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Principal Quasi-Baerness of Skew Power Series Rings

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Abstract: Let R be a ring such that all left semicentral idempotents are central and α a weakly rigid endomorphism of R. It is shown that the skew power series ring $R[[x;\alpha]]$ is right p.q.Baer if and only if R is right p.q.Baer and any countable family of idempotents in R has a generalized join in $I(R)$, where $I(R)$ is the set of all idempotents of R.

Key words: weakly rigid endomorphism; p.q.Baer ring; skew power series ring.
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1. Introduction

Throughout this paper, R denotes a ring with unity and $C(R)$ the set of all central elements of R. For a nonempty subset Y of R, $r_R(Y)$ denotes the right annihilator of Y in R.

Recall that R is (quasi-) Baer if the right annihilator of every nonempty subset (every right ideal) of R is generated by an idempotent. In [9] Kaplansky introduced Baer rings to abstract various properties of AW^*-algebras and von Neumann algebras. Clark defined quasi-Baer rings in [7] and used them to characterize when a finite dimensional algebra with unity over an algebraically closed field is isomorphic to a twisted matrix units semigroup algebra. Further work on Baer rings and quasi-Baer rings appears in [1-4, 8]. As a generalization of quasi-Baer rings, G. F. Birkenmeier, J. Y. Kim and J. K. Park in [5] introduced the concept of principally quasi-Baer rings. A ring R is called right principally quasi-Baer (or simply right p.q.Baer) if the right annihilator of a principal right ideal of R is generated by an idempotent. Similarly, left p.q.Baer rings can be defined. A ring is called p.q.Baer if it is both right and left p.q.Baer. Observe that every biregular ring and every quasi-Baer ring are p.q.Baer rings. For more details and examples of right p.q.Baer rings[5,6].

It was proved in [3, Theorem 1.8] that a ring R is quasi-Baer if and only if $R[X]$ is quasi-Baer if and only if $R[[X]]$ is quasi-Baer, where X is an arbitrary nonempty set of not necessarily commuting indeterminates. If R is a reduced ring, then R is Baer if and only if $R[X]$ is Baer if and only if $R[[X]]$ is Baer [3, Corollary 1.10]. If R is commutative and $(S,\leq)$ is a strictly totally ordered monoid, then it is shown in [11, Theorem 7] that R is Baer if and only if $[[R^{S,\leq}]]$, the ring of generalized power series with coefficients in R and exponents in S, is Baer. It was proved in [6,

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Theorem 2.1] that a ring R is right p.q.Baer if and only if $R[x]$ is right p.q.Baer. If R is an α-rigid ring, then it was shown in [8, Corollary 15] that R is a right p.q.Baer ring if and only if $R[x;\alpha,\delta]$ is a right p.q.Baer ring. For skew power series ring $R[[x;\alpha]]$, it was shown in [8, Theorem 21 and Corollary 22] that if α is a rigid endomorphism of R, then R is a Baer (a quasi-Baer, resp.) ring if and only if $R[[x;\alpha]]$ is a Baer (a quasi-Baer, resp.) ring. Also an example was given in [8] which shows that there exists a commutative von Neumann regular ring R (hence p.q.Baer) such that the ring $R[[x;\alpha]]$ is not right p.q.Baer. Thus a natural question of characterization of the right p.q.Baerness of skew power series ring $R[[x;\alpha]]$ is raised. In this paper, we give a necessary and sufficient condition for some rings under which the ring $R[[x;\alpha]]$ is right p.q.Baer. We show that for a ring R with $S_\ell(R)\subseteq C(R)$ and for a weakly rigid endomorphism α of R, $R[[x;\alpha]]$ is right p.q.Baer if and only if R is right p.q.Baer and any countable family of idempotents in R has a generalized join in $I(R)$.

2. Weakly rigid endomorphism

Let α be an endomorphism of R. According to [8] and [10], α is called a rigid endomorphism if $r\alpha(r)=0$ implies $r=0$ for $r\in R$. A ring R is called to be α-rigid if there exists a rigid endomorphism α of R. Clearly, any rigid endomorphism is a monomorphism and any α-rigid ring is reduced. Generalizing these concepts, we give the following definition.

