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Existence of Solutions to a Class of Higher-Order Singular Boundary Value Problem for One-Dimensional p-Laplacian

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Abstract: This paper deals with the existence of positive solutions for the problem

$$\begin{cases} (\Phi_p(x^{(n-1)}(t)))' + f(t, x, \ldots, x^{(n-1)}) = 0, \quad 0 < t < 1, \\ x^{(i)}(0) = 0, \quad 0 \le i \le n-3, \\ x^{(n-2)}(0) - B_0(x^{(n-1)}(0)) = 0, \quad x^{(n-2)}(1) + B_1(x^{(n-1)}(1)) = 0, \end{cases}$$

where $\Phi_p(s) = |s|^{p-2}s, p > 1$. f may be singular at $x^{(i)} = 0, i = 0, \ldots, n-2$. The proof is based on the Leray-Schauder degree and Vitali's convergence theorem.

Key words: singular higher-order differential equation; positive solution; Vitali's convergence theorem.
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1. Introduction

Let $J = [0, 1]$, $R_- = (-\infty, 0)$, $R_+ = (0, \infty)$, and $R_0 = R \setminus \{0\}$.

We investigate the existence of positive solutions for singular boundary value problem (BVP)

$$(\Phi_p(x^{(n-1)}(t)))' + f(t, x(t), \ldots, x^{(n-1)}(t)) = 0, \quad 0 < t < 1, \tag{1.1}$$

$$x^{(i)}(0) = 0, \ 0 \le i \le n-3, \ x^{(n-2)}(0) - B_0(x^{(n-1)}(0)) = 0, \ x^{(n-2)}(1) + B_1(x^{(n-1)}(1)) = 0, \tag{1.2}$$

where $n \ge 2$, and the nonlinear term f satisfies local Carathédory conditions on $J \times D (f \in \mathrm{Car}(J \times D))$ with

$$D = \underbrace{R_+ \times \cdots \times R_+}_{n-2} \times R,$$

and may be singular at $x^{(i)} = 0, i = 0, \ldots, n-2$.

Definition 1.1 *A function $x \in AC^{n-1}(J)$ (i.e. x has absolutely continuous $(n-1)^{st}$ derivative on J) is said to be a solution of BVP (1.1), (1.2), if $x^{(i)}(t) > 0$ on $(0, 1]$ for $0 \le i \le n-2$, and x satisfies the boundary condition (1.2) and the equation (1.1) a.e. on J.*

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This paper is mainly motivated by two aspects. On the one hand, when nonlinear term f has no singularity, many excellent results have been obtained, please see [1],[5]. For example, He and Ge in [5] studied the following boundary value problem

$$\begin{cases} (g(u'))' + e(t)f(u) = 0, \\ u(0) - B_0(u'(0)) = 0, \quad u(1) + B_1(u'(1)) = 0, \end{cases} \tag{1.3}$$

where $f \in C([0,\infty),[0,\infty))$. They proved that, under some assumptions, BVP (1.3) has at least one or two positive solutions. On the other hand, when nonlinearity f may be singular in phase variables, there are few papers on the existence results for boundary value problem. So far, singular boundary value problem for differential equations with Lidstone and (n,p) boundary conditions have been studied by Agarwal et al., see [3], [4].

To the best of our knowledge, the solvability of boundary value problem (1.1), (1.2) has not been studied till now. The purpose of this paper is to establish an existence result for problem (1.1), (1.2). Our method is based on Leray-Schauder degree theory and Vitali's convergence theorem. The approaches to estimate a priori bound of the solutions to boundary value problem (1.1), (1.2) are different from the corresponding ones of the past work [1], [2], [3] and [4].

From now on, $\|x\|_0 = \max\{|x(t)| : t \in J\}$ stands for the norm in $C^0(J)$. For any measurable set $\mathcal{M} \subset \mathcal{R}, \mu(\mathcal{M})$ denotes the Lebesgue measure of $\mathcal{M}$.

