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On the Vertex Strong Total Coloring of Halin-Graphs

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Abstract: A proper k-total coloring f of the graph $G(V, E)$ is said to be a k-vertex strong total coloring if and only if for every $v \in V(G)$, the elements in $N[v]$ are colored with different colors, where $N[v] = \{u|uv \in V(G)\} \cup \{v\}$. The value $\chi_T^{vs}(G) = \min\{k|$ there is a k-vertex strong total coloring of $G\}$ is called the vertex strong total chromatic number of G. For a 3-connected plane graph $G(V, E)$, if the graph obtained from $G(V, E)$ by deleting all the edges on the boundary of a face f_0 is a tree, then $G(V, E)$ is called a Halin-graph. In this paper, $\chi_T^{vs}(G)$ of the Halin-graph $G(V, E)$ with $\Delta(G) \geq 6$ and some special graphs are obtained. Furthermore, a conjecture is initialized as follows: Let $G(V, E)$ be a graph with the order of each component are at least 6, then $\chi_T^{vs}(G) \leq \Delta(G) + 2$, where $\Delta(G)$ is the maximum degree of G.

Key words: Halin-graph; coloring problem; vertex strong total coloring; total coloring problem.
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1. Introduction

The coloring problem is one of the most important problems in the graph theory. As an extension of the classical coloring problem, the strong coloring problems, which were firstly presented by M. Aigner et al.[1,2] and F. Harary[11], are of significance both in theory and in practice. Although it is more difficult than the classical coloring problem, some meaningful results about the vertex distinguishing edge coloring were obtained recently. For example, M. Aigner[1,2], O. Favaron, et al.[9] and A. C. Burns[7] studied the strong edge-coloring for general graphs and obtained some results. C. Bazgan, et al.[4] studied the vertex-distinguishing proper coloring of graphs with large minimum degree and [5] that of general graphs. P. N. Balister, et al.[3] studied the vertex distinguishing colorings of graphs with $\Delta(G) = 2$. P. Wittmann[13] studied the vertex-distinguishing edge-colorings of 2-regular graphs. R. A. Brualdi and J. J. Quinn Massey[6] studied the Incidence and strong edge colorings of graphs. Z. Zhang and L. Liu[14] have studied the adjacent vertex distinguishing edge coloring (also says the adjacent strong edge coloring) of graphs, and presented the adjacent vertex distinguishing edge coloring conjecture.

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In this paper, as another extension of strong coloring problems, we propose a new concept, vertex strong total coloring, of graph and study the vertex strong total coloring of Halin-graph and some special graphs.

Let $G(V,E)$ be a graph with vertices set $V(G)$ and edges set $E(G)$. $G[S]$ denotes the subgraph of the graph $G(V,E)$ induced by set $S \subset V(G)$ or $S \subset E(G)$. $N(v)$ denotes the adjacent vertices set of vertex $v \in V(G)$, $N[v] = N(v) \cup \{v\}$. V_k denotes the vertices set of k-degree of the graph $G(V,E)$, where k is a positive integer. For a face f_0 of a plane graph, $V(f_0)$ denotes the set of vertices on the boundary of the face f_0.

Definition 1.1 *A proper k-total coloring f of graph $G(V,E)$ is said to be a k-vertex strong total coloring of $G(V,E)$, abbreviated as k-VSTC, if the elements in $N[v]$ are colored with different colors for every $v \in V(G)$. The integer $\chi_T^{vs}(G) = \min\{k|$ there is a k-VSTC of $G\}$ is called the vertex strong total chromatic number of graph $G(V,E)$.*

Definition 1.2[10] *Let $G(V,E)$ be a 3-connected plane graph. If the graph obtained from $G(V,E)$ by deleting all edges on the boundary of a face f_0 is a tree, then $G(V,E)$ is called a Halin-graph, and f_0 is called the outer face of G (others the inner face), the vertices in $V(f_0)$ are called outer vertex (others the inner vertex).*

It follows from the Definition 1.2 that the degree of all vertices of outer face f_0 of a Halin–graph is equal to 3.

