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Exponential Attractor for Davey-Stewartson Equation in a Banach Space

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Abstract: In this paper, the existence of the exponential attractor of Davey-Stewartson equation is considered and its estimation of fractal dimension is obtained in a Banach subspace X_p^α of $L^p(\Omega)$.

Key words: Davey-Stewartson equation; exponential attractor; Banach space.
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1. Introduction

In this paper, we will study the Davey-Stewartson equation:

$$\frac{\partial A}{\partial t}-a\frac{\partial^2 A}{\partial x^2}-b\frac{\partial^2 A}{\partial y^2}=\chi A-\beta|A^2|A+\gamma QA,\quad t>0,(x,y)\in\Omega,\tag{1}$$

$$\frac{\partial^2 Q}{\partial x^2}+\frac{\partial^2 Q}{\partial y^2}=\frac{\partial^2}{\partial y^2}(|A|^2),\quad t>0,(x,y)\in\Omega,\tag{2}$$

$$A(t,x,y)=0,Q(t,x,y)=0,\quad t\geq 0,(x,y)\in\partial\Omega,\tag{3}$$

$$A(0,x,y)=A_0(x,y),\quad (x,y)\in\Omega,\tag{4}$$

where $a=a_1+ia_2, b=b_1+ib_2, \beta=\beta_1+i\beta_2, \gamma=\gamma_1+i\gamma_2$ and $\chi=\chi_1+i\chi_2$ are complex constants, and $\Omega\subset R^2$ is a smooth bounded domain. In order to ensure the uniqueness of Q, we must add $\int_\Omega Q(t,x,y)\mathrm{d}x\mathrm{d}y=0$.

We consider $-\Delta$ as an isomorphism from $W^{2,p}(\Omega)$ into $L^p(\Omega)(1<p<+\infty)$ and $(-\Delta)^{-1}$ is its inverse. From (2) and (3) we can solve Q in terms of A.

$$Q=-(-\Delta)^{-1}\frac{\partial^2(|A|^2)}{\partial y^2}\triangleq E(|A|^2).$$

Thus we can reduce (1) and (2) into a nonlocal nonlinear Schrödinger (or Ginzburg-Landau)-like equation

$$\frac{\partial A}{\partial t}-a\frac{\partial^2 A}{\partial x^2}-b\frac{\partial^2 A}{\partial y^2}=\chi A-\beta\mid A\mid^2 A-\gamma AE(|A|^2),\quad t>0,(x,y)\in\Omega,\tag{5}$$

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$$A(t,x,y)=0,\quad t\geq 0,(x,y)\in\partial\Omega, \tag{6}$$

$$A(0,x,y)=A_0(x,y),\quad (x,y)\in\Omega. \tag{7}$$

By Sobolev inequality, there exists a minimal $C(p)>0(1<p<+\infty)$ such that

$$\|\frac{\partial u}{\partial y^2}\|_p\leq C(p)\|\Delta u\|_p,\qquad u\in C_0^\infty(\Omega)$$

where $\|.\|_p$ denotes the norm in $L^p(\Omega)$. Thus

$$\|E(u)\|_p\leq C(p)\|u\|_p,\quad u\in C_0^\infty(\Omega) \tag{8}$$

and $E=(-\Delta)^{-1}\frac{\partial^2}{\partial y^2}$ can be extended to a bounded linear operator on $L^p(\Omega)(1<p<+\infty)$ with norm C(p). In fact, $\frac{\partial^2}{\partial y^2}$ and Δ can commute in $W^{2,p}(\Omega)$, and $(-\Delta)^{-1}$ is linearly bounded from $L^p(\Omega)$ into $W^{2,p}(\Omega)$. Therefore $\overline{E}=\frac{\partial^2}{\partial y^2}(-\Delta)^{-1}$ is the extension of E on $L^p(\Omega)$ with $\|\overline{E}\|_{L(L(\Omega))}$. The system (1),(2) was firstly derived by Davey etc[1] to model the evolution of a three-dimensional disturbance in the nonlinear regime of plane Poiseuille flow. $A(t,x,y)$ stands for the complex amplitude, and $Q(t,x,y)$ describes the perturbation of the real relocity. In recent years, Davey-Stewartson equations have drawn much attention of many physicists and mathematicians. The local and global existence, stability of plane wave solutions,solitons, lump solutions, nature of solutions that develop singularities and the long time behavior of solutions have been studied by many authors, such as Davey, Hocking and Stewartson[1], C.A.Holmes[6], Ghidaglia and Saut[4], Anker and Freeman[2], Ablowitz and Fokas[3], Tsutsumi[8], Hayashi and Saut[5], Linaves and Ponce[7], Boling Guo and Yongsheng Li[9] etc (cf. references therein). In recent years, Professor Dai and many other authors have studied the exponential attractor in Hilbert spaces sufficiently[9−12]. But in this paper, we will study the exponential attractor of the system (1)–(4) via (5)–(7)in Banach space. We will prove that, if the parameters satisfy the following conditions