The following assumptions imposed upon the function in (1.1) will be used in the paper:

(H$_1$) $f \in \mathrm{Car}(J \times D)$ and there exist nonnegative functions $\varphi \in L^1(J), q_i \in L^\infty(J), \quad i = 0,\dots,n-2, \varphi(t) \not\equiv 0, h_j \in C(J\times R), \quad j = 0,\dots,n-1$, and non-increasing nonnegative function $\omega_i \in L^1(R_+), 0 \le i \le n-2$ such that for $(t,x) \in J \times D$,

$$f(t,x_0,\dots,x_{n-1}) = \varphi(t) + \sum_{i=0}^{n-2} q_i(t)\omega_i(|x_i|) + \sum_{i=0}^{n-1} h_i(t,x_i),$$

and h_i satisfies

$$\lim_{|x_i|\to\infty} \sup_{t\in[0,1]} \frac{h_i(t,x_i)}{\Phi_p(|x_i|)} = \alpha_i \ge 0, \quad \alpha_i \in (0,1), 0 \le i \le n-1, \tag{1.4}$$

ω_i satisfies

$$\omega_i(xy) \le \Lambda\omega_i(x)\omega_i(y) \ \text{ for } x,y \in (0,\infty), \Lambda > 0 \text{ is a positive constant.} \tag{1.5}$$

$$\int_0^1 \omega_i\left(\int_0^t (t-s)^{n-3-i}s(1-s)\mathrm{d}s\right)\mathrm{d}t < \infty, 0 \le i \le n-3, \quad \int_0^1 \omega_{n-2}(s(1-s))\mathrm{d}s < \infty. \tag{1.6}$$

(H$_2$) $B_i(v), i = 0,1$ are both nondecreasing, continuous, odd functions defined on $(-\infty,\infty)$. At least one of them satisfies the condition that there exists $m > 0$, such that $0 \le B_i(v) \le mv$.

2. Auxiliary results

Lemma 2.1 *Let $\varphi \in L_1(J)$ be nonnegative and $\varphi(t) \not\equiv 0$. Suppose that $x \in AC^{n-1}(J)$ satisfies (1.2) and*

$$0 \le -(\Phi_p(x^{(n-1)}(t)))', \quad t \in J. \tag{2.1}$$

Then we have for $t \in J$,

$$x^{(i)}(t) \geq \|x^{(n-2)}\| \int_0^t (t-s)^{n-3-i} s(1-s)\mathrm{d}s,\ 0 \leq i \leq n-3, \quad x^{(n-2)}(t) \geq \|x^{(n-2)}\| t(1-t).$$

Lemma 2.2 *Let $\varphi \in L_1(J)$ be nonnegative and $\varphi(t) \not\equiv 0$. Then there exists a positive constant $c = c(\varphi)$ such that for each function $x \in AC^{n-1}(J)$ satisfying (1.2) and*

$$\varphi(t) \leq -(\Phi_p(x^{(n-1)}(t)))' \ \text{ for a.e. } \ t \in J,$$

the estimate $\|x^{(n-2)}\| \geq c$ holds.

Remark 2.1 Suppose (H_1) and (H_2) hold. It follows from Lemmas 2.1 and 2.2 that for any solution of BVP (1.1), (1.2), there exists $c = c(\varphi)$ such that

$$|x^{(i)}(t)| \geq \frac{c}{(n-3-i)!} \int_0^t (t-s)^{n-3-i} s(1-s)\mathrm{d}s, \quad i = 0, \ldots, n-3, \quad x^{(n-2)}(t) \geq ct(1-t).$$

For each $m \in N$, define $\mathcal{X}_m$ and $f_m \in \mathrm{Car}(J \times R^n)$ by the formulas

$$\mathcal{X}_m(u) = \begin{cases} u, & \text{for } \ u \geq \frac{1}{m}, \\ \frac{1}{m}, & \text{for } \ u < \frac{1}{m}, \end{cases}$$

and

$$f_m(t, x_0, x_1, \ldots, x_{n-1}) = \varphi(t) + \sum_{i=0}^{n-2} q_i(t)\omega_i(\mathcal{X}_m(x_i)) + \sum_{i=0}^{n-1} h_i(t, x_i) \tag{2.2}$$

for $(t, x_0, \ldots, x_{n-1}) \in J \times R^n$. Hence

$$0 < \varphi(t) \leq f_m(t, x_0, \ldots, x_{n-1}) \leq \varphi(t) + \sum_{i=0}^{n-2} q_i(t)\omega_i(|x_i|) + \sum_{i=0}^{n-1} h_i(t, x_i) \tag{2.3}$$

for a.e. $t \in J$ and each $(x_0, \ldots, x_{n-2}, x_{n-1}) \in R_0^{n-1} \times R$.