Definition 1 *Let α be an endomorphism of R. α is called a weakly rigid endomorphism if*

(1) α is a monomorphism, and

(2) if $a,b\in R$ are such that $ab=0$ then $a\alpha(b)=\alpha(a)b=0$.

Example 2 (1) Clearly the identity map of R is weakly rigid.

(2) Let α be a rigid endomorphism of R. It was shown in [8] that if $ab=0$ then $a\alpha^n(b)=\alpha^n(a)b=0$ for any positive integer n. Thus any rigid endomorphism is weakly rigid. But the converse is not true. For example, suppose that the ring R is not reduced, then the identity map of R is weakly rigid but not rigid.

(3) Let β be a weakly rigid endomorphism of ring R_0 and S a ring. Set $R_1=R_0\oplus S$, the direct sum of rings R_0 and S. Define an endomorphism α of R_1 via

$$\alpha(r,s)=(\beta(r),s).$$

Then it is easy to see that α is a weakly rigid endomorphism of R_1. If β is not rigid, or S is a ring with a nonzero nilpotent element, then α is not rigid.

Proposition 3 *Let α be an endomorphism of R. Then α is rigid if and only if α is weakly rigid and R is reduced.*

Proof Let α be an endomorphism of the reduced ring R. If α is weakly rigid and $r\in R$ is such that $r\alpha(r)=0$, then $\alpha(r)\alpha(r)=0$. Thus $\alpha(r)=0$ since R is reduced. Hence $r=0$ since α is a monomorphism. This means that α is rigid. Conversely, if α is rigid, then, by [8], R is reduced.

Thus the result follows.

3. The right p.q.Baerness of $R[[x;\alpha]]$

Recall from [3] an idempotent $e \in R$ is left (resp. right) semicentral in R if $ere = re$ (resp. $ere = er$), for all $r \in R$. Equivalently, $e^2 = e \in R$ is left (resp. right) semicentral if eR (resp. Re) is an ideal of R. Since the right annihilator of a right ideal is an ideal, we see that the right annihilator of a principal right ideal is generated by a left semicentral idempotent in a right p.q.Baer ring. The set of all left semicentral idempotents of R is denoted by $\mathcal{S}_\ell(R)$. The following result is a generalization of [6, Proposition 1.5].

Lemma 4 *Let α be a weakly rigid endomorphism of R. If $e(x) = e_0 + e_1x + \ldots + e_nx^n + \ldots \in R[[x;\alpha]]$ is a left semicentral idempotent of $R[[x;\alpha]]$, then*

(1) e_0 is a left semicentral idempotent of R.

(2) $e_0e_i = e_i, e_ie_0 = 0$, for $i = 1, 2, \cdots$.

(3) $e(x)R[[x;\alpha]] = e_0R[[x;\alpha]]$.

Proof We complete the proof by adapting the proof of [6, Proposition 1.4].

Let $r \in R$. Since $re(x) = e(x)re(x)$, we have

$$\sum_{k=0}^{\infty} re_kx^k = \sum_{k=0}^{\infty}\left(\sum_{i+j=k} e_ix^ire_jx^j\right) = \sum_{k=0}^{\infty}\left(\sum_{i+j=k} e_i\alpha^i(re_j)\right)x^k.$$

