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Base and Subbase in I-Fuzzy Topological Spaces

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Abstract: In this paper, we give the concepts of the base and subbase in I-fuzzy topological spaces, and use them to score continuous mapping and open mapping. We also study the base and subbase in the product space of I-fuzzy topological spaces.

Key words: I-fuzzy topology; I-fuzzy quasi-coincident neighborhood system; base; subbase; product space.
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1. Introduction

In 1980, Höhle[1] introduced the concept of the fuzzy measurable spaces with the idea of giving degrees in [0,1] to some topological terms rather than 0 and 1. In 1991, from a logical point view, Ying[2] introduced the concept of fuzzifying topology and gave its base and subbase, which is established on the crisp sets not on the fuzzy set. It is valued to note that Ying[2] also introduced another fuzzy topological space which is called bifuzzy topological space (in this paper we call it I-fuzzy topological space). Briefly speaking, an I-fuzzy topology on a set X assigns to every fuzzy set on X a certain degree of being open, other than being definitely open or not. He only gave the concept of I-fuzzy topology, but did not study it. It is very meaningful to study this kind of fuzzy topological space, so Fang[3] introduced the I-fuzzy quasi-coincident neighborhood system in I-fuzzy topological spaces. In order to study the I-fuzzy topological spaces, we use the I-fuzzy quasi-coincident neighborhood system to give the concepts of base and subbase of I-fuzzy topology and use them to depict the continuous and open mapping. We also study the product space of I-fuzzy topological spaces and gain many important results.

Let X be a universal discourse and the family of all fuzzy sets on X will be denoted by I^X (where $I=[0,1]$). By 0_X and 1_X, we denote respectively the constant fuzzy set on X taking the value 0 and 1. The set of all fuzzy points x_λ (i.e., a fuzzy set $A\in I^X$ such that $A(x)=\lambda\neq 0$ and $A(y)=0$ for $y\neq x$) is denoted by $pt(I^X)$. Given a mapping $f:X\to Y$, we write $f^{\leftarrow}$ for the function $I^Y\to I^X$ defined by $f^{\leftarrow}(A)=A\circ f$ and write $f^{\rightarrow}$ for the function $I^X\to I^Y$ defined by $f^{\rightarrow}(A)(y)=\bigvee_{f(x)=y}A(x)$ for all $A\in I^X$, $y\in Y$.

Definition 1.1[7] *Let $x_\lambda\in pt(I^X)$ and $A,B\in I^X$. We say x_λ quasi-coincides with A, or say x_λ is quasi-coincident with A, denoted by $x_\lambda qA$, if $A(x)+\lambda>1$; say A quasi-coincides with B*

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at x, or say A is quasi-coincident with B at x, AqB at x for short, if $A(x)+B(x)>1$; say A quasi-coincides with B, denoted by AqB if A is quasi-coincident with B at some point $x\in X$. Relation "does not quasi-coincide with" or "is not quasi-coincident with" is denoted by $\neg q$.

Definition 1.2[1] *An I-fuzzy topology on a set X is a function $\tau: I^X\to I$ such that*

(1) $\tau(1_X)=\tau(0_X)=1$;

(2) $\forall U,V\in I^X$, $\tau(U\bigwedge V)\geq \tau(U)\bigwedge\tau(V)$;

(3) $\forall U_j\in I^X$, $j\in J$, $\tau(\bigvee_{j\in J}U_j)\geq\bigwedge_{j\in J}\tau(U_j)$.

And (X,τ) is called I-fuzzy topological space (ifts, for short). Furthermore, if $\tau(\lambda)=1$ for all constant mapping λ from X to I, (X,τ) is called stratified I-fuzzy topological space.

Definition 1.3[3] *Suppose $\tau: I^X\to I$ is an I-fuzzy topology on X. $\forall x_\lambda\in pt(I^X), U\in I^X$, let*

$$Q_{x_\lambda}(U)=\begin{cases}\bigvee_{x_\lambda qV\leq U}\tau(V), & x_\lambda qU,\\ 0, & x_\lambda\neg qU.\end{cases}$$

The set of $Q=\{Q_{x_\lambda}: x_\lambda\in pt(I^X)\}$ is called I-fuzzy quasi-coincident neighborhood system of T on X.

