\lambda_0 \in \sigma(T)$.

Theorem 1 *Browder's theorem holds for T if and only if $\sigma(T) = \sigma_1(T)$.*

Proof Suppose that Browder's theorem holds for T. Since $\sigma(T)\backslash\sigma_1(T) \subseteq \sigma(T)\backslash\sigma_w(T) = P_{00}(T) \subseteq \sigma_1(T)$, it follows that $\sigma(T)\backslash\sigma_1(T) = \emptyset$, which means $\sigma(T) = \sigma_1(T)$.

For the inverse, suppose that $\sigma(T) = \sigma_1(T)$. For any $\lambda_0 \in \sigma(T)\backslash\sigma_w(T)$, $T-\lambda_0 I$ is Weyl. By the perturbation theorem of Fredholm operator, there exists $\varepsilon > 0$ such that $T - \lambda I$ is Fredholm operator with $ind(T-\lambda I) = ind(T-\lambda_0 I)$ and $N(T-\lambda I) \subseteq \bigcap_{n=1}^{\infty} R[(T-\lambda I)^n]$ if $0 < |\lambda - \lambda_0| < \varepsilon$, then $\lambda \in \rho_1(T)$ for $0 < |\lambda - \lambda_0| < \varepsilon$. Thus $\lambda \in \rho_1(T) = \rho(T)$ if $0 < |\lambda - \lambda_0| < \varepsilon$, which means that λ_0 is an isolated point of $\sigma(T)$. Then we get that $\lambda_0 \in P_{00}(T)$, so Browder's theorem holds for T. The completes the proof.

Corollary 1 *If Browder's theorem holds for $T \in B(X)$ and $S \in B(Y)$ and $f \in \ calH(T)$, then*

$$\text{Browder's theorem holds for } f(T) \Longleftrightarrow \sigma_1(f(T)) = f(\sigma_1(T))$$

and

$$\text{Browder's theorem holds for } T \oplus S \Longleftrightarrow \sigma_1(T) \cup \sigma_1(S) = \sigma_1(T \oplus S).$$

So if T obeys Browder's theorem and $f \in (T)$ is injective, then Browder's theorem holds for $f(T)$.

Proof By Theorem 1, Browder's theorem holds for $f(T) \Longleftrightarrow \sigma_1(f(T)) = \sigma(f(T)) = f(\sigma(T)) = f(\sigma_1(T))$. And Browder's theorem holds for $T \oplus S \Longleftrightarrow \sigma_1(T \oplus S) = \sigma(T \oplus S) = \sigma(T) \cup \sigma(S) = \sigma_1(T) \cup \sigma_1(S)$.

Theorem 2 *If T obeys Weyl's theorem and T is isoloid, then the following statements are equivalent:*

(1) $\sigma_1(f(T)) = f(\sigma_1(T))$ for every $f \in \mathcal{H}(\sigma(T))$;

(2) Weyl's theorem holds for $f(T)$ for every $f \in \mathcal{H}(\sigma(T))$.

Proof Weyl's theorem induces Browder's theorem, so by Corollary 1, we have that $(2) \Longrightarrow (1)$.

$(1) \Longrightarrow (2)$. By Corollary 1, for any $f \in \mathcal{H}(\sigma(T))$, Browder's theorem holds for $f(T)$, that is $\sigma(f(T)) \backslash \sigma_w(f(T)) \subseteq \pi_{00}(f(T))$. In the following we will prove $\pi_{00}(f(T)) \subseteq \sigma(f(T)) \backslash \sigma_w(f(T))$. Let $\mu_o \in \pi_{00}(f(T))$, suppose

$$f(T) - \mu_0 I = a(T - \lambda_1 I)(T - \lambda_2 I) \cdots (T - \lambda_n I) g(T),$$