$$[\mathrm{H}]\qquad k=\min\{a_1,b_1\}>0,\beta_1>0,\beta_1+C(2)\gamma_1>0,\chi_1>0,$$

the system (5)–(7) has an exponential attractor in a Banach subspace of $L^p(\Omega)$, where $C(2)$ is $C(p)$ when $p=2$.

In the second part of this thesis, we give some preliminaries, and in the third part we obtain the main results.

Throughout this paper, $W^{s,p}(\Omega)$ and $W_0^{s,p}$ denote the usual Sobolev spaces. $H^s(\Omega)=W^{s,2}(\Omega)$, $\|.\|_{s,p}$ is the norm of $W^{s,p}(\Omega)$, $\|.\|_p=\|.\|_{0,p}$, and $\overline{A}$ is the complex conjugate of A. C is a common constant and may assume different values in different formula.

2. Preliminaries

Definition 1[12] *Suppose E is a Banach space. We say that a compact set M is the exponential attractor of $(S(t),B)$, if $\mathcal{A}\subseteq\mathcal{M}\subset\mathcal{B}$ and*

1) $S(t)M\subseteq M,\quad \forall t\geq o;$

2) The fractal dimension of M is finite, i.e., $d_f(M) < \infty$;

3) M attracts exponentially all the obits from B, i.e., there exist C_0, C_1 such that

$$\text{dist}_E(S(t)B, M) \le C_0 e^{-C_1 t}, \quad \forall t > 0,$$

where $S(t)$ is the semigroup of solution operators and B is the compact absorbing set of $S(t)$ in E.

Definition 2[12] *If for every $\delta \in (0, \frac{1}{4})$, there exists an orthogonal projector with rank N such that for all $u, v \in B$,*

$$\|S(t_*)u - S(t_*)v\|_E \le \delta \|u - v\|_E, \tag{9}$$

or

$$\|Q_N(S_(t_*)u - S(t_*)v)\|_E \le \|P_N(S(t_*)u - S(t_*)v)\|_E, \tag{10}$$

then we say $S(t)$ is squeezing. Here $t_* > 0$ is a constant, P_N is an orthogonal projector and $Q_N = I - P_N$.

Definition 3[12] *For every u, v in the compact set B, if there exists a bounded function $l(t)$ such that*

$$\|S(t)u - S(t)v\|_E \le l(t)\|u - v\|_E, \tag{11}$$

then we say $S(t)$ is Lipschitz continuous in B and call $l(t)$ the Lipschitz constant of $S(t)$. Here $l(t)$ does not depend on u and v.

Now we give a proposition:

Proposition 1[12] *Suppose $S(t)$ is squeezing and Lipschitz continuous, then exists an exponential attractor M for $(S(t), B)$ and*

$$M = \bigcup_{0 \le t \le t_*} S(t) M_*, \tag{12}$$

where

$$M_* = \mathcal{A} \bigcup (\bigcup_{j=1}^{\infty} \bigcup_{k=1}^{\infty} S(t_*)^j)(E^{(k)}). \tag{13}$$

Moreover,

$$d_F(M) \le C N_0 + 1, \tag{14}$$

$$\text{dist}_E(S(t)B, M) \le C_o e^{-C_1 t}, \tag{15}$$

where N_0,$E^{(k)}$ are defined as in [12]; C, C_0,C_1 have no connection with the elements of B; and t_* is a positive constant.