Thus $re_k = \sum_{i+j=k} e_i\alpha^i(re_j)$ for any $k = 0, 1, \cdots$. From $re_0 = e_0re_0$ it follows that $e_0 \in \mathcal{S}_\ell(\mathcal{R})$, so part (1) is satisfied. If we multiply equation $re_1 = e_1\alpha(re_0) + e_0re_1$ on the right by e_0, then $re_1e_0 = e_1\alpha(re_0)e_0 + e_0re_1e_0 = e_1\alpha(re_0)e_0 + re_1e_0$. Thus $e_1\alpha(re_0)e_0 = 0$. Since α is weakly rigid, we have $e_1\alpha(re_0) = e_1\alpha(re_0e_0) = e_1\alpha(re_0)\alpha(e_0) = 0$. Thus $re_1 = e_0re_1$. Taking $r = 1$, we obtain $e_0e_1 = e_1$ and $e_1\alpha(e_0) = 0$. Now assume that k is a positive integer such that

$$e_i\alpha^i(e_0) = 0, \quad e_0e_i = e_i,$$

for all $1 \le i < k$. Then from $e_i\alpha^i(e_0) = 0$ and from the weak rigidness of α, it follows that $\alpha^i(e_ie_0) = \alpha^i(e_i)\alpha^i(e_0) = 0$ for all $1 \le i < k$. Since α is a monomorphism, we have $e_ie_0 = 0$. Thus $\alpha^j(e_i)e_0 = 0$ for all $1 \le i < k$ and all j. Now multiplying equation $re_k = \sum_{i+j=k} e_i\alpha^i(re_j)$ on the right by e_0, we obtain $re_ke_0 = \sum_{i+j=k} e_i\alpha^i(re_j)e_0 = e_k\alpha^k(re_0)e_0 + e_0(re_k)e_0 = e_k\alpha^k(re_0)e_0 + (re_k)e_0$. Thus $e_k\alpha^k(re_0)e_0 = 0$. Since α is weakly rigid, we have $e_k\alpha^k(re_0) = e_k\alpha^k(re_0)\alpha^k(e_0) = 0$. Thus $e_k\alpha^k(e_0) = 0$. Also $re_k = \sum_{\substack{i+j=k \\ i\neq k}} e_i\alpha^i(re_j)$. Multiplying on the left by e_0, by hypothesis, we have

$$e_0re_k = \sum_{\substack{i+j=k \\ i\neq k}} e_0e_i\alpha^i(re_j) = \sum_{\substack{i+j=k \\ i\neq k}} e_i\alpha^i(re_j) = re_k.$$

Taking $r = 1$ yields $e_0e_k = e_k$. By induction, part (2) is satisfied. Now it is easy to see that $e(x)e_0 = e_0$ and $e_0e(x) = e(x)$. Hence $e(x)R[[x;\alpha]] = e_0R[[x;\alpha]]$.

Let $I(R)$ be the set of all idempotents of R. Let $\{e_0, e_1, \ldots\}$ be a countable family of idempotents of R. We say $\{e_0, e_1, \ldots\}$ has a generalized join in $I(R)$ if there exists an idempotent $e \in I(R)$ such that

(1) $e_iR(1-e) = 0$, and

(2) if $f \in I(R)$ is such that $e_iR(1-f) = 0$, then $eR(1-f) = 0$.

If α is a rigid endomorphism of R, then it was shown in [8] that R is a Baer (a quasi-Baer, resp.) ring if and only if $R[[x;\alpha]]$ is a Baer (a quasi-Baer, resp.) ring. Also an example was given in [8] to show that there exists a reduced right p.q.Baer ring R such that $R[[x;\alpha]]$ is not a right p.q.Baer ring. Here we have

Theorem 5 *Let R be a ring with $\mathcal{S}_\ell(\mathcal{R}) \subseteq \mathcal{C}(\mathcal{R})$ and α a weakly rigid endomorphism of R. Then the following conditions are equivalent:*

(1) $R[[x;\alpha]]$ is right p.q.Baer;

(2) R is right p.q.Baer and any countable family of idempotents in R has a generalized join in $I(R)$.