Proposition 1.4[3] *$Q=\{Q_{x_\lambda}: x_\lambda\in pt(I^X)\}$ of maps $Q_{x_\lambda}: I^X\to I$ defined in Definition 1.3 satisfies: $\forall U,V\in I^X$,*

(1) $Q_{x_\lambda}(1_X)=1, Q_{x_\lambda}(0_X)=0$;

(2) $Q_{x_\lambda}(U)>0\Rightarrow x_\lambda qU$;

(3) $Q_{x_\lambda}(U\bigwedge V)=Q_{x_\lambda}(U)\bigwedge Q_{x_\lambda}(V)$;

(4) $Q_{x_\lambda}(U)=\bigvee_{x_\lambda qV\leq U}\bigwedge_{y_\mu qV}Q_{y_\mu}(V)$.

Definition 1.5[4] *Given an I-fuzzy topological space (X,T) and subset $Y\subseteq X$. We call (Y,T_Y) (where $T_Y(U)=\bigvee\{T(V): V\in I^X, V|Y=U\}$, and we know that T_Y is an I-fuzzy topology on Y) the subspace of (X,T).*

Definition 1.6 *Let $f: X\to Y$ be bijection. If $f^{\to}:(X,\tau)\to(Y,\delta)$ satisfies the following conditions:*

(1) $f^{\to}:(X,\tau)\to(Y,\delta)$ *is continuous, i.e., for each* $B\in I^Y$, $\delta(B)\leq\tau(f^{\leftarrow}(B))$;

(2) $f^{\to}:(X,\tau)\to(Y,\delta)$ *is open, i.e., for each* $U\in I^X$, $\tau(U)\leq\delta(f(U))$,

then f is called an I-fuzzy homeomorphism.

2. Base and subbase

Definition 2.1 *Let τ be an I-fuzzy topology on X and $\mathcal{B}: I^X\to I$ with $\mathcal{B}\leq\tau$ (in pointwise sense). $\mathcal{B}$ is called a base of τ if $\mathcal{B}$ satisfies the following condition*

$$\forall U\in I^X,\forall x_\lambda\in pt(I^X), Q_{x_\lambda}(U)\leq\sup_{x_\lambda qV\leq U}\mathcal{B}(V).$$

Lemma 2.2 *Let (X,τ) be an I-fuzzy topological space. Then $\tau(A)=\inf_{x_\lambda qA}Q_{x_\lambda}(A)$ for every $A\in I^X$.*

Proof Straightforward.

Theorem 2.3 *A map $\mathcal{B}: I^X \to I$ is a base of τ iff $\tau(A) = \sup_{\vee_{\lambda\in\Lambda} B_\lambda = A} \inf_{\lambda\in\Lambda} \mathcal{B}(B_\lambda)$ for each $A \in I^X$, where the expression $\sup_{\vee_{\lambda\in\Lambda} B_\lambda = A} \inf_{\lambda\in\Lambda} \mathcal{B}(B_\lambda)$ will be denoted by $\mathcal{B}^{(\sqcup)}(A)$, i.e., $\mathcal{B}^{(\sqcup)}: I^X \to I$ s.t. $\mathcal{B}^{(\sqcup)} = \tau$.*

Proof It is similar with the Proof of Ying's Theorem 4.1 in [2].

Theorem 2.4 *Suppose that $\mathcal{B}: I^X \to I$. Then $\mathcal{B}$ is a base of some I-fuzzy topology if and only if*

(1) $\mathcal{B}^{(\sqcup)}(1_X) = 1$;

(2) $\mathcal{B}(A) \wedge \mathcal{B}(B) \le \sup_{x_\lambda q C \le A\wedge B} \mathcal{B}(C)$ if $x_\lambda q(A \wedge B)$.

Proof It is similar with the proof of Ying's Theorem 4.2 in [2].