Consider auxiliary regular differential equation

$$(\Phi_p(x^{(n-1)}(t)))' + f_m(t, x(t), \ldots, x^{(n-1)}(t)) = 0 \tag{2.4}$$

and

$$(\Phi_p(x^{(n-1)}(t)))' + \lambda f_m(t, x(t), \ldots, x^{(n-1)}(t)) = 0, \quad \lambda \in [0,1] \tag{2.5}$$

depending on the parameter $m \in N$.

Lemma 2.3 *Let x be a solution of the following BVP*

$$\begin{cases} (\Phi_p(x^{(n-1)}(t)))' + h(t) = 0, \quad 0 < t < 1, \\ x^{(i)}(0) = 0,\ 0 \leq i \leq n-3,\ x^{(n-2)}(0) - B_0(x^{(n-1)}(0)) = 0, \quad x^{(n-2)}(1) + B_1(x^{(n-1)}(1)) = 0. \end{cases} \tag{2.6}$$

Then x can be uniquely expressed as

$$x(t) = \int_0^t \frac{(t-s)^{n-3}}{(n-3)!} \left\{ B_0(\sigma_x) + \int_0^s \Phi_q \left[\Phi_p(\sigma_x) - \int_0^r h(\theta)\mathrm{d}\theta \right] \mathrm{d}r \right\} \mathrm{d}s. \tag{2.7}$$

where σ_x satisfied

$$B_0(\sigma_x)+\int_0^1\Phi_q\left[\Phi_p(\sigma_x)-\int_0^r h(\theta)\mathrm{d}\theta\right]\mathrm{d}r+B_1\circ\Phi_q\left[\Phi_p(\sigma_x)-\int_0^1 h(\theta)\mathrm{d}\theta\right]=0.$$

Lemma 2.4 *Let $m\in N$. If there exists a positive constant K such that*

$$\|x^{(j)}\|_0\le K,\ 0\le j\le n-1 \tag{2.8}$$

for any solution x of BVP (1.2), (2.5) with $\lambda\in[0,1]$, then BVP (1.2), (2.4) has a solution x satisfying (2.8).

For convenience, we write

$$\Gamma:=\int_0^1\left(\varphi(s)+\Lambda\sum_{i=0}^{n-3}q_i(s)\omega_i\left(\frac{c}{(n-3-i)!}\right)\omega_i\left(\int_0^s(s-\theta)^{n-3-i}\theta(1-\theta)\mathrm{d}\theta\right)+\right.$$
$$\left.q_{n-2}(s)\Lambda\omega_{n-2}(c)\omega_{n-2}(s(1-s))\right)\mathrm{d}s.$$

Lemma 2.5 *Let assumptions (H_1), (H_2) be satisfied. Furthermore, suppose the following inequality (H_3) $\sum_{i=0}^{n-2}\alpha_i\Phi_p\left(\frac{m+1}{(n-2-i)!}\right)+\alpha_{n-1}<1$ holds. Then there exists a positive constant P (independent of m) such that $\|x^{(j)}\|_0\le P, 0\le j\le n-1$ for any solution x of BVP (1.2), (2.5) with $m\in N$.*

Proof Let x be a solution of BVP (1.2), (2.5) for some $m\in N$.