Proof (1)$\Longrightarrow$(2). Suppose that $R[[x;\alpha]]$ is right p.q.Baer. Let a be an element of R. Then there exists a left semicentral idempotent $e(x) = e_0 + e_1x + \cdots + e_nx^n + \cdots \in R[[x;\alpha]]$ such that $r_{R[[x;\alpha]]}(aR[[x;\alpha]]) = e(x)R[[x;\alpha]]$. From Lemma 4, $e(x)R[[x;\alpha]] = e_0R[[x;\alpha]]$. Thus $r_{R[[x;\alpha]]}(aR[[x;\alpha]]) = e_0R[[x;\alpha]]$. It is clearly that $aRe_0 = 0$. Thus $e_0 \in r_R(aR)$. Hence $e_0R \subseteq r_R(aR)$. Conversely, suppose that $p \in r_R(aR)$. Then for any $f(x) = \sum_{i=0}^{\infty} a_ix^i \in R[[x;\alpha]]$, $af(x)p = \sum_{i=0}^{\infty} aa_ix^ip = \sum_{i=0}^{\infty} aa_i\alpha^i(p)x^i$. Since $aa_ip = 0$, we have $aa_i\alpha^i(p) = 0$ by the weak rigidness of α. Thus $af(x)p = 0$, which implies that $p \in r_{R[[x;\alpha]]}(aR[[x;\alpha]])$ and hence $p = e_0p \in e_0R$. Therefore $r_R(aR) \subseteq e_0R$. This shows that $r_R(aR) = e_0R$. Thus R is right p.q.Baer.

Now suppose that $\{e_0, e_1, \ldots\}$ is a countable set of idempotents of R. Set

$$\varphi(x) = e_0 + e_1x + e_2x^2 + \ldots \in R[[x;\alpha]].$$

Since $R[[x;\alpha]]$ is right p.q.Baer, there exists a left semicentral idempotent $e(x) \in R[[x;\alpha]]$ such that $r_{R[[x;\alpha]]}(\varphi(x)R[[x;\alpha]]) = e(x)R[[x;\alpha]]$. Let $e(x) = f_0+f_1x+f_2x^2+\cdots$. Then, by Lemma 4, f_0 is an idempotent of R and $e(x)R[[x;\alpha]] = f_0R[[x;\alpha]]$. Thus $r_{R[[x;\alpha]]}(\varphi(x)R[[x;\alpha]]) = f_0R[[x;\alpha]]$. For any $r \in R$, $0 = \varphi(x)rf_0 = e_0rf_0+e_1\alpha(rf_0)x+e_2\alpha^2(rf_0)x^2+\cdots$. Thus $e_i\alpha^i(rf_0) = 0$ for every $i = 0, 1, \cdots$. Let $g = 1 - f_0$. Then $e_i\alpha^i(r(1-g)) = 0$ for any $r \in R$. Thus $\alpha^i(e_i)\alpha^i(r(1-g)) = 0$ by the weak rigidness of α. Hence $e_iR(1-g) = 0$ since α is a monomorphism. Suppose that h is an idempotent of R such that $e_iR(1-h) = 0$. Then $e_ir(1-h) = 0$ for any $r \in R$. Since α is weakly rigid, we have $e_ir\alpha^k(1-h) = 0$ for any $r \in R$. Thus, for any $a \in R$ and for any $\psi(x) = a_0 + a_1x + a_2x^2 + \cdots \in R[[x;\alpha]]$,

$$\varphi(x)\psi(x)a(1-h) = \sum_{k=0}^{\infty}\left(\sum_{i+j=k} e_i\alpha^i(a_j)\alpha^k(a(1-h))\right)x^k$$

$$= \sum_{k=0}^{\infty}\left(\sum_{i+j=k} e_i(\alpha^i(a_j)\alpha^k(a))\alpha^k(1-h)\right)x^k = 0.$$

This means that $a(1-h) \in r_{R[[x;\alpha]]}(\varphi(x)R[[x;\alpha]])$ for any $a \in R$. Thus $a(1-h) = f_0 a(1-h)$, which implies that $ga(1-h) = 0$ for any $a \in R$. Thus $gR(1-h) = 0$. Hence g is a generalized join of the set $\{e_0, e_1, \ldots\}$.