Definition 2.5 *Let $\varphi: I^X \to I$ be a map. φ is called a subbase of τ if $\varphi^{(\sqcap)}$: $I^X \to I$ is a base, where $\varphi^{(\sqcap)}(A) = \sup_{(\sqcap)_{\lambda\in\Lambda} B_\lambda = A} \inf_{\lambda\in\Lambda} \varphi(B_\lambda)$ for all $A \in I^X$ with $(\sqcap)$ standing for "finite intersection".*

Theorem 2.6 *$\varphi: I^X \to I$ is a subbase of some I-fuzzy topology if and only if $\varphi^{(\sqcup)}(1_X) = 1$, where $\varphi^{(\sqcup)}$ is defined by $\varphi^{(\sqcup)}(A) = \sup_{\vee_{\lambda\in\Lambda} B_\lambda = A} \inf_{\lambda\in\Lambda} \varphi(B_\lambda)$.*

Proof The proof of the sufficiency is similar with the proof of Ying's Theorem 4.3 in [2]. Now we prove the necessity of this theorem. From the definition of subbase and Theorem 2.4, we have $(\varphi^{(\sqcap)})^{(\sqcup)}(1_X) = 1$. Thus $\forall \alpha < 1$,

$$\alpha < (\varphi^{(\sqcap)})^{(\sqcup)}(1_X) = \sup_{\bigvee_{\lambda\in\Lambda} B_\lambda = 1_X} \inf_{\lambda\in\Lambda} \varphi^{(\sqcap)}(B_\lambda) = \sup_{\bigvee_{\lambda\in\Lambda} B_\lambda = 1_X} \inf_{\lambda\in\Lambda} \sup_{(\sqcap)_{\beta\in\Lambda_\lambda} C_{\lambda\beta} = B_\lambda} \inf_{\beta\in\Lambda_\lambda} \varphi(C_{\lambda\beta}).$$

Then there exist $\{B_\lambda\}_{\lambda\in\Lambda}$ with $\bigvee_{\lambda\in\Lambda} B_\lambda = 1_X$ and $\{C_{\lambda\beta}\}_{\beta\in\Lambda_\lambda}$ with $(\sqcap)_{\beta\in\Lambda_\lambda} C_{\lambda\beta} = B_\lambda$ for each $\lambda \in \Lambda$ such that $\alpha \le \varphi(C_{\lambda\beta})$. Thus $\bigvee_{\lambda\in\Lambda,\beta\in\Lambda_\lambda} C_{\lambda\beta} = 1_X$ and $\alpha \le \bigwedge_{\lambda\in\Lambda,\beta\in\Lambda_\lambda} \varphi(C_{\lambda\beta})$. So $\alpha \le \sup_{\bigvee_{\lambda\in\Lambda} B_\lambda = 1_X} \inf_{\lambda\in\Lambda} \varphi(B_\lambda) = \varphi^{(\sqcup)}(1_X)$. By the arbitrariness of α, we have $\varphi^{(\sqcup)}(1_X) = 1$.

3. Applications

Theorem 3.1 *Let $f^{\to}: (X,\tau) \to (Y,\delta)$ be a map and δ be generated by its subbase φ. If $\varphi(U) \le \tau(f^{\leftarrow}(U))$ for each $U \in I^Y$, then $f^{\to}$ is continuous.*

Proof Let $U \in I^Y$. Because δ is generated by its subbase φ, we have

$$\begin{aligned}\delta(U) &= \sup_{\bigvee_{\lambda\in J} A_\lambda = U} \inf_{\lambda\in\Lambda} \sup_{(\sqcap)_{\mu\in\Lambda_\lambda} B_\mu = A_\lambda} \inf_{\mu\in\Lambda_\lambda} \varphi(B_\mu)\\ &\le \sup_{\bigvee_{\lambda\in\Lambda} A_\lambda = U} \inf_{\lambda\in\Lambda} \sup_{(\sqcap)_{\mu\in\Lambda_\lambda} B_\mu = A_\lambda} \inf_{\mu\in\Lambda_\lambda} \tau(f^{\leftarrow}(B_\mu))\\ &\le \sup_{\bigvee_{\lambda\in\Lambda} A_\lambda = U} \inf_{\lambda\in\Lambda} \tau(f^{\leftarrow}(A_\lambda))\end{aligned}$$

$$\leq \sup_{\bigvee_{\lambda\in\Lambda} A_\lambda = U} \tau(f^{\leftarrow}(\bigvee_{\lambda\in\Lambda} A_\lambda)) = \tau(f^{\leftarrow}(U)).$$

