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Global Attractors of Strong Solutions for the Beam Equation of Memory Type

MA Qiao-zhen
(College of Mathematics and Information Science, Northwest Normal University, Gansu 730070, China)
(E-mail: maqzh@nwnu.edu.cn)

Abstract: The model with linear memory arise in the case of a generalized Kirchhoff viscoelastic bar, where a bending-moment relation with memory was considered. In this paper, the exponential decay is proved if the memory kernal satisfies the condition of the exponential decay. Furthermore, we show that the existence of strong global attractor by verifying the condition (C) introduced in [3].

Key words: beam equation; linear memory; global attractors.
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1. Introduction

A general Kirchhoff viscoelastic beam model is mentioned in [1]

$$u_{tt}-\gamma_0\triangle u_{tt}+\gamma_1\triangle^2u(t)+\psi*\triangle^2u(t)-\gamma_2\triangle u+\varphi*\triangle u(t)=0, \tag{1}$$

where $\gamma_0,\ \gamma_1$ and γ_2 are non-negative constants, ψ and φ are memory kernals, and u describes the transversal motion of the beam. Furthermore, they discussed the polynomial decay of the energy and the non-exponential stability of the solutions for $\gamma_0=\gamma_2=0,\ \psi\equiv 0$. In this paper, we investigate the nonlinear equation with linear damping

$$u_{tt}+\delta u_t+\phi(0)\triangle^2 u+\int_0^\infty \phi'(s)\triangle^2 u(t-s)\mathrm{d}s+g(u)=h,\ \text{in }\Omega\times\mathbb{R}^+ \tag{2}$$

with the following boundary conditions: The first case is simply supported at both ends

$$u(x,t)=\triangle u(x,t)=0,\quad x\in\Gamma,\quad t\in\mathbb{R}. \tag{3}$$

Another case is fixed at both ends

$$u(x,t)=\nabla u(x,t)=0,\quad x\in\Gamma,\quad t\in\mathbb{R}. \tag{4}$$

The initial conditions are given by

$$u(x,t)=u_0(x,t),\quad x\in\Omega,\quad t\le 0, \tag{5}$$

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where $\Omega \subset \mathbb{R}^2$ is a bounded domain with smooth boundary Γ. $\phi(0),\ \phi(\infty) > 0$ and $\phi'(s) \leq 0$ for every $s \in \mathbb{R}^+$. Similar problem was also studied by C.Giorgi, J.E.M.Rivera and V.Pata in [2]. But in any case, all the results on the existence of global attractors for the dynamical system were obtained only in the weak Sobolev spaces. We will prove that in the stronger Hilbert spaces by verifying the condition (C) which arised by [3]. For other methods proving the attractors, see [4–6].

2. Preliminaries

Remark 1 In the following we only study equation (2) with (3) and (5).

Analogous to [2], we define

$$\eta^t(x,s) = u(x,t) - u(x,t-s). \tag{6}$$

We set for simplicity $\mu(s) = -\phi'(s)$ and $\phi(\infty) = 1$. In view of (6), adding and subtracting the term $\triangle^2 u$, equation (2) is transformed into the system

$$\begin{cases} u_{tt} + \delta u_t + \triangle^2 u + \displaystyle\int_0^\infty \mu(s) \triangle^2 \eta^t(s)\mathrm{d}s + g(u) = h, \\ \eta_t + \eta_s = u_t, \end{cases} \tag{7}$$

where the second equation is obtained by differentiating (6). The corresponding intial-boundary value conditions are then given by

$$\begin{cases} u(x,t) = \triangle u(x,t) = 0, \quad x \in \Gamma, \quad t \geq 0, \\ \eta^t(x,s) = \triangle \eta^t(x,s) = 0, \quad x \in \Gamma, \quad t \geq 0, \quad s \in \mathbb{R}^+, \\ u(x,0) = u_1(x), \quad u_t(x,0) = u_2(x), \quad x \in \Omega, \\ \eta^0(x,s) = \eta_0(x,s), \quad (x,s) \in \Omega \times \mathbb{R}^+, \end{cases} \tag{8}$$

where

$$\begin{cases} u_1(x) = u_0(x,0), \quad u_2(x) = \partial_t u_0(x,t)|_{t=0}, \\ \eta_0(x,s) = u_0(x,0) - u_0(x,-s). \end{cases}$$

Assume that the nonlinear function $g \in C^2(\mathbb{R},\mathbb{R})$ satisfying the following conditions:

(g1) $\liminf\limits_{|s|\to\infty} \frac{G(s)}{s^2} \geq 0, \qquad G(s) = \int_0^s g(\tau)\mathrm{d}\tau;$

(g2) $\limsup\limits_{|s|\to\infty} \frac{|g'(s)|}{|s|^\gamma} = 0, \quad \forall\ 0 \leq \gamma < \infty;$

(g3) There exists $C_1 > 0$, such that $\liminf\limits_{|s|\to\infty} \frac{sg(s) - C_1 G(s)}{s^2} \geq 0.$

The memory kernel μ is required to satisfy the following assumptions:

(h1) $\mu \in C^1(\mathbb{R}^+) \cap L^1(\mathbb{R}^+),\ \ \mu(s) \geq 0,\ \ \mu'(s) \leq 0,\ \ \forall\ s \in \mathbb{R}^+;$

(h2) $\int_0^\infty \mu(s)\mathrm{d}s = M > 0;$

(h3) $\mu'(s) + \alpha\mu(s) \leq 0,\ \ \forall\ s \in \mathbb{R}^+,\ \ \alpha > 0.$