where $a, \lambda_1, \lambda_2, \cdots, \lambda_n \in \sigma(T)$ and $g(T)$ if invertible. Since μ_0 is the isolated point in $\sigma(f(T))$, we konw $\lambda_i \in \mathrm{iso}\sigma(T)$. With $N(T - \lambda_i I) \subseteq N(f(T) - \mu_0 I)$, then $n(T - \lambda_i I) < \infty$. The fact that T is isoloid asserts that $\lambda_i \in \pi_{00}(T)$ for all λ_i. Weyl's theorem holds for T, then $T - \lambda_i I$ is Weyl operator, thus $f(T) - \mu_0 I$ is Weyl operator, which means that $\mu_0 \in \sigma(f(T)) \backslash \sigma_w(f(T))$. Now we get that Weyl's theorem holds for $f(T)$. This completes the proof.

3. Browder's theorem and Weyl's theorem II

Suppose $\rho_3(T) = \{\lambda \in \mathbf{C}, n(T - \lambda I) < \infty$ and there exists $\varepsilon > 0$ such that $T - \mu I$ is Weyl operator and $N(T - \mu I) \subseteq \bigcap_{n=1}^{\infty} R[(T - \mu I)^n]$ for $0 < |\mu - \lambda| < \varepsilon\}$. Let $\sigma_3(T) = \mathbf{C} \backslash \rho_3(T)$. Then $\rho(T) \subseteq \rho_b(T) \subseteq \rho_w(T) \subseteq \rho_3(T)$.

Theorem 3 *T is isoloid and Weyl's theorem holds for T if and only if $\sigma_b(T) = \sigma_3(T)$.*

Proof Suppose that T is isoloid and Weyl's theorem holds for T. We only need to prove $\sigma_b(T) \subseteq \sigma_3(T)$.

Suppose $\lambda_0 \in \sigma_b(T)$ but λ_0 is not in $\sigma_3(T)$, then $n(T - \lambda_0 I) < \infty$ and there exists $\varepsilon > 0$ such that $T - \mu I$ is Weyl operator and $N(T - \mu I) \subseteq \bigcap_{n=1}^{\infty} R[(T - \mu I)^n]$ if $0 <$

$|\mu-\lambda_0|<\varepsilon$. Since Weyl's theorem holds for T, it follows that $T-\mu I$ is Browder operator if $0<|\mu-\lambda_0|<\varepsilon$. Then

$$N(T-\mu I)=N(T-\mu I)\cap\bigcap_{n=1}^{\infty}R[(T-\mu I)^n]=\{0\}.$$

Thus $T-\mu I$ is invertible if $0<|\mu-\lambda_0|<\varepsilon$, which means that $\lambda_0\in \mathrm{iso}\sigma(T)$. T is isoloid implies $0<n(T-\lambda_0 I)<\infty$. We now get that $\lambda_0\in\pi_{00}(T)=\sigma(T)\backslash\sigma_w(T)$, that is $T-\lambda_0 I$ is Browder operator. It is a contradiction. Then we have $\sigma_3(T)=\sigma_b(T)$.

Conversely, suppose that $\sigma_b(T)=\sigma_3(T)$. By definition of $\rho_3(T)$, we know that $[\sigma(T)\backslash\sigma_w(T)]\cup\pi_{00}(T)\subseteq\rho_3(T)=\rho_b(T)$, so $\sigma(T)\backslash\sigma_w(T)=\pi_{00}(T)$, which means that Weyl's theorem holds for T. In the following, we will prove that T is isoloid. Let $\lambda_0\in\mathrm{iso}\sigma(T)$, if $N(T-\lambda_0 I)=\{0\}$, by definition of $\rho_3(T)$, $\lambda_0\in\rho_3(T)=\rho_b(T)$. Then $T-\lambda_0 I$ is invertible, which is in contradiction to the fact that $\lambda_0\in\sigma(T)$. This completes the proof.

