第25卷第4期 数学研究与评论 Vol.25, No.4
2005年11月 JOURNAL OF MATHEMATICAL RESEARCH AND EXPOSITION Nov., 2005

**Article ID:** 1000-341X(2005)04-0634-01 **Document code:** A

# Elimination and Identities with Integral Sign

*LIU Hong-mei, WANG Tian-ming*
(Dept. of Appl. Math., Dalian University of Technology, Liaoning 116024, China)
(E-mail: yijianmei99@sina.com)

**MSC(2000):** 05A19
**CLC number:** O157

In this paper, our discussion is based on Zeilberg's basic idea[1] and use an elimination in the non-commutative Weyl algebra to get the differential operator. Thereby we can obtain the algorithm of proving identities of the form $\int_{-\infty}^{\infty}F(x,y)\mathrm{d}y=a(x)$.

Firstly, we give the remainder formula in the non-commutative algebra $C\langle D_x,D_y,x\rangle$ generated by $D_x,D_y$ and $x$, where $D_x,D_y$ are differential operators, $D_xx=xD_x+1, D_yy=yD_y+1$, and a basic theorem.

**Remainder formula** For two given polynomials $A,B\in C\langle D_x,D_y,x\rangle$, $\deg(A,x)\geq\deg(B,x)$, there exist $u,v\in C\langle D_x,D_y,x\rangle$ such that $uA=vB+r$, where $\deg(r,x)<\deg(B,x)$.

**Basic theorem** *Let $A,B\in C\langle D_x,D_y,x\rangle$, there always exist $u,v\in C\langle D_x,D_y,x\rangle$ such that $uA=vB$.*

From the above theories we have established algorithms Getuv and GetR to get $u,v$ and $R$, and set up a Maple program that implements the two algorithms , and we have checked the following example .

**Example** Prove the identity $\int_{-\infty}^{\infty}exp(-x^2/y^2-y^2)\mathrm{d}y=\sqrt{\pi}\exp(-2x)$ after the appropriate initial conditions are checked.

Input: $F:=\exp(-x^2/y^2-y^2)$ , $a(x):=\sqrt{\pi}\exp(-2x)$.

Output: $F(x,y)$ is annihilated by the following y-free operator $A(x,D_x)=-96x-96D_xx^2-40D_x^3x^2-40xD_x^2-24D_x^4x+16D_x^5x^2+16x^3D_x^4-32x^3D_x^2-2x^3D_x^6$ . $a(x)$ is annihilated by the operator $\mathrm{conj}(x,D_x)=D_x+2$. Now $A(x,D_x)=S(x,D_x)\mathrm{conj}(x,D_x)$ (check!), where $S(x,D_x)=-2(D_x^3-2D_x^2-4D_x+8)D_x^2x^3+2(5D_x^3-4D_x^2-24)D_xx^2+(-4D_x^3-8D_x^2-24D_x-48)x-4D_x^2-24$. So the identity is true!

## References:

[1] ALMKVIST G, ZEILBERGER D. *The method of differentiating under the integral sign* [J]. J. Symbolic Comput., 1990, **10**: 571–591.
[2] HERBERT S W, ZEILBERGER D. *An algorithmic proof theory for hypergeometric (ordinary and "q") multisum/integral identities* [J]. Invent. Math., 1992, **108**: 575–633.

**Received date:** 2004-03-25
**Foundation item:** Supported by Wu Foundation and NSF of Hainan Province