Step 1. It follows from (H_2) that at least one of B_i satisfies $B_i(x)\le mx$. Without loss of generality, we suppose $B_0(x)\le mx$ holds, so

$$x^{(n-2)}(t)=B_0(x^{(n-1)}(0))+\int_0^t x^{(n-1)}(s)\mathrm{d}s\le(m+1)\|x^{(n-1)}\|_0,\quad t\in J.$$

$$x^{(i)}(t)=\int_0^t\frac{(t-\theta)^{n-3-i}}{(n-3-i)!}x^{(n-2)}(\theta)\mathrm{d}\theta\le\frac{m+1}{(n-2-i)!}\|x^{(n-1)}\|_0,\ t\in J,\ 0\le i\le n-2. \tag{2.9}$$

Step 2. There exists a positive constant P such that $\|x^{(n-1)}\|_0\le P$.

Let $\varepsilon>0$ be sufficiently small such that $\sum_{i=0}^{n-2}(\alpha_i+\varepsilon)\Phi_p\left(\frac{m+1}{(n-2-i)!}\right)+(\alpha_{n-1}+\varepsilon)<1$. Then for this $\varepsilon>0$, there is a $\delta>0$ such that

$$|h_i(t,x_i)|<(\alpha_i+\varepsilon)\Phi_p(|x_i|)\text{ uniformly for }t\in[0,1],\text{ and }|x_i|>\delta,\ 0\le i\le n-2. \tag{2.10}$$

Let, for $i=0,\dots,n-1$,

$$\Delta_{1,i}=\{t:t\in[0,1],|x_i(t)|\le\delta\},\quad\Delta_{2,i}=\{t:t\in[0,1],|x_i(t)|>\delta\},\quad h_{\delta,i}=\max_{t\in[0,1],|x_i|\le\delta}h_i(t,x_i).$$

There exists $\xi\in[0,1]$ such that $x^{(n-1)}(\xi)=0$ since $-(\Phi_p(x^{(n-1)}(t)))'\ge\varphi(t)\ge0$ and boundary condition (1.2).

On the one hand, integrating on both sides of (2.5) from t to ξ, ($t \in [0,\xi]$), using (2.3), Remark 2.1, (2.9) and (2.10) one obtains

$$\Phi_p\left(x^{(n-1)}(t)\right) \leq \Gamma + \sum_{i=0}^{n-1} h_{\delta,i} + \sum_{i=0}^{n-2}(\alpha_i+\varepsilon)\Phi_p\left(\frac{m+1}{(n-2-i)!}\right)\Phi_p(\|x^{(n-1)}\|_0) + (\alpha_{n-1}+\varepsilon)\Phi_p(\|x^{(n-1)}\|_0) \quad \text{for } t\in[0,\xi]. \tag{2.11}$$

The assumptions of this theorem now imply the result.

On the other hand, integrating both sides of (2.5) from ξ to t ($t \in [\xi,1]$), similar to the above process, we obtain (2.11) for $t\in[\xi,1]$. Thus $\|x^{(n-1)}\|_0 \leq P$. By (H_1) we have $P<\infty$.

Step 3. By Step 1 and Step 2, it is clear that $\|x^{(i)}\|_0 \leq P$, $i=0,1,\ldots,n-1$. The proof is complete.

Lemma 2.6 *Let assumptions (H_1), (H_3) be satisfied. Suppose that $\{x_m\}$ is a sequence of solutions to BVP (2.4), (1.2) for each $m\in N$. Then the sequence*

$$\{f_m(t,x_m(t),\ldots,x_m^{(n-1)}(t))\}\subset L_1(J)$$

is uniformly absolutely continuous on J, that is, for each $\varepsilon>0$ there exists $\delta>0$ such that

$$\int_{\mathcal{M}} f_m(t,x_m(t),\ldots,x_m^{(n-1)}(t))\mathrm{d}t<\varepsilon$$

for any measurable set $\mathcal{M}\subset\mathcal{J},\mu(\mathcal{M})<\delta$.