Proof We utilize that B is compact in E and it is positive invariant for $S(t)$ with B instead of X in [15], and note that $S(t)$ is Lipschitz continuous and has a squeezing property in B, then this theorem is proved by the same method of the proof of Theorem in [15].

3. Main results

The functional setting of Davey-Stewartson system has the form

$$A_t + L_p A = f(A), \quad A(0) = A_0, \tag{16}$$

where

$$L_p = -a\frac{\partial^2}{\partial x^2} - b\frac{\partial^2}{\partial y^2} \tag{17}$$

is a differential operator on $X_p = L^p(\Omega)$ with domain $D(L_p) = W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)$, and

$$f(A) = \chi A - \beta|A|^2 A - \gamma A E(|A|^2). \tag{18}$$

It is obvious that, under the assumption [H], the spectra lies in the half plane $\{Z|\mathrm{Re}Z \geq w, w > 0\}$. The symbol of L_p is $L(\xi) = -a\xi_1^2 - b\xi_2^2$ and its real part $\mathrm{Re}L(\xi) \leq -k\xi^2$. Thus $L(\xi)$ is a strongly elliptic polynomial and $-L_p = -L(i\frac{\partial}{\partial x}, i\frac{\partial}{\partial y})$ generates a bounded analytic semigroup $e^{-L_p t}$ on X_p. Therefore we can define the functional powers L_p^α of L_p with domain $X_p^\alpha = D(L_p^\alpha)$, $\alpha > 0$. The semigroup $e^{-L_p t}$ satisfies the following properties:

$$\|e^{-L_p t} A\|_{X_p^\alpha} \leq M_\alpha e^{-wt} \|A\|_{X_p^\alpha}, \quad \forall A \in X_p^\alpha, \ t \leq 0, \tag{19}$$

$$\|e^{-L_p t} A\|_{X_p^\alpha} \leq M_\alpha t^{-\alpha} e^{-wt} \|A\|_{X_p}, \quad \forall A \in X_p^\alpha, \ t > 0, \tag{20}$$

for some $w > 0$ and $M_\alpha > 0$. For $0 \leq \alpha \leq 1$,

$$W_0^{2\alpha,p}(\Omega) \subset D(L_p^\alpha) \subset W^{2\alpha,p}, \quad 0 \leq \alpha \leq 1, \tag{21}$$

$$W_0^{1,p}(\Omega) \cap W^{2\alpha,p}(\Omega) = D(L_p^\alpha), \quad \frac{1}{2} \leq \alpha \leq 1. \tag{22}$$

The abstract Cauchy problem (16) can be equivalently expressed in an integral form

$$A(t) = e^{-L_p t} A_0 + \int_0^t e^{-L_p(t-s)} f(A(s)) \mathrm{d}s. \tag{23}$$

Throughout this paper, we denote $\alpha_1 = \frac{2}{3p}$, $\alpha_2 = \max\{\frac{1}{2}, \frac{1}{p}\}$. Then $0 < \alpha_1 < \alpha_2 < 1$, and

$$X_p^\alpha \hookrightarrow \begin{cases} L^{3p}(\Omega), & \text{for } \alpha \geq \alpha_1, 1 < p < \infty, \\ L^\infty(\Omega) & \text{for } \alpha > \alpha_2 \text{ when } p > 2, \text{ and for } \alpha > \alpha_2 \text{ when } 1 < p \leq 2, \\ L^q(\Omega) & \text{for } \alpha \geq \alpha_2 \text{ and any } 1 < q < \infty \text{ when } 1 < p \leq 2. \end{cases} \tag{24}$$

Therefore the nonlinear mapping $f(A)$ is locally Lipschitz continuous from X_p^α into X_p for $\alpha \geq \alpha_1$ and

$$\|f(A)\|_{X_p} \leq C_\alpha(\|A\|_{X_p^\alpha} + \|A\|_{X_p^\alpha}^3), \tag{25}$$

$$\|f(A_1) - f(A_2)\|_{X_p^\alpha} \leq C(\alpha, R)\|A_1 - A_2\|_{X_p^\alpha}. \tag{26}$$