(2)$\Longrightarrow$(1). Suppose that $\varphi(x) = r_0 + r_1x + r_2x^2 + \ldots \in R[[x;\alpha]]$. Then there exist idempotents e_i, $i = 0, 1, \ldots$, such that $r_R(r_iR) = e_iR$. By the hypothesis, the set $\{1 - e_i | i = 0, 1, \ldots\}$ has a generalized join f. Thus $(1-e_i)R(1-f) = 0$. For any $\lambda(x) = a_0 + a_1x + a_2x^2 + \ldots \in R[[x;\alpha]]$,

$$\varphi(x)\lambda(x)(1-f) = \sum_{k=0}^{\infty}\left(\sum_{i+j=k} r_i\alpha^i(a_j)\alpha^k(1-f)\right)x^k.$$

Since $(1-e_i)a_j(1-f) = 0$, we have $(1-e_i)a_j\alpha^j(1-f) = 0$ by weak rigidness of α. Thus $a_j\alpha^j(1-f) = e_ia_j\alpha^j(1-f)$. Again by weak rigidness of α and $r_ie_i = 0$, it follows that $r_i\alpha^i(e_i) = 0$. Thus

$$\begin{aligned}\varphi(x)\lambda(x)(1-f) &= \sum_{k=0}^{\infty}\left(\sum_{i+j=k} r_i\alpha^i(a_j\alpha^j(1-f))\right)x^k \\ &= \sum_{k=0}^{\infty}\left(\sum_{i+j=k} r_i\alpha^i(e_ia_j\alpha^j(1-f))\right)x^k \\ &= \sum_{k=0}^{\infty}\left(\sum_{i+j=k} r_i\alpha^i(e_i)\alpha^i(a_j\alpha^j(1-f))\right)x^k = 0.\end{aligned}$$

This means that $(1-f)R[[x;\alpha]] \leq r_{R[[x;\alpha]]}(\varphi(x)R[[x;\alpha]])$.

Suppose that $\psi(x) = p_0 + p_1x + p_2x^2 + \ldots \in r_{R[[x;\alpha]]}(\varphi(x)R)$. Then from $\varphi(x)R\psi(x) = 0$ it follows that

$$\sum_{i+j=k} r_i\alpha^i(ap_j) = 0, \quad k = 0, 1, 2, \ldots,$$

where a is an arbitrary element of R. Thus, since $r_0ap_0 = 0$, one has $p_0 \in r_R(r_0R) = e_0R$. Let $a' \in R$ and take $a = a'e_0$ in $r_1\alpha(ap_0) + r_0ap_1 = 0$. Then $r_1\alpha(a'e_0p_0) + r_0a'e_0p_1 = 0$. But $r_0a'e_0p_1 = 0$. So $r_1\alpha(a'e_0p_0) = 0$. Since $e_0p_0 = p_0$, we have $r_1\alpha(a'p_0) = 0$. Since α is weakly rigid, it follows that $\alpha(r_1)\alpha(a'p_0) = 0$. Thus $r_1a'p_0 = 0$, which implies that $p_0 \in r_R(r_1R) = e_1R$. Also $r_0ap_1 = 0$ for any $a \in R$. This means that $p_1 \in r_R(r_0R) = e_0R$.

Now assume that

$$p_i \in e_jR, \quad i + j = 0, 1, 2, \cdots, k-1.$$

Let $a' \in R$ and take $a = a'e_0$ in $\sum_{i+j=k} r_i\alpha^i(ap_j) = 0$. Then, since $r_0a'e_0p_k = 0$, we have

$$\begin{aligned}&r_1\alpha(a'p_{k-1}) + \cdots + r_{k-1}\alpha^{k-1}(a'p_1) + r_k\alpha^k(a'p_0) \\ &= r_1\alpha(a'e_0p_{k-1}) + \ldots + r_{k-1}\alpha^{k-1}(a'e_0p_1) + r_k\alpha^k(a'e_0p_0) = 0.\end{aligned}$$