Thus $f^{\rightarrow}$ is continuous by the definition, as desired.

Theorem 3.2 *Let $f: X \to Y$ be a map and $f^{\rightarrow}: (X,\tau) \to (Y,\delta)$, where τ is generated by its base $\mathcal{B}$. If $\mathcal{B}(U) \leq \delta(f^{\rightarrow}(U))$ for all $U \in I^X$, then $\tau(U) \leq \delta(f^{\rightarrow}(U))$, i.e., $f^{\rightarrow}$ is open.*

Proof Let $W \in I^X$. By Theorem 2.3 we have

$$\begin{aligned}\tau(W) &= \sup_{\bigvee_{\lambda\in\Lambda} A_\lambda = W} \inf_{\lambda\in\Lambda} \mathcal{B}(A_\lambda) \leq \sup_{\bigvee_{\lambda\in\Lambda} A_\lambda = W} \inf_{\lambda\in\Lambda} \delta(f^{\rightarrow}(A_\lambda)) \\ &\leq \sup_{\bigvee_{\lambda\in\Lambda} A_\lambda = W} \delta(f^{\rightarrow}(\bigvee_{\lambda\in\Lambda} A_\lambda)) = \delta(f^{\rightarrow}(W)).\end{aligned}$$

Remark Note that $\bigwedge_{\lambda\in J} f^{\rightarrow}(A_\lambda) = f^{\rightarrow}(\bigwedge_{\lambda\in J} A_\lambda)$ for every $\{A_\lambda : \lambda \in J\} \subseteq I^X$ if $f: X \to Y$ is bijection. Hence, in Theorem 3.2, if $f: X \to Y$ is bijection and τ is generated by its subbase φ, and $\varphi(U) \leq \delta(f^{\rightarrow}(U))$ for every $U \in I^X$, then $f^{\rightarrow}: (X,\tau) \to (Y,\delta)$ is open.

Lemma 3.3 *Let $f^{\rightarrow}: (X,\tau) \to (Y,\delta)$ be I-fuzzy continuous and $Z \subseteq X$. Then $(f|Z)^{\rightarrow}: (Z, \tau|Z) \to (Y,\delta)$ is continuous, where $(f|Z)(x) = f(x)$ for $x \in Z$ and $(\tau|Z)(A) = \bigvee\{\tau(U) : U|Z = A\}$ for $A \in I^Z$.*

Proof We note that $(f|Z)^{\leftarrow}(W) = f^{\leftarrow}(W)|Z$ if $W \in I^Z$. Therefore, $\forall W \in I^Z$, we have

$$\begin{aligned}(\tau|Z)((f|Z)^{\leftarrow}(W)) &= \bigvee\{\tau(U) : U|Z = (f|Z)^{\leftarrow}(W)\} \\ &\geq \tau(f^{\leftarrow}(W)) \geq \delta(W).\end{aligned}$$

This implies that $(f|Z)^{\rightarrow}$ is continuous.