For $\alpha \geq \alpha_2$, when $p > 2$ and for $\alpha > \alpha_2$, when $1 < p \leq 2$,X_p^α is an algebra and

$$\|f(A)\|_{X_p} \leq C_\alpha(\|A\|_{X_p^\alpha} + \|A\|_{X_p^\alpha}^3) \tag{27}$$

for all A_1,$A_2 \in X_p^\alpha$ with $\|A_K\|_{X_p^\alpha} \leq R$, $k = 1, 2$. Especially $f(A)$ is C^∞ mapping in $D(L_p^j)$, for integer $j \geq 1$.

We have known the following results[9],

Proposition 2[9] *Let [H] hold and $\alpha \geq \alpha_1 = \frac{2}{3p}$. For any $A_0 \in X_p^\alpha$, system(5)–(7) has a unique global solution*

$$A \in C([o,+\infty); X_p^\alpha) \bigcap \bigcap_{j,k=1}^{\infty} C^k((0,+\infty); D(L_p^j)). \tag{28}$$

Thus the system (5)–(7) defines a continuous semigroup $S(t)$ on X_p^α.

Proposition 3[9] *Let [H] hold and $\alpha_1 = \frac{2}{3p}, \alpha_2 = \max\{1/2, 1/p\}$, then $S(t)$, the semigroup generated by the system (5)–(7), has a compact absorbing set B_0 and has a global attractor $\mathcal{A}$.*

In the following we give the main results of this paper:

Theorem 1 *Let $A_1(t)$,$A_2(t)$ be two solutions of problem (5)–(7) in B. We denote $w = A_1 - A_2$. If*

$$\|w(t)\|_{X_p^\alpha} \leq l(t)\|w(0)\|_{X_p^\alpha}, \tag{29}$$

$$\|Q_N W(t)\|_{X_p^\alpha} \leq (C_2 e^{-C_3 t} + C_4 \lambda_{N+1}^{-\alpha} e^{C_5 \lambda_{N+1}^{-\alpha} t})\|w(0)\|_{X_p^\alpha}, \tag{30}$$

then $S(t)$ is Lipschitz continuous and squeezing on B. Therefore, $S(t)$ has an exponential attractor M in Banach subspace X_p^α and its fractal dimension is finite, i.e., $d_f(M) \leq CN_0 + 1$. Thus $d_F(\mathcal{A}) \leq CN_0 + 1$, where $\mathcal{A}$ is the global attractor of system (5)–(7) in X_p^α and N_0 satisfies

$$\lambda_{N_0} \geq \max\{(\frac{C_5}{C_3}\ln(32\sqrt{2}C_2))^{1/\alpha}, (32\sqrt{2}C_4 e)^{1/\alpha}\}. \tag{31}$$

Proof From Definitions 1 and 3, we easily know that $S(t)$ is Lipschitz continuous on B. Now we start to prove the squeeze property. If $\|P_N w\|_{X_p^\alpha} < \|Q_N w\|_{X_p^\alpha}$, because A^α and P_N,Q_N can commute, we have $\|w\|_{X_p^\alpha}{}^2 = \|P_N w\|_{X_p^\alpha}{}^2 + \|Q_N w\|_{X_p^\alpha}{}^2 \leq 2\|Q_N w\|_{X_p^\alpha}^2$ from (30)

$$\|w\|_{X_p^\alpha} \leq \sqrt{2}(C_2 e^{-C_3 t} + C_4 \lambda_{N+1}^{-\alpha} e^{C_5 \lambda_{N+1}^{-\alpha} t})\|w(0)\|_{X_p^\alpha}.$$

We choose an appropriate t_*,$t_* \geq \frac{1}{C_3}\ln(32\sqrt{2}C_2)$, therefore we have $\sqrt{2}C_2 e^{-C_3 t_*} < \frac{1}{32}$ when $t \geq t_*$. Now we fix t_* and choose N_0 such that $\lambda_{N_0} \geq \max\{(\frac{C_5}{C_3}\ln(32\sqrt{2}C_2))^{1/\alpha}, (32\sqrt{2}C_4 e)^{1/\alpha}\}$, then when $N \geq N_0$

$$\sqrt{2}C_4 \lambda_{N+1}^{-\alpha} e^{C_5 \lambda_{N+1}^{-\alpha} t} < \frac{1}{32}, \quad \forall t \geq t_*,$$

$$\|w(t)\|_{X_p^\alpha} < \frac{1}{16}\|w(0)\|_{X_p^\alpha}, \quad \forall t \geq t_*, N \geq N_0.$$