Let $b \in R$ and take $a' = be_1$. Then, since $r_1 be_1 p_{k-1} = 0$, we have $r_1\alpha(be_1p_{k-1}) = 0$ by the weak rigidness of α. Thus

$$\begin{aligned} &r_2\alpha^2(bp_{k-2}) + \ldots + r_{k-1}\alpha^{k-1}(bp_1) + r_k\alpha^k(bp_0) \\ &= r_2\alpha^2(be_1p_{k-2}) + \ldots + r_{k-1}\alpha^{k-1}(be_1p_1) + r_k\alpha^k(be_1p_0) = 0. \end{aligned}$$

Continuing in this manner, we have $r_k\alpha^k(cp_0) = r_k\alpha^k(ce_{k-1}p_0) = 0$, where c is an arbitrary element of R. This implies that

$$r_{k-1}\alpha^{k-1}(cp_1) = 0, \ \cdots, \ r_1\alpha(cp_{k-1}) = 0, \ r_0cp_k = 0.$$

From the weak rigidness of α, it follows that $\alpha^i(r_icp_{k-i}) = \alpha^i(r_i)\alpha^i(cp_{k-i}) = 0$ for any $i = 0, 1, \cdots, k$. Thus $r_icp_{k-i} = 0$, $i = 0, 1, \cdots, k$. Thus $p_{k-i} \in r_R(r_iR) = e_iR$, $i = 0, 1, \cdots, k$. Therefore, by induction, we have $p_i \in e_jR$, for any $i, j = 0, 1, \cdots$, and so

$$p_i = e_jp_i, \quad i, j = 0, 1, \cdots.$$

Suppose that $r_R(p_iR) = f_iR$, where f_i is a left semicentral idempotent of R. Since e_j is left semicentral, by the hypothesis, e_j is central. Thus we have $p_ir = e_jp_ir = p_ire_j$, which implies that $1 - e_j \in f_iR$. Thus $1 - e_j = f_i(1 - e_j)$ for any i,j. So $(1 - e_j)R(1 - f_i) = 0$. Since f is a generalized join of $\{1 - e_i | i = 0, 1, \ldots\}$, it follows that $fR(1 - f_i) = 0$ for any i. Hence

$$\begin{aligned} p_i &= p_i - p_if_i = p_i(1 - f_i) = (1 - f_i)p_i \\ &= (1 - f)(1 - f_i)p_i \in (1 - f)R. \end{aligned}$$

So $\psi(x) \in (1 - f)R[[x; \alpha]]$. Now it is easy to see that

$$\begin{aligned} (1 - f)R[[x; \alpha]] &\leq r_{R[[x;\alpha]]}(\varphi(x)R[[x; \alpha]]) \leq r_{R[[x;\alpha]]}(\varphi(x)R) \\ &\leq (1 - f)R[[x; \alpha]], \end{aligned}$$

which implies that $r_{R[[x;\alpha]]}(\varphi(x)R[[x; \alpha]]) = (1 - f)R[[x; \alpha]]$. Hence $R[[x; \alpha]]$ is right p.q.Baer.

Let R be an abelian ring (i.e., every idempotent of R is central). Then $I(R)$ is a Boolean algebra where $e \leq f$ means $ef = e$, and where the join, meet and complement are given by $e \vee f = e + f - ef$, $e \wedge f = ef$, and $e' = 1 - e$ respectively.

Corollary 6 *Let R be an abelian ring and α a weakly rigid endomorphism of R. Then the following conditions are equivalent:*

(1) $R[[x; \alpha]]$ is right p.q.Baer;

(2) R is right p.q.Baer and any countable family of idempotents in R has a join in $I(R)$.

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斜幂级数环的主拟 Baer 性

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摘要: 设 R 是环，并且 R 的左半中心幂等元都是中心幂等元， α 是 R 的一个弱刚性自同态. 本文证明了斜幂级数环 $R[[x,\alpha]]$ 是右主拟 Baer 环当且仅当 R 是右主拟 Baer 环，并且 R 的任意可数幂等元集在 $I(R)$ 中有广义交，其中 $I(R)$ 是 R 的幂等元集.

关键词: 弱刚性自同态；主拟 Baer 环；斜幂级数环.