Theorem 3.4 *Let τ be an I-fuzzy topology on X, $\mathcal{B}$ be the base of τ, $Y \subseteq X$ and $\mathcal{B}|Y$ be defined by $\mathcal{B}|Y(U) = \bigvee\{\mathcal{B}(W) : W|Y = U\}$ for every $U \in I^Y$. Then $\mathcal{B}|Y$ is a base of $\tau|Y$.*

Proof Let $U \in I^Y$. By Theorem 2.3, we have

$$(\tau|Y)(U) = \bigvee_{P|Y=U} \tau(P) = \sup_{P|Y=U} \sup_{\bigvee_{\lambda\in\Lambda} A_\lambda = P} \inf_{\lambda\in\Lambda} \mathcal{B}(A_\lambda).$$

To show that $\mathcal{B}|Y$ is a base of $\tau|Y$, we need to prove the following equality

$$\sup_{V|Y=U} \sup_{\bigvee_{\lambda\in\Lambda} A_\lambda = V} \inf_{\lambda\in\Lambda} \mathcal{B}(A_\lambda) = \sup_{\bigvee_{\lambda\in\Lambda} B_\lambda = U} \inf_{\lambda\in\Lambda} \sup_{W|Y=B_\lambda} \mathcal{B}(W).$$

In one hand, if $V \in I^X$ with $V|Y = U$ and $\bigvee_{\lambda\in\Lambda} A_\lambda = V$, we have $\bigvee_{\lambda\in\Lambda}(A_\lambda|Y) = U$. Let $B_\lambda = A_\lambda|Y$, thus $\bigvee_{\lambda\in\Lambda} B_\lambda = U$. Moreover,

$$\sup_{\bigvee_{\lambda\in\Lambda} B_\lambda = U} \inf_{\lambda\in\Lambda} \sup_{W|Y=B_\lambda} \mathcal{B}(W) \geq \inf_{\lambda\in\Lambda} \mathcal{B}(A_\lambda).$$

Hence

$$\sup_{V|Y=U}\ \sup_{\bigvee_{\lambda\in\Lambda}A_\lambda=V}\ \inf_{\lambda\in\Lambda}\mathcal{B}(A_\lambda)\leq \sup_{\bigvee_{\lambda\in\Lambda}B_\lambda=U}\ \inf_{\lambda\in\Lambda}\ \sup_{W|Y=B_\lambda}\mathcal{B}(W).$$

On the other hand, we assume that $\mu\in(0,1]$ and

$$\mu<\sup_{\bigvee_{\lambda\in\Lambda}B_\lambda=U}\ \inf_{\lambda\in\Lambda}\ \sup_{W|Y=B_\lambda}\mathcal{B}(W).$$

Then there exists a family of $\{B_\lambda\}_{\lambda\in\Lambda}\subseteq I^Y$ such that

(1) $\bigvee_{\lambda\in\Lambda}B_\lambda=U$;

(2) $\forall\lambda\in\Lambda$, there exists $W_\lambda\in I^X$ with $W_\lambda|Y=B_\lambda$ such that $\mu<\mathcal{B}(W_\lambda)$. Let $V=\bigvee_{\lambda\in\Lambda}W_\lambda$. Then $V|Y=U$ and $\bigwedge_{\lambda\in\Lambda}\mathcal{B}(W_\lambda)\geq\mu$. Therefore

$$\sup_{V|Y=U}\ \sup_{\bigvee_{\lambda\in\Lambda}A_\lambda=V}\ \inf_{\lambda\in\Lambda}\mathcal{B}(A_\lambda)\geq\mu.$$

By the arbitrariness of μ, we obtain that

$$\sup_{V|Y=U}\ \sup_{\bigvee_{\lambda\in\Lambda}=V}\ \inf_{\lambda\in\Lambda}\mathcal{B}(A_\lambda)\geq \sup_{\bigvee_{\lambda\in\Lambda}B_\lambda=U}\ \inf_{\lambda\in\Lambda}\ \sup_{W|Y=B_\lambda}\mathcal{B}(W).$$

Thus we complete the proof.

Definition 3.5 *Let $\{(X_\alpha,\tau_\alpha)\}_{\alpha\in J}$ be a family of I-fuzzy topological spaces and $P_\beta:\prod_{\alpha\in J}X_\alpha\to X_\beta$ be the projection. The I-fuzzy topology whose subbase is defined by*

$$\forall W\in I^{\prod_{\alpha\in J}X_\alpha},\varphi(W)=\sup_{\alpha\in J}\ \sup_{P_\alpha^{\leftarrow}(U)=W}\tau_\alpha(U)$$

is called the product I-fuzzy topology of $\{\tau_\alpha:\alpha\in J\}$ and denoted by $\prod_{\alpha\in J}\tau_\alpha$, and

$$(\prod_{\alpha\in J}X_\alpha,\prod_{\alpha\in J}\tau_\alpha)$$

is called the product space of $\{(X_\alpha,\tau_\alpha)\}_{\alpha\in J}$.