Corollary 2 *Suppose $T\in B(X)$ and $S\in B(Y)$ are all isoloid operators. If Weyl's theorem holds for T and S and if $f\in\mathcal{H}(T)$, then*

$$\textit{Weyl's theorem holds for } f(T)\Longleftrightarrow\sigma_3(f(T))=f(\sigma_3(T))\Longleftrightarrow f(\sigma_3(T))\subseteq\sigma_3(f(T))$$

and

$$\textit{Weyl's theorem holds for } T\oplus S\Longleftrightarrow\sigma_3(T)\cup\sigma_3(S)=\sigma_3(T\oplus S).$$

Proof If T is an isoloid operator, then for any $f\in\mathcal{H}(T)$, $f(T)$ is an isoloid operator. In fact, if $\mu\in\mathrm{iso}\sigma(f(T))$ and suppose $\mu=f(\lambda)$, then $\lambda\in iso\sigma(T)$. Since T is isoloid, it follows that $n(T-\lambda I)>0$. By $N(T-\lambda I)\subseteq N(f(T)-\mu I)$, we get $n(f(T)-\mu I)>0$, which means that $f(T)$ is an isoloid operator. Using the same way, we can prove that: if T and S are all isoloid operators , then $T\oplus S$ is an isoloid operator.

By contrast ([5],Theorem 9.8.2), the spectral mapping theorem holds for the Browder spectrum, and the Browder spectrum of a direct sum is the union of the Browder spectrum of the components.

So

$$\begin{aligned}&\text{Weyl's theorem holds for } f(T)\\&\Longleftrightarrow\sigma_3(f(T))=\sigma_b(f(T))=f(\sigma_b(T))=f(\sigma_3(T))\\&\Longrightarrow f(\sigma_3(T))\subseteq\sigma_3(f(T)).\end{aligned}$$

If $f(\sigma_3(T))\subseteq\sigma_3(f(T))$, then $\sigma_b(f(T))=f(\sigma_b(T))=f(\sigma_3(T))\subseteq\sigma_3(f(T))$, thus $\sigma_b(f(T))=\sigma_3(f(T))$, which means that Weyl's theorem holds for $f(T)$.

Also

$$\text{Weyl's theorem holds for } T\oplus S\Longleftrightarrow\sigma_3(T\oplus S)=\sigma_b(T\oplus S)=\sigma_b(T)\cup\sigma_b(S)=\sigma_3(T)\cup\sigma_3(S).$$

Theorem 4 *If Browder's theorem holds for T, then the following statements equivalent:*

(1) $f(\sigma_3(T)) \subseteq \sigma_3(f(T))$ for every $f \in \mathcal{H}(\sigma(T))$;

(2) Browder's theorem holds for $f(T)$ for every $f \in \mathcal{H}(\sigma(T))$.

Proof (1) $\Longrightarrow$ (2). Let $\mu_0 \in \sigma(f(T))\backslash\sigma_w(f(T))$, that is $f(T) - \mu_0 I$ is a Weyl operator. Suppose

$$f(T) - \mu_0 I = a(T - \lambda_1 I)(T - \lambda_2 I) \cdots (T - \lambda_n I)g(T),$$

where $a, \lambda_1, \lambda_2, \cdots, \lambda_n \in \mathbf{C}$ and $g(T)$ is invertible. Then $T - \lambda_i I$ is Fredholm operator for all λ_i. Since μ_0 is not in $\sigma_3(f(T))$, it follows that μ_0 is not in $f(\sigma_3(T))$, that is $\lambda_i \in \rho_3(T)$. By perturbation theorem of Fredholm operator, for any λ_i, there exists $\varepsilon_i > 0$ such that $\mathrm{ind}(T - \lambda_i I) = ind(T - \lambda_i' I)$ if $0 < |\lambda_i' - \lambda_i| < \varepsilon_i$. Let ε_i is small enough, by $\lambda_i \in \rho_3(T)$, then $\mathrm{ind}(T - \lambda_i I) = \mathrm{ind}(T - \lambda_i' I) = 0$, which means that $T - \lambda_i I$ is Weyl. Since Browder's theorem holds for T, it follows that $T - \lambda_i I$ is Browder. Then $f(T) - \mu_0 I$ is a Browder's operator.