It is obvious that $\chi_T^{vs}(G) \geq \Delta(G)$ for a simple connected graph $G(V,E)$ with $|V(G)| \geq 3$, where $\Delta(G)$ is the maximum degree of the graph $G(V,E)$.

Conjecture 1.1 (VSTCC) *Let $G(V,E)$ be a graph such that the order of each component is at least 6, then $\chi_T^{vs}(G) \leq \Delta(G)+2$, where $\Delta(G)$ is the maximum degree of G.*

For the other terminologies we refer to references [8], [11].

2. Main results

We have proved in another paper that the following theorem holds.

Theorem 2.1 *(1) For a cycle C_n, we have*

$$\chi_T^{vs}(C_n) = \begin{cases} 3, & \text{for } 2n \equiv 0 (\bmod\ 3), \\ 4, & \text{for } 2n \not\equiv 0 (\bmod\ 3) \text{ and } p \neq 5, \\ 5, & \text{for } n = 5. \end{cases}$$

(2) For a complete bipartite graph $K_{m,n}$, we have $\chi_T^{vs}(K_{m,n}) = m+n$, $m+n \geq 3$.

(3) For a tree $G(V,E)$ with $|V(G)| \geq 3$, we have $\chi_T^{vs}(G) = \Delta(G)+1$.

(4) For a complete graph K_n, we have

$$\chi_T^{vs}(K_n) = \begin{cases} n, & \text{for } n \equiv 1 (\bmod\ 2), \\ n+1, & \text{for } n \equiv 0 (\bmod\ 2). \end{cases}$$

Halin-graph is an important graph class, and its many parameters have been studied in references.

Lemma 2.1[15] *Let $G(V,E)$ be a Halin-graph, then the following four statements hold in graph $G(V,E)$:*

(1) The degrees of all outer vertices are equal to 3.

(2) Any two interior faces have exactly one common edge and each interior face has exactly one common edge with outer face.

(3) If $G \ncong W_p$, which is a wheel of order p, then there are at least two interior vertices of G, and there always exists an interior vertex w which adjacent to only one interior vertex.

(4) If $G \ncong W_p$, then there always exists such a vertex w satisfying the statement 3 of Lemma 2.1 that $N_G(w) = \{u, v_1, v_2, \ldots, v_k\}$, $xv_1, v_ky \in E(G)$, $v_1 \neq x$, $v_k \neq y$, $2 \leq k \leq \Delta(G) - 1$, and graphs

$$G_1 = G - \{v_1, v_2, \ldots, v_k\} + \{xw, yw\},$$

$$G_2 = G - \{v_i, v_{i+1}, \ldots, v_j\} + \{v_{i-1}v_{j+1}\}, \quad 2 \leq i \leq j < k, k \geq 3$$

are also Halin-graph, where u is the interior vertex adjacent to w and $v_1, v_2, \ldots, v_k$ are outer vertices adjacent to w.

Lemma 2.2 *For a wheel graph W_p with $p = |V(W_p)| \geq 6$, we have $\chi_T^{vs}(W_p) = \Delta(W_p) + 1 = p$.*

Proof It is clear that $\chi_T^{vs}(W_p) \geq \Delta(W_p) + 1$ for a wheel graph W_p with $p = |V(W_p)| \geq 6$. We now prove that there exists a $\Delta(W_p) + 1$-VSTC of W_p. Let $V(W_p) = \{v_1, v_2, \ldots, v_p\}$, $E(W_p) = \{v_iv_p, i = 1, 2, \ldots, p-1, v_jv_{j+1}, j = 1, 2, \ldots, p-2, v_{p-1}v_1\}$. Then a $\Delta(W_p) + 1$-VSTC of W_p can be defined as follows:

$$f(v_p) = p, f(v_i) = i, i = 1, 2, \ldots, p-1;$$

$$f(v_iv_p) = i+1, i = 1, 2, \ldots, p-2, f(v_{p-1}v_p) = 1;$$

$$f(v_iv_{i+1}) = i+3, i = 1, 2, \ldots, p-3, f(v_{p-3}v_{p-2}) = 1, f(v_{p-2}v_{p-1}) = 2, f(v_{p-1}v_1) = 3.$$