3. Existence results

Theorem 3.1 *Suppose that the assumptions (H_1), (H_2) and (H_3) are satisfied. Then there exists at least one positive solution for BVP (1.1), (1.2).*

Proof For each $m\in N$, there exists a solution x_m of BVP (1.2), (2.4) by Lemmas 2.4 and 2.5. Consider the solution sequence $\{x_m\}$. Lemma 2.5 shows that $\{x_m\}$ is bounded in $C^{n-1}(J)$. We will show $\{x_m\}$ is equi-continuous on J. For $t_1,t_2\in J, t_1<t_2$, we have

$$\left|\Phi_p(x_m^{(n-1)}(t_2))-\Phi_p(x_m^{(n-1)}(t_1))\right| \leq \int_{t_1}^{t_2} f_m(s,x_m(s),\ldots,x_m^{(n-1)}(s))\mathrm{d}s.$$

Lemma 2.6 implies that $\{f_m(t,x_m(t),\ldots,x_m^{(n-1)}(t))\}$ is uniformly absolutely continuous on J. This implies that the sequence $\left\{\Phi_p\left(x_m^{(n-1)}\right)\right\}_{m\in N_0}$ is equi-continuous on J, that is, $\{x_m^{(n-1)}\}$ is equi-continuous on J. Thus, Arzelà-Ascoli theorem guarantees the existence of a subsequence (we still denote it as $\{x_m^{(i)}\}$) converging in $C^{n-1}(J), i=0,\ldots,n-1$ to $x\in C^{n-1}(J)$. Clearly, x satisfies the boundary condition (1.2). Lemmas 2.1 and 2.2 mean that

$$\begin{aligned} &x_m^{(i)}(t)\geq \frac{c}{(n-3-i)!}\int_0^t (t-s)^{n-3-i}s(1-s)\mathrm{d}s,\quad 0\leq i\leq n-3,\\ &x_m^{(n-2)}(t)\geq ct(1-t),\ t\in J. \end{aligned} \tag{3.1}$$

Lemma 2.2 means that c is independent of m, so (3.1) gives that

$$x^{(i)}(t) \geq \frac{c}{(n-3-i)!}\int_0^t (t-s)^{n-3-i}s(1-s)\mathrm{d}s, \quad 0 \leq i \leq n-3, \quad x^{(n-2)}(t) \geq ct(1-t),\ t \in J.$$

Finally, let us show that $x \in AC^{n-1}(J)$ fulfills (1.1) a.e. on J.

From $f_m \in \mathrm{Car}(J \times R^n)$, and their construction, it follows that there exists $\mathcal{M} \in J$, $\mu(\mathcal{M}) = 0$ such that $f_m(t,\cdot,\ldots,\cdot)$ is continuous on R^n for each $t \in J \setminus \mathcal{M}$, which implies that

$$\lim_{k\to\infty} f_m(t, x_m(t), \ldots, x_m^{(n-1)}(t)) = f(t, x(t), \ldots, x^{(n-1)}(t))$$

for $t \in J\backslash\mathcal{M}\cup\{\prime\}$. By Lemma 2.6 $\{f_m(t, x_m(t), \ldots, x_m^{(n-1)}(t))\}$ is uniformly absolutely continuous on J. By the Vitali's Convergence theorem, $f \in L_1(J)$ and for $t \in J$,

$$\lim_{m\to\infty}\int_0^t f_m(s, x_m(s), \ldots, x_m^{(n-1)}(s))\mathrm{d}s = \int_0^t f(s, x(s), \ldots, x^{(n-1)}(s))\mathrm{d}s.$$

Define the operator $L : C^{n-1}([a,b]) \to C^{n-1}([a,b])$ by

$$(Lu_m^{(n-2)})(t) = u_m^{(n-2)}(a) + \int_a^t \Phi_q\left(\sigma_{u_m} - \int_0^s f_m(r, u_m(r), \ldots, u_m^{(n-1)}(r))\mathrm{d}r\right)\mathrm{d}s,$$

where σ_{u_m} satisfies

$$\int_a^b \Phi_q\left(\sigma_{u_m} - \int_0^s f_m(r, u_m(r), \ldots, u_m^{(n-1)}(r))\mathrm{d}r\right)\mathrm{d}s = u_m^{(n-2)}(b) - u_m^{(n-2)}(a).$$