(2) $\Longrightarrow$ (1). Suppose $\mu_0 \in f(\sigma_3(T))$ and μ_0 is not in $\sigma_3(f(T))$. Let $\mu_0 = f(\lambda_1)$, where $\lambda_1 \in \sigma_3(T)$. Suppose

$$f(T) - \mu_0 I = a(T - \lambda_1 I)^{n_1}(T - \lambda_2 I)^{n_2} \cdots (T - \lambda_m I)^{n_m} g(T),$$

where $a, \lambda_1, \lambda_2, \cdots, \lambda_m \in \mathbf{C}$, $\lambda_i \neq \lambda_j$ and $g(T)$ is invertible. Use ([6], Satz 80.1) to drive

$$N(f(T) - \mu_0 I) = N(T - \lambda_1 I)^{n_1} \oplus N(T - \lambda_2 I)^{n_2} \oplus \cdots \oplus N(T - \lambda_m I)^{n_m},$$

then $n(T - \lambda_1 I) < \infty$. In the following we will prove that $\lambda_1 \in \rho_3(T)$. By $\mu_0 \in \rho_3(f(T))$, there exists $\delta > 0$ such that $f(T) - \mu I$ is Weyl and $N(f(T) - \mu I) \subseteq \bigcap_{n=1}^{\infty} R[(f(T) - \mu I)^n]$ if $0 < |\mu - \mu_0| < \delta$. Since f is analytic, for λ_1, there exists $\varepsilon > 0$ such that $0 < |f(\lambda_1') - f(\lambda)| = |f(\lambda_1') - \mu_0| < \delta$ if $0 < |\lambda_1' - \lambda_1| < \varepsilon$. Then $f(T) - f(\lambda_1')I$ is Weyl and $N[f(T) - f(\lambda_1')I] \subseteq \bigcap_{n=1}^{\infty} R[(f(T) - f(\lambda_1')I)^n]$. Let

$$f(T) - f(\lambda_1')I = b(T - \lambda_1' I)^{t_1}(T - \lambda_2' I)^{t_2} \cdots (t - \lambda_k' I)^{t_k} h(T),$$

where $h(T)$ is invertible. Since Browder's theorem holds for $f(T)$, we get that $f(T) - f(\lambda_1')I$ is a Browder operator. Then $f(T) - f(\lambda_1')I$ is invertible ([T],Lemma 3.4). Thus $T - \lambda_i' I$ is invertible. Now we have that for λ_1, there exist $\varepsilon > 0$ such that $T - \lambda_1' I$ is invertible if $0 < |\lambda_1' - \lambda_1| < \varepsilon$, so $\lambda_1 \in \rho_3(T)$. It is a contradiction.

Corollary 3 *If T is isoloid and Weyl's theorem holds for T, then the following statements are equivalent:*

(1) $f(\sigma_3(T)) = \sigma_3(f(T))$ for every $f \in \mathcal{H}(\sigma(T))$;

(2) Browder's theorem holds for $f(T)$ for every $f \in \mathcal{H}(\sigma(T))$;

(3) Weyl's theorem holds for $f(T)$ for every $f \in \mathcal{H}(\sigma(T))$.

Proof (1) $\Longleftrightarrow$ (3) See Corollary 2.

(1) $\Longrightarrow$ (2) See Theorem 4.