Thus we get $\|w(t)\|_{X_p^\alpha} < \frac{1}{8}\|w(0)\|_{X_p^\alpha}$ and conclude the proof of squeeze property. We use the proof in [13] and get $d_F(M) \leq CN_0 + 1$, where C is an absolute constant. Especially, we obtain $d_F(\mathcal{A}) \leq CN_0 + 1$. Consequently, the Proof of Theorem 1 is complete.

Theorem 2 *Let [H] hold, system (5)–(7) has an exponential attractor M in a Banach subspace X_p^α and its fractal dimension is finite, i.e., $d_F(M) \leq CN_0 + 1$, where N_0 satisfies*

$$\lambda_{N_0} \geq \max\{1, 2^{\frac{11}{2}+\frac{a}{\beta}} C_2 (2^{\frac{11}{2}+\frac{a}{\beta}} M_\alpha)^{C_3} M_\alpha\}, \tag{32}$$

where C_2, C_3 are defined in (42), (43); λ_{N_0} is the N_0th eigenvalue of L_p; β is any real number in $(\alpha, 1)$; and M_α is a constant that only depends on α. Especially, we have an estimation of the fractal dimension of attractor $\mathcal{A}$: $d_F(\mathcal{A}) \le CN_0 + 1$.

Before proving Theorem 2, we give two lemmas[15]:

Lemma 1 *Suppose B is a compact positive invariant set of $S(t)$ in X_p^α and $A(t)$ is any function in B, then $\|A(t)\|_{X_p^\alpha}$ is continuous in t.*

Lemma 2 *Suppose $0 < \alpha < 1$, $g(s)$ is a positive function and is continuous about s, if*

$$g(t) \le C_0 e^{-\lambda t} + C_1 \int_0^t e^{-\lambda(t-s)}(t-s)^{-\alpha} g(s)\mathrm{d}s, \tag{33}$$

$$g(s) \le 2^{\frac{\alpha}{\beta}}(e^{-\lambda t} + (1 + \frac{\lambda\alpha\Gamma_0^{-1}}{\beta-\alpha})^{\frac{\alpha-\beta}{\beta}} e^{\Gamma_1 t})C_0, \tag{34}$$

$$\Gamma_0 = C^{\frac{\beta}{\beta-\alpha}}(2\Gamma(1-\beta))^{\frac{\alpha}{\beta-\alpha}}(\frac{\alpha}{\beta\lambda})^{\frac{\alpha(1-\beta)}{\beta-\alpha}}, \tag{35}$$

$$\Gamma_1 = (2\Gamma(1-\beta))^{\frac{\alpha}{\beta}}(\frac{\alpha}{\beta\lambda})^{\frac{\alpha(1-\beta)}{\beta}} C_1. \tag{36}$$

The proofs of these two lemmas are very similar to that in [15], and are omitted. Now we begin to prove Theorem 2.

Proof We must prove two points: (i) $S(t)$ has a positive compact set B in X_p^α, and

(ii) $w(t)$ must satisfy the conditions of Theorem 1.

From Proposition 3, B_0 is a compact absorbing set in X_p^α of $S(t)$, therefore there exists $t_0 > 0$ such that $S(t)B_0 \subseteq B_0$ when $t \ge t_0$. Choosing $B = \overline{\bigcup_{0\le t\le t_0} S(t)B_0}$, then we can easily prove that B is the compact positive invariant set of $S(t)$ in X_p^α. By the definition of B_0, indeed, we know that there exists $t_0(B_0)$ such that $S(t)B_0 \subseteq B_0$, $\forall t \ge t_0(B_0)$, denoting $t = kt_0(B_0) + t_1, 0 \le t_1 \le t_0(B_0)$, thus we get

$$\begin{aligned} S(t)B &= \overline{\bigcup_{t_1\le s\le t_0(B_0)} S(t)S(kt_0(B_0))B_0} \bigcup \overline{\bigcup_{0\le s\le t_1} S(s)S((k+1)t_0(B_0))B_0} \\ &\subseteq \overline{\bigcup_{t_1\le s\le t_0(B_0)} S(s)B_0} \bigcup \overline{\bigcup_{0\le s\le t_1} S(s)B_0} \subseteq B. \end{aligned} \tag{37}$$

The absorbing property is clear and the proof of (i) is complete.