Let $u_m \to u$ uniformly on $[a,b]$. If we show $\lim_{m\to\infty}\sigma_{u_m} = \sigma_u$, then this together with the continuity of Φ_q implies that $L : C[a,b] \to C[a,b]$ is continuous. First notice

$$\begin{aligned}
&\int_a^b \Phi_q\left(\sigma_{u_m} - \int_0^s f_m(r, u_m(r), \ldots, u_m^{(n-1)}(r))\mathrm{d}r\right)\mathrm{d}s - \\
&\quad\int_a^b \Phi_q\left(\sigma_u - \int_0^s f(r, u(r), \ldots, u^{(n-1)}(r))\mathrm{d}r\right)\mathrm{d}s \\
&= u_m^{(n-2)}(b) - u_m^{(n-2)}(a) - u^{(n-2)}(b) + u^{(n-2)}(a).
\end{aligned}$$

The mean value theorem for integrals then implies that there exists $\eta_m \in [a,b]$ with

$$\begin{aligned}
&(b-a)\left[\Phi_q\left(\sigma_{u_m} - \int_0^{\eta_m} f_m(r, u_m(r), \ldots, u_m^{(n-1)}(r))\mathrm{d}r\right)\mathrm{d}s - \right. \\
&\quad\left.\Phi_q\left(\sigma_u - \int_0^{\eta_m} f(r, u(r), \ldots, u^{(n-1)}(r))\mathrm{d}r\right)\mathrm{d}s\right] \\
&= u_m^{(n-2)}(b) - u_m^{(n-2)}(a) - u^{(n-2)}(b) + u^{(n-2)}(a).
\end{aligned}$$

Since $u_m^{(i)} \to u^{(i)}$ uniformly on $[a,b]$, $i = 0, \ldots, n-1$, we have $\lim_{m\to\infty}\sigma_{u_m} = \sigma_u$.

Now $u_m^{(i)} \to u^{(i)}$ uniformly on $[a,b]$, $i = 0, \ldots, n-2$, and $Lu_m = u_m$, yields $Lu = u$, i.e.

$$(\Phi_p(u^{(n-1)}(t)))' + f(t, u(t), \ldots, u^{(n-1)}(t)) = 0, \quad a \leq t \leq b.$$

We can do this argument for each $t \in (0,1)$ and so

$$(\Phi_p(u^{(n-1)}(t)))' + f(t, u(t), \ldots, u^{(n-1)}(t)) = 0,$$

for a.e. $t \in (0,1)$. Therefore, u is a solution of BVP (1.1), (1.2). □

Remark 3.1 If we replace boundary condition (1.2) by

$$x^{(i)}(0) = 0, \quad 0 \le i \le n-3, \quad x^{(n-1)}(0) = 0, \quad x^{(n-2)}(1) + B_1(x^{(n-1)}(1)) = 0, \tag{3.2}$$

or

$$x^{(i)}(0) = 0, \quad 0 \le i \le n-3, \quad x^{(n-2)}(0) - B_0(x^{(n-1)}(0)) = 0, \quad x^{(n-1)}(1) = 0, \tag{3.3}$$

under the assumptions (H_1)–(H_3), there exists at least one positive solution to BVP (1.1), (3.2) and BVP (1.1), (3.3), respectively.

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具有 p-Laplace 算子的一类高阶奇异边值问题解的存在性

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摘要: 本文研究下面问题的正解

$$\begin{cases} (\Phi_p(x^{(n-1)}(t)))' + f(t, \ldots, x^{(n-1)}) = 0, \quad 0 < t < 1, \\ x^{(i)}(0) = 0, \quad 0 \le i \le n-3, \\ x^{(n-2)}(0) - B_0(x^{(n-1)}(0)) = 0, \quad x^{(n-2)}(1) + B_1(x^{(n-1)}(1)) = 0, \end{cases}$$

其中 $\Phi_p(s) = |s|^{p-2}s, p > 1$. f 在点 $x^{(i)} = 0, i = 0, \ldots, n-2$ 可能是奇异的. 证明建立在 Leray-Schauder 拓扑度和 Vitali 收敛定理的基础上.

关键词: 高阶奇异微分方程; 正解; Vitali 收敛定理.