Clearly, f is a $\Delta(W_p) + 1$-VSTC of W_p. The lemma is thus proved. □

Theorem 2.2 *Let $G(V,E)$ be a Halin-graph with $\Delta(G) \geq 6$. Then $\chi_T^{vs}(G) = \Delta(G) + 1$.*

Proof We prove this theorem by using induction method on $|V(G)|$. The proof of $\Delta(G) \geq 6$ is the same as that of $\Delta(G) = 6$. Therefore, without lose of generality, we just prove the case of $\Delta(G) = 6$. In the following process, $C = \{1, 2, 3, 4, 5, 6, 7\}$ denotes a set of seven colors. W denotes a set consisting of all vertices w in the statements 3 and 4 of Lemma 2.1.

Assume that the Theorem 2.2 holds for $|V(G)| < p$. We now prove that it holds for $|V(G)| = p$.

For a k-vertex strong total coloring f of graph $G(V,E)$, we use $f(s)$ to denote the color of the element $s \in V \cup E$. In the following process, we consider the vertex $w \in W$, of which the degree is minimum.

Case 1. When $d(w)=3$, we denote $N_G(w)=\{u,v_1,v_2\}$ and define a new graph as

$$G'=G-\{v_1,v_2\}+\{xw,yw\}.$$

It follows from Lemma 2.1 that G' is also a Halin-graph with $\Delta(G')=\Delta(G)$ and $|V(G')|<|V(G)|=p$. By the induction hypothesis, G' has a $(\Delta(G)+1)$-VSTC g. Based on the $(\Delta(G)+1)$-VSTC g of G', we now define a $(\Delta(G)+1)$-VSTC f of graph $G(V,E)$ as follows:

$$\begin{aligned}
&f(v_1v_2)=g(w), f(v_1)\in C\setminus\{g(w),g(u),g(x_1),g(x),g(y)\},\\
&f(v_2)\in C\setminus\{g(w),g(u),g(x),f(v_1),g(y),g(y_1)\},\\
&f(xv_1)\in C\setminus\{g(x_1x),g(x),f(v_1),f(v_1v_2)\},\\
&f(yv_2)\in C\setminus\{g(y_1y),g(y),f(v_2),g(v_1,v_2)\},\\
&f(v_1w)\in C\setminus\{g(wu),g(w),f(v_1),f(xv_1)\},\\
&f(v_2w)\in C\setminus\{g(wu),g(w),f(v_2),f(v_1w),f(yv_2)\}.
\end{aligned}$$

The colors of other elements of the graph $G(V,E)$ are the same as that of g. Obviously, f is a $(\Delta(G)+1)$-VSTC of G.

Case 2. When $d(w)=4$, we denote $N_G(w)=\{u,v_1,v_2,v_3\}$ and define a new graph as

$$G'=G-\{v_1,v_2,v_3\}+\{xw,yw\}.$$

It follows from the Lemma 2.2 that G' is also a Halin-graph with $\Delta(G')=\Delta(G)$ and $|V(G')|<|V(G)|=p$. By the induction hypothesis, there exists a $(\Delta(G)+1)$-VSTC g of G'. Based on the $(\Delta(G)+1)$-VSTC g of G', we now define a $(\Delta(G)+1)$-VSTC f of G as follows:

$$\begin{aligned}
&f(v_1)\in C\setminus\{g(w),g(x_1),g(x),g(u)\},\\
&f(v_2)\in C\setminus\{g(w),g(u),g(x),f(v_1),g(u_1)\},\\
&f(v_3)\in C\setminus\{g(y_1),g(y),f(u),g(w),f(v_1),f(v_2)\}.
\end{aligned}$$

For each edge in $\{wv_1,wv_2,wv_3\}$, except the un-colored adjacent or incident vertices and edges on the boundary of f_0, the number of its incident or adjacent elements is at most 6. Hence, there is always one color to color edges wv_1, wv_2 and wv_3, respectively. For each edge in $\{xv_1,v_1v_2,v_2v_3,v_3y\}$, the number of its incident or adjacent elements is also at most 6. Therefore, there is always one color to color xv_1, v_1v_2, v_2v_3 and v_3y, respectively. The colors of other elements of G are the same as that of g of G'. Obviously, f is a $(\Delta(G)+1)$-VSTC of G.