(2) $\Longrightarrow$ (1). We only need to prove that $\sigma_3(f(T)) \subseteq f(\sigma_3(T))$. Let $\mu_0 \in \sigma_3(f(T))$ and μ_0 is not in $f(\sigma_3(T))$. Suppose

$$f(T) - \mu_0 I = a(T - \lambda_1 I)^{n_1}(T - \lambda_2 I)^{n_2} \cdots (T - \lambda_m I)^{n_m} g(T),$$

where $a, \lambda_1, \lambda_2, \cdots, \lambda_m \in \mathbf{C}$, $\lambda_i \neq \lambda_j$ and $g(T)$ is invertible. Since

$$N(f(T) - \mu_0 I) = N(T - \lambda_1 I)^{n_1} \oplus N(T - \lambda_2 I)^{n_2} \oplus \cdots \oplus N(T - \lambda_m I)^{n_m},$$

it follows that $n(f(T) - \mu_0 I) < \infty$. For λ_i, there exists $\varepsilon_i > 0$ such that $T - \lambda_i' I$ is Weyl and $N(T - \lambda_i' I) \subseteq \bigcap_{n=1}^{\infty} R[(T - \lambda_i' I)^n]$. Since Browder's theorem holds for T, we know that $T - \lambda_i' I$ is invertible. Then $\lambda_i \in \mathrm{iso}\sigma(T)$. Using the fact that T is isoloid, it follows that $0 < n(T - \lambda_i I) < \infty$. Then $\lambda_i \in \pi_{00}(T)$. The fact that Weyl's theorem holds for T induces that $T - \lambda_i I$ is Browder, so $f(T) - \mu_0 I$ is Browder, therefore $\mu_0 \in \rho_3(f(T))$. It is in contradiction to the fact that $\mu_0 \in \sigma_3(f(T))$. This completes the proof.

Remark From the proof of Corollary 3, we find that: if T is isoloid and Browder's theorem holds for T, then (1) and (2) are equivalent.

References:

[1] BERBERIAN S K. *An extension of Weyl's theorem to a class of not necessarily normal operators* [J]. Michigan Math. J., 1969, **16**: 273–279.

[2] BERBERIAN S K. *The Weyl spectrum of an operator* [J]. Indiana Univ. Math. J., 1970, **20**: 529–544.

[3] COBURN L A. *Weyl's theorem for nonnormal* operators [J]. Michigan Math. J., 1966, 285–288.

[4] DJORDJEVIĆ S V, DJORDJEVIĆ D S. *Weyl's theorems:continuity of the spectrum and quasihyponormal operators* [J]. Acta Sci. Math., (Szeged), 1998, **64**: 259–269.

[5] HARTE R E. *Invertibility and Singularity for Bounded Linear Operators* [M]. Dekker, New York, 1988. MR 89d: 47001.

[6] HEUSER H. *Funktionalanalysis* [M]. 2nd. ed. Stuttgart, 1986.

[7] ISTRÁTESCU V I. *On Weyl's spectrum of an operator I* [J]. Rev. Ronmaine Math. Pure Appl., 1972, **17**: 1049–1059.

[8] OBERAI K K. *On the Weyl* spectrum [J]. Illinois J. Math., 1974, **18**: 208–212.

[9] RAKOČEVIČ V. *Operators obeying a-Weyl's theorem* [J]. Rev. Roumaine Math. Pures Appl., 1989, **34**(10): 915–919.

[10] TAYLOR A E. *Theorems on ascent,descent,nullity and defect of linear operators* [J]. Math. Ann., 1966, **163**: 18–49.

[11] WEYL H. *Über beschränkte quadratische Formen, deren Differenz vollstetig ist* [J]. Rend. Circ. Mat. Palermo, 1909, **27**: 373–392.

Browder 定理和 Weyl 定理

曹 小 红[1], 郭 懋 正[2], 孟 彬[2]

(1. 陕西师范大学数学与信息科学学院, 陕西 西安 710062; 2. 北京大学数学科学学院, 北京 100871)

摘 要: 本文通过定义两个新的谱集, 给出了 Browder 定理和 Weyl 定理对算子 T 以及 $f(T)$ 成立的充要条件, 其中 $f \in \mathcal{H}(\sigma(T))$, $\mathcal{H}(\sigma(T))$ 表示在谱集 $\sigma(T)$ 的开邻域上解析的函数的全体.

关键词: Browder 定理; Weyl 定理; 谱.