(ii) Suppose $A_1(t), A_2(t)$ are the two solutions with initial values $A_{1,0}, A_{2,0}$ respectively, from $A_{1,0}, A_{2,0} \in B$, we get $A_1(t), A_2(t) \in B$. For any $t \ge 0$, denoting $w(t) = A_1(t) - A_2(t)$, then we obtain

$$w_t + L_p = f(A_1) - f(A_2), \quad w(0) = w_0. \tag{38}$$

By using (19)–(21),(23) and constant variation formula

$$w(t) = e^{-L_p(t-s)}w(\delta) + \int_\delta^t e^{-L_p(t-s)}(f(A_1) - f(A_2))\mathrm{d}s, \text{ for } \delta \le t \le T,$$

then

$$\|w(t)\|_{X_p^\alpha} \le M_\alpha e^{-\omega(t-\delta)}\|w(\delta)\|_{X_p^\alpha} + \int_\delta^t M_\alpha e^{-\omega(t-s)}(t-s)^{-\alpha}\|f(A_1)-f(A_2)\|_{X_p}\mathrm{d}s,$$

using (26)

$$\|w(t)\|_{X_p^\alpha} \le M_\alpha e^{-\omega(t-\delta)}\|w(\delta)\|_{X_p^\alpha} + C(\alpha,R)\int_\delta^t M_\alpha e^{-\omega(t-s)}(t-s)^{-\alpha}\|w(t)\|_{X_p^\alpha}\mathrm{d}s, \tag{39}$$

choosing $\delta = 0$,

$$\|w(t)\|_{X_p^\alpha} \le M_\alpha e^{-\omega t}\|w(0)\|_{X_p^\alpha} + C(\alpha,R)\int_0^t M_\alpha e^{-\omega(t-s)}(t-s)^{-\alpha}\|w(t)\|_{X_p^\alpha}\mathrm{d}s. \tag{40}$$

Using Lemma 2 and choosing $C_0 = M_\alpha\|w(0)\|_{X_p^\alpha}$, $C_1 = M_\alpha C(\alpha,R)$, we complete the first point of Theorem 1.

If $w(t) \in Q_N X_p^\alpha$, then $\|e^{-Q_N L_p t}w\|_{X_p^\alpha} \le M_\alpha e^{-\lambda_{N+1}t}\|w(t)\|_{x_p^\alpha}$, where λ_{N+1} is the $(N+1)$th eigenvalue of L_p. Similar to the computation process of (42), we obtain

$$\|w(t)\|_{X_p^\alpha} \le M_\alpha e^{-\lambda^{N+1}t}\|Q_N w_0\|_{X_p^\alpha} + M_\alpha(\alpha,R)\int_0^t e^{-\lambda_{N+1}(t-s)}(t-s)^{-\alpha}\|w(t)\|_{X_p^\alpha}\mathrm{d}s.$$

Choosing $g(s) = \|w(s)\|_{X_p^\alpha}, C_0 = M_\alpha\|Q_N w_0\|_{X_p^\alpha}, C_1 = M_\alpha C(\alpha,R)$, using Lemma 2, we get

$$\|w(t)\|_{X_p^\alpha} \le 2^{\frac{\alpha}{\beta}}(e^{-\lambda_{N+1}t} + (1+\frac{\alpha\lambda_{N+1}\Gamma_0^{-1}}{\beta-\alpha})^{\frac{\alpha-\beta}{\beta}}e^{\Gamma_1 t})M_\alpha\|Q_N w_0\|_{X_p^\alpha}$$