Definition 3.6 *Suppose that $\{(X_\alpha,\tau_\alpha)\}_{\alpha\in J}$ is a family of I-fuzzy topological spaces and $P_\beta:\prod_{\alpha\in J}X_\alpha\to X_\beta$ be the projection. Let $y=(y_\alpha)_{\alpha\in J}\in\prod_{\alpha\in J}X_\alpha$ be fixed point. We define a subset $X_\beta(y)\subseteq\prod_{\alpha\in J}X_\alpha$ as follows*

$$X_\beta(y)=\{x=(x_\alpha)_{\alpha\in J}\in\prod_{\alpha\in J}X_\alpha:\alpha\neq\beta,x_\alpha=y_\alpha\}.$$

Then $(X_\beta(y),\prod_{\alpha\in J}\tau_\alpha|X_\beta(y))$ is called a factor space of $(\prod_{\alpha\in J}X_\alpha,\prod_{\alpha\in J}\tau_\alpha)$ parallel to X_β through y.

Theorem 3.7 *Let $(X_\beta(y),\prod_{\alpha\in J}\tau_\alpha|X_\beta(y))$ be a stratified factor space of $(\prod_{\alpha\in J}X_\alpha,\prod_{\alpha\in J}\tau_\alpha)$ parallel to X_β through $y=(y_\alpha)_{\alpha\in J}$. Then $(P_\beta|X_\beta(y))^{\rightarrow}:(X_\beta(y),\prod_{\alpha\in J}\tau_\alpha|X_\beta(y))\to(X_\beta,\tau_\beta)$ is I-fuzzy homeomorphism.*

Proof According to Definition 3.5, the subbase of the product I-fuzzy topology, denoted by φ, is as follows

$$\forall U \in I^{\prod_{\alpha\in J} X_\alpha}, \varphi(U) = \sup_{\alpha\in J} \sup_{P_\alpha^{\leftarrow}(W)=U} \tau_\alpha(W).$$

Thus the base of the product I-fuzzy topology, denoted by $\mathcal{B}$, should be as follows

$$\forall U \in I^{\prod_{\alpha\in J} X_\alpha}, \mathcal{B}(U) = \sup_{(\sqcap)_{\lambda\in\Lambda} A_\lambda = U} \inf_{\lambda\in\Lambda} \varphi(A_\lambda).$$

On account of Theorem 3.4, $\mathcal{B}|X_\beta(y)$ is a base of $\prod_{\alpha\in J}\tau_\alpha|X_\beta(y)$. Note that

$$P_\beta|X_\beta(y) : X_\beta(y) \to X_\beta$$

is bijection and continuous. To prove that $(P_\beta|X_\beta(y))^{\rightarrow} : (X_\beta(y), \prod_{\alpha\in J}\tau_\alpha|X_\beta(y)) \to (X_\beta, \tau_\beta)$ is homeomorphism, considering Definition 1.6 and Theorem 3.2, we only prove that $\forall U \in I^{X_\beta(y)}$,

$$(\mathcal{B}|X_\beta(y))(U) \le \tau_\beta((P_\beta|X_\beta(y))^{\rightarrow}(U)).$$

If $(\mathcal{B}|X_\beta(y))(U) = 0$, then the inequality is obvious. For other case, we take any $\mu \in (0,1]$ such that

$$\begin{aligned}\mu &< (\mathcal{B}|X_\beta(y))(U) = \sup\{\mathcal{B}(W) : W|X_\beta(y) = U\}\\ &= \sup_{W|X_\beta(y)=U} \sup_{(\sqcap)_{\lambda\in\Lambda} A_\lambda = W} \inf_{\lambda\in\Lambda} \varphi(A_\lambda).\end{aligned}$$