Case 3. When $d(w)=5$, we denote $N_G(w)=\{u,v_1,v_2,v_3,v_4\}$ and define a new graph as

$$G'=G-\{v_1,v_2,v_3,v_4\}+\{xw,yw\}.$$

It follows from Lemma 2.2 that G' is also a Halin-graph with $\Delta(G')=\Delta(G)$ and $|V(G')|<|V(G)|=p$. By the induction hypothesis, there exists a $(\Delta(G)+1)$-VSTC g of G'. It is obvious that $g(x),g(y)\notin\{g(u),g(w)\}$.

Based on g of graph $G'(V,E)$, we now define a $(\Delta(G)+1)$-VSTC f of the graph $G(V,E)$ as follows:

$$\begin{aligned}
&f(v_3)=g(x), f(v_2)=g(y), f(xv_1)=g(xw), f(yv_4)=g(yw),\\
&f(v_1)\in C\setminus\{g(x_1),g(x),g(w),g(u),f(v_2)\},\\
&f(v_4)\in C\setminus\{g(y_1),g(y),g(w),g(u),f(v_1),f(v_3)\},\\
&f(v_1w)\in C\setminus\{g(wu),g(w),f(v_1),f(xv_1)\},\\
&f(v_2w)\in C\setminus\{g(wu),g(w),f(v_2),f(v_1w)\},\\
&f(v_4w)\in C\setminus\{g(wu),f(v_1w),f(v_2w),f(v_4),g(w),g(v_4y)\},\\
&f(v_3w)\in C\setminus\{g(wu),f(v_1w),f(v_2w),f(v_4w),g(w),f(v_3)\}.
\end{aligned}$$

For each edge in $\{v_1v_2,v_2v_3,v_3v_4\}$, the number of its incident or adjacent elements is at most 6. So there is always one color to color v_1v_2, v_2v_3 and v_3v_4, respectively. The colors of other elements of G are same as that of g of G'. Obviously, f is a $(\Delta(G)+1)$-VSTC of G.

Case 4. When $d(w)=6$. Denotes $N_G(w)=\{u,v_1,v_2,v_3,v_4,v_5\}$. We define a new graph as

$$G'=G-\{v_2,v_3,v_4\}+\{v_1v_5\}.$$

It follows from Lemma 2.2 that G' is also a Halin-graph with $\Delta(G')=\Delta(G)$ and $|V(G')|<|V(G)|=p$. By the induction hypothesis, there exists a $(\Delta(G)+1)$-VSTC g of graph $G'(V,E)$. Based on the g, we now define a $(\Delta(G)+1)$-VSTC f as follows: Let

$$\begin{aligned}
&f(v_2)\in C\setminus\{g(w),g(u),g(x),g(v_1),g(v_5)\},\\
&f(v_4)\in C\setminus\{g(w),g(u),g(y),g(v_1),g(v_5),f(v_2)\},\\
&f(v_3)\in C\setminus\{g(w),g(u),g(v_1),f(v_2),f(v_4),g(v_5)\}.
\end{aligned}$$

Subcase 4.1 If $\{f(v_2),f(v_3),f(v_4)\}=\{g(v_1w),g(uw),g(v_5w)\}$, then

$$\{g(v_1),g(u),g(v_5)\}\cap\{g(v_1w),g(uw),g(v_5w)\}=\emptyset.$$

Firstly, let $f(v_2w)=g(v_1), f(v_3w)=g(u), f(v_4w)=g(v_5)$.

For each edge in $\{v_1v_2,v_2v_3,v_3v_4,v_4v_5\}$, the number of its incident or adjacent elements is at most 6. Hence there is always one color to color v_1v_2, v_2v_3, v_3v_4 and v_4v_5, respectively. The colors of other elements of G are the same as that of g of G'. Obviously, f is a $(\Delta(G)+1)$-VSTC of G.