for

$$\begin{aligned}(1+\frac{\alpha\Gamma_0^{-1}\lambda_{N+1}}{\beta-\alpha})^{\frac{\alpha-\beta}{\beta}} &\le (\frac{\alpha}{\beta-\alpha}\Gamma_0^{-1}\lambda_{N+1})^{\frac{\alpha-\beta}{\beta}}\\ &\le 2^{\frac{\alpha}{\beta}}\alpha^{\frac{2\alpha}{\beta}-\alpha-1}\beta^{\alpha-\frac{\alpha}{\beta}}\beta-\alpha)^{1-\frac{\alpha}{\beta}}M_\alpha C(\alpha,R)\Gamma(1-\beta)^{\frac{\alpha}{\beta}}\lambda_{N+1}^{-1}\\ &= C_2\lambda_{N+1}^{-1},\end{aligned} \tag{41}$$

where

$$C_2 = 2^{\frac{\alpha}{\beta}}\alpha^{\frac{2\alpha}{\beta}-\alpha-1}\beta^{\alpha-\frac{\alpha}{\beta}}(\beta-\alpha)^{1-\frac{\alpha}{\beta}}M_\alpha C(\alpha,R)\Gamma(1-\beta)^{\frac{\alpha}{\beta}}. \tag{42}$$

Thus

$$\|w(t)\|_{X_p^\alpha} \le 2^{\frac{\alpha}{\beta}}M_\alpha(e^{-\lambda_{N+1}t} + C_2\lambda_{N+1}^{-1}e^{\Gamma_1 t})\|Q_N w_0\|_{X_p^\alpha}$$

where

$$\begin{aligned}\Gamma_1 &= 2^{\frac{\alpha}{\beta}}(\alpha^{1-\beta}\Gamma(1-\beta))^{\frac{\alpha}{\beta}}\beta^{\alpha-\frac{\alpha}{\beta}}M_\alpha C(\alpha,R)\lambda_{N+1}^{\frac{-\alpha}{\beta}} = C_3\lambda_{N+1}^{\frac{-\alpha}{\beta}},\\ C_3 &= 2^{\frac{\alpha}{\beta}}(\alpha^{1-\beta}\Gamma(1-\beta))^{\frac{\alpha}{\beta}}\beta^{\alpha-\frac{\alpha}{\beta}}M_\alpha C(\alpha,R).\end{aligned} \tag{43}$$

Using Theorem 1

$$\|w(t)\|_{X_p^\alpha} \le (2^{\frac{\alpha}{\beta}}M_\alpha e^{-\lambda_{N+1}t} + 2^{\frac{\alpha}{\beta}}C_2 M_\alpha\lambda_{N+1}^{-1}e^{C_3\lambda_{N+1}^{\frac{-\alpha}{\beta}}t})\|w(0)\|_{X_p^\alpha},$$

choosing $t_* = \ln(2^{\frac{11}{2}+\frac{\alpha}{\beta}} M_\alpha)$ and an appropriate big N such that $\lambda_{N+1} > 1$, we obtain $2^{\frac{\alpha}{\beta}} M_\alpha e^{-\lambda_{N+1} t} \geq \frac{1}{32}$ when $t \geq t_*$; choosing N_0 again, such that

$$\lambda_{N_0} \geq \max\{1, 2^{\frac{11}{2}+\frac{\alpha}{\beta}} C_2 (2^{\frac{11}{2}+\frac{\alpha}{\beta}})^{C_3} M_\alpha\},$$

then

$$\|w(t)\|_{X_p^\alpha} \leq \frac{1}{16} \|w(0)\|_{X_p^\alpha} < \frac{1}{8} \|w(0)\|_{X_p^\alpha}$$

when $N \geq N_0$.

This proves the squeezing property of $S(t)$ and the supper-boundary of fractal dimension of exponential attractor. We complete the proof of Theorem 2.

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Davey-Stewartson 方程在 Banach 空间中的指数吸引子

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摘要: 我们考虑了 DS 方程在 Banach 空间 X_p^α 中的指数吸引子，并且得到其分形维度估计.

关键词: Davey-Stewartson 方程; 指数吸引子; Banach 空间.