Therefore, there exist $W \in I^{\prod_{\alpha\in J} X_\alpha}$ and a finite family of $\{A_\lambda : \lambda \in J\}$ fulfilling the following conditions:

(1) $W|X_\beta(y) = U$ and $(\sqcap)_{\lambda\in J} A_\lambda = W$;

(2) $\forall \lambda \in J$, there exist $\alpha(\lambda) \in J$ and $V(\alpha(\lambda)) \in I^{X_{\alpha(\lambda)}}$ such that $P_{\alpha(\lambda)}^{\leftarrow}(V(\alpha(\lambda))) = A_\lambda$ and $\tau_{\alpha(\lambda)}(V(\alpha(\lambda))) > \mu$.

Clearly, we have $W = (\sqcap)_{\lambda\in J} P_{\alpha(\lambda)}^{\leftarrow}(V(\alpha(\lambda)))$. We will show $(\mathcal{B}|X_\beta(y))(U) \le \tau_\beta((P_\beta|X_\beta(y))^{\rightarrow}(U))$ in the following two cases

Case 1. If there exists $\alpha(\lambda)$ with $V(\alpha(\lambda)) = 0_{X_{\alpha(\lambda)}}$, then $\tau_\beta((P_\beta|X_\beta(y))^{\rightarrow}(U)) = \tau_\beta(0_{X_\beta}) = 1$.

Case 2. Now we assume that $V(\alpha(\lambda)) \ne 0_{X_{\alpha(\lambda)}}$ for every $\lambda \in J$. If $\beta \notin \{\alpha(\lambda) : \lambda \in J\}$, then $(P_\beta|X_\beta(y))^{\rightarrow}(U)$ is constant. Hence $\tau_\beta((P_\beta|X_\beta(y))^{\rightarrow}(U)) = 1$. Otherwise, if $\beta \in \{\alpha(\lambda) : \lambda \in J\}$, for example $\beta = \alpha_0(\lambda)$, then we have

$$(P_\beta|X_\beta(y))^{\rightarrow}(U) = c \bigwedge V(\alpha_0(\lambda)),$$

where c is a constant map on $X_\beta(y)$ such that

$$c(z) = \bigwedge\{V(\alpha)(z_\alpha) : \alpha \in \{\alpha(\lambda) : \lambda \in J\} - \{\beta\}\}$$

for every $z \in X_\beta(y)$. Therefore,

$$\begin{aligned}\tau_\beta((P_\beta|X_\beta(y))^{\rightarrow}(U)) &= \tau_\beta(c \bigwedge V(\alpha_0(\lambda)) \ge \tau_\beta(c) \bigwedge \tau_\beta(V(\alpha_0(\lambda)))\\ &= \tau_\beta(V(\alpha_0(\lambda))) > \mu,\end{aligned}$$

where $\tau_\beta(c) = 1$. Considering the arbitrariness of μ, we conclude that

$$\forall U \in I^{X_\beta(y)}, (\mathcal{B}|X_\beta(y))(U) \leq \tau_\beta((P_\beta|X_\beta(y))^{\rightarrow}(U)).$$

Corollary 3.8 *Let* $(\prod_{\alpha\in J} X_\alpha, \prod_{\alpha\in J}\tau_\alpha)$ *be the product space of a family of* I*-fuzzy topological spaces* $\{(X_\alpha, \tau_\alpha)\}_{\alpha\in J}$*. Then* (X_α, τ_α) *is* I*-fuzzy homeomorphism to a subspace of the product space whenever* (X_α, τ_α) *is stratified.*

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I-Fuzzy 拓扑空间中的基与子基

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摘要: 本文给出了 I-fuzzy 拓扑空间中基与子基的合理定义，对连续映射和开映射进行了刻画. 此外，文中界定了乘积空间中基与子基并研究了乘积空间与因子空间的关系.

关键词: I-fuzzy 拓扑； I-fuzzy 重域系统；基；子基；乘积空间.