Subcase 4.2 $\{f(v_2),f(v_3),f(v_4)\}\neq\{g(v_1w),g(uw),g(v_5w)\}$.

Subcase 4.2.1 If $\{g(v_1),g(v_5),g(u)\}=\{g(v_1w),g(uw),g(v_5w)\}$, then let

$$f(v_2w)=g(v_3), f(v_3w)=f(v_4), f(v_4w)=f(v_2).$$

For each edge in $\{v_1v_2,v_2v_3,v_3v_4,v_4v_5\}$, the number of its incident or adjacent elements is at most 6. Therefore, there is always one color to color v_1v_2, v_2v_3, v_3v_4 and v_4v_5, respectively. The colors of other elements of G are the same as that of g of G'. Obviously, f is a $(\Delta(G)+1)$-VSTC of G.

Subcase 4.2.2 $\{g(v_1), g(v_5), g(u)\} \neq \{g(v_1w), g(uw), g(v_5w)\}$.

Subcase 4.2.2.1 If $\{g(v_1), g(v_5), g(u)\} \notin \{g(v_1w), g(uw), g(v_5w)\}$, the proof is similar as that of Subcase 4.1, and is omitted here.

Subcase 4.2.2.2 If $|\{g(v_1), g(v_5), g(u)\} \cap \{g(v_1w), g(uw), g(v_5w)\}| = 1$, without lose of generality, assume that $g(v_1) \in \{g(v_1w), g(uw), g(v_5w)\}$, $g(v_5)$ and $g(u) \notin \{g(v_1w), g(uw), g(v_5w)\}$. Then there are exactly two elements of $\{f(v_2), f(v_3), f(v_4)\}$ in the set $\{g(v_1w), g(uw), g(v_5w)\}$. Without lose of generality, assume that $f(v_3)$ and $f(v_4) \in \{g(v_1w), g(uw), g(v_5w)\}$, $f(v_2) \notin \{g(v_1w), g(uw), g(v_5w)\}$, then let

$$f(v_2w) = g(u), f(v_3w) = g(v_5), f(v_4w) = f(v_2).$$

For each edge in $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$, the number of its incident or adjacent elements is at most 6. Hence there is always one color to color v_1v_2, v_2v_3, v_3v_4 and v_4v_5, respectively. The colors of other elements of G are same as that of g of G'. Obviously, f is a $(\Delta(G)+1)$-VSTC of G.

Subcase 4.2.2.3 If there are exactly two elements of $\{g(v_1), g(u), g(v_5)\}$ in the set $\{g(v_1w), g(uw), g(v_5w)\}$. Without lose of generality, assume that

$$\{g(v_1), g(u)\} \subset \{g(v_1w), g(uw), g(v_5w)\}, \quad g(v_5) \notin \{g(v_1w), g(uw), g(v_5w)\}.$$

Then there is exactly one element of $\{f(v_2), f(v_3), f(v_4)\}$ in the set $\{g(v_1w), g(uw), g(v_5w)\}$. Without lose of generality, assume that $f(v_2) \in \{g(v_1w), g(uw), g(v_5w)\}$, $g(v_3)$ and $f(v_4) \notin \{g(v_1w), g(uw), g(v_5w)\}$. Then let

$$f(v_2w) = g(v_5), f(v_4w) = f(v_3), f(v_3w) = f(v_4).$$

For each edge in $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$, the number of its incident or adjacent elements is at most 6. Hence there is always one color to color v_1v_2, v_2v_3, v_3v_4 and v_4v_5, respectively. The colors of other elements of G are the same as that of g of G'. Obviously, f is a $(\Delta(G)+1)$-VSTC of G.

From what was discussed above, the proof is thus complete. $\square$

Follows from the proving process of Theorem 2.2, the following theorem also holds.

Theorem 2.3 *If G is a Halin graph of $\Delta(G) \leq 5$, then $\chi_T^{vs}(G) \leq 6$.*

References:

[1] AIGNER M, TRIESCH E, TUZA Z. *Irregular assignments and vertex-distinguishing edge-colorings of graphs* [J]. Ann. Discrete Math., 1990, **52**(1): 1–9.

[2] AIGNER M, TRIESCH E. *Irregular assignments of trees and forests* [J]. SIAM J. Discrete Math., 1990, **3**(2): 439–449.

[3] BALISTER P N, BOLLOBAS B, SCHELP R H. *Vertex distinguishing colorings of graphs with* $\Delta(G) = 2$ [J]. Discrete Math., 2002, **252**(1): 17–29.
[4] BAZGAN C, HARKAT-BENHAMDINE A, LI Hao. et al. *A note on the vertex-distinguishing proper coloring of graphs with large minimum degree* [J]. Discrete Math., 2001, **236**: 37–42.
[5] BAZGAN C, HARKAT-BENHAMDINE A, LI Hao. et al. *On the vertex-distinguishing proper edge-colorings of graphs* [J]. J. Combin. Theory Ser. B, 1999, **75**(2): 288–301.
[6] BRUALDI R A, MASSEY J J Q. *Incidence and strong edge colorings of graphs* [J]. Discrete Math., 1993, **122**(1): 51–58.
[7] BURNS A C. *Vertex-distinguishing proper edge-colorings* [J]. J. Graph Theory, 1997, **22**(1): 73–82.
[8] CHARTRAND G, LESNIAK L. *Graphs and Digraphs* [M]. Monterey: Edition Wadsworth Brooks/Cole, 1986.
[9] FAVARON O , LI H, SCHELP R H. *Strong edge colorings of graphs* [J]. Discrete Math., 1996, **159**(1): 103–109.
[10] HALIN R. *Stuties on minimal n-connected graph* [A]. Comb. Math. and Its Applications, Proc. Conf. Oxford, 1969.
[11] HARARY F. *Conditional Colorability in Craphs, in: Graph and Applications. Proc. First Colorado Symp, Graph Theory* [M]. New York: John & Wiley Inc., 1985.
[12] LIU Lin-zhong, ZHANG Zhong-fu, WANG Jian-fang. *The adjacent strong edge chromatic number of outerplanar graphs with* $\Delta(G) \leq 4$ [J]. Appl. Math. J. Chinese Univ. Ser. A, 2000, **15**(2): 139–146. (in Chinese)
[13] WITTMANN P. *Vertex-distinguishing edge-colorings of 2-regular graphs* [J]. Discrete Math., 1997, **79**(3): 265–277.
[14] ZHANG Zhong-fu, LIU Lin-zhong, WANG Jian-fang. *Adjacent strong edge coloring of graphs* [J]. Appl. Math. Lett., 2002, **15**(5): 623–626.
[15] ZHANG Zhong-fu, LIU Lin-zhong. *A note on the total chromatics number of Halin-graph with* $\Delta(G) \geq 4$ [J]. Appl. Math. Lett., 1998, **11**(1): 23–27.
[16] ZHANG Zhong-fu, LIU Lin-zhong. *On the complete chromatic number of Halin graphs* [J]. Acta Math. Appl. Sinica (English Ser.), 1997, **13**(1): 102–106.

Halin- 图的点强全染色

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摘要: 图 $G(V,E)$ 的一个 k- 正常全染色 f 叫做一个 k- 点强全染色当且仅当对任意 $v \in V(G)$, $N[v]$ 中的元素被染不同色，其中 $N[v] = \{u|uv \in V(G)\}\bigcup\{v\}$. $\chi_T^{vs}(G) = \min\{k|$ 存在图 G 的 k-点强全染色 $\}$ 叫做图 G 的点强全色数. 对 3- 连通平面图 $G(V,E)$, 如果删去面 f_0 边界上的所有点后的图为一个树图，则 $G(V,E)$ 叫做一个 Halin- 图. 本文确定了最大度不小于 6 的 Halin-图和一些特殊图的的点强全色数 $\chi_T^{vs}(G)$, 并提出了如下猜想：设 $G(V,E)$ 为每一连通分支的阶不小于 6 的图，则 $\chi_T^{vs}(G) \leq \Delta(G)+2$, 其中 $\Delta(G)$ 为图 $G(V,E)$ 的最大度.

关键词: Halin- 图；图染色；点强全染色；全染色.