We write $H = L^2(\Omega)$, $V = H_0^2(\Omega)$. The scalar product and the norm on H and V are denoted by $(\cdot,\cdot)$, $|\cdot|$ and $((\cdot,\cdot))$, $\|\cdot\|$ respectively, where

$$(u,v) = \int_\Omega u(x)v(x)\mathrm{d}x, \qquad ((u,v)) = \int_\Omega \triangle u(x) \triangle v(x)\mathrm{d}x.$$

Define $D(A)=\{v\in V,\ Av\in H\}$, where $A=\triangle^2$. For the operator A, we assume that

$$A:\quad \begin{array}{c} V\longrightarrow V^*,\\ D(A)\rightarrow H\end{array}$$

are isomorphism, and there exists $\alpha>0$ such that $(Au,\ u)\geq \alpha\|u\|^2$, $\forall\ u\in V$. We also define the power A^s of A for $s\in\mathbb{R}$ which operates on the spaces $D(A^s)$, and we write $V_{2s}=D(A^s),\ s\in\mathbb{R}$. This is a Hilbert space for the scalar product and the norm as follows

$$(u,v)_{2s}=(A^su,A^sv),\quad \|u\|_{2s}=((u,u)_{2s})^{\frac{1}{2}},\ \ \forall\ u,\ \ v\in D(A^s),$$

and A^r is an isomorphism from $D(A^s)$ onto $D(A^{s-r}),\ \ \forall\ s,\ \ r\in\mathbb{R}$. It is clear that $D(A^0)=H$, $D(A^{\frac{1}{2}})=V,\ D(A^{-\frac{1}{2}})=V^*$ and $D(A)\subset V\subset H=H^*\subset V^*$, where $H^*,\ V^*$ are the dual of $H,\ V$ respectively, and each space is dense in the following one and the injections are continuous. Using the Poincáre inequality we have

$$\|v\|\geq\lambda_1|v|,\ \ \forall\ v\in V, \tag{9}$$

where λ_1 denotes the first eigenvalue of $A^{\frac{1}{2}}$.

In view of (h1), let $L^2_\mu(\mathbb{R}^+,H^2_0)$ be the Hilbert space of H^2_0-valued functions on $\mathbb{R}^+$, endowed with the following inner product and the norm

$$(\varphi,\psi)_{\mu,V}=\int_0^\infty\mu(s)(\triangle\varphi(s),\triangle\psi(s))\mathrm{d}s$$

and

$$|\varphi|^2_{\mu,V}=(\varphi,\varphi)_{\mu,V}=\int_0^\infty\mu(s)\|\varphi\|^2\mathrm{d}s.$$

Finally, we introduce the following Hilbert spaces:

$$\mathcal{H}_0=V\times H\times L^2_\mu(\mathbb{R}^+,V),\quad \mathcal{H}_1=D(A)\times V\times L^2_\mu(\mathbb{R}^+,D(A)).$$

According to the classical Faedo-Galerkin method it is easy to obtain the existence and uniqueness of solutions and the continuous dependence to the initial value, so we omit it and only give the following theorem:

Theorem 1[2,4] *Let (g1)–(g3) and (h1) hold. Then given any time interval* $I=[0,T]$*, problem (7)–(8) has a unique solution* (u,u_t,η) *in* I *with initial data* $(u_1,u_2,\eta_0)\in\mathcal{H}_0$*, and the mapping* $\{u_1,u_2,\eta_0\}\rightarrow\{u(t),u_t(t),\eta^t(s)\}$ *is continuous in* $\mathcal{H}_0$*. If, furthermore,* $(u_1,u_2,\eta_0)\in\mathcal{H}_1$*, then we have a unique solution* $(u,u_t,\eta)\in C(I,\mathcal{H}_1)$*, and the mapping above is continuous in* $\mathcal{H}_1$*, too.*

Thus, it admits to define a C^0 semigroup

$$S(t):\ \ \{u_1,u_2,\eta_0\}\rightarrow\{u(t),u_t(t),\eta^t(s)\},\ \ t\in\mathbb{R}^+,$$

and they map $\mathcal{H}_0,\ \mathcal{H}_1$ into themselves, respectively.

In addition, the following abstract results will be used in our consideration:

Definition 1[3] *A C^0 semigroup $\{S(t)\}_{t\geq 0}$ in a Banach space X is said to satisfy condition (C) if for any $\varepsilon>0$ and for any bounded set B of X, there exists $t(B)>0$ and a finite dimensional subspace X_1 of X, such that $\{\|PS(t)x\|_X,\ x\in B,\ t\geq t(B)\}$ is bounded and*

$$\|(I-P)S(t)x\|_X <_X \varepsilon,\quad \text{for}\quad t\geq t(B),\quad x\in B,$$

where $P:\ X\to X_1$ is a bounded projector.

Theorem 2[3] *Let $\{S(t)\}_{t\geq 0}$ be a C^0 semigroup in a Hilbert space M. Then $\{S(t)\}_{t\geq 0}$ has a global attractor if and only if*

(1) $\{S(t)\}_{t\geq 0}$ satisfies condition (C);

(2) there exists a bounded absorbing subset B of M.

3. Bounded absorbing set in $\mathcal{H}_1$

For simplicity, we denote $\phi(u)=\int_\Omega G(u(x))\mathrm{d}x$.

First, taking the inner product of the first equation of (7) with $v=u_t+\sigma u$ in H, after computation we conclude

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\|u\|^2+|v|^2)+\sigma\|u\|^2+(\delta-\sigma)|v|^2-\sigma(\delta-\sigma)(u,v)+ \\ (\eta,v)_{\mu,V}+(g(u),v)=(h,v). \tag{10}$$

Combining with the second equation of (7) we have

$$(\eta,v)_{\mu,V}=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}|\eta|^2_{\mu,V}+(\eta,\eta_s)_{\mu,V}+\sigma(\eta,u)_{\mu,V}. \tag{11}$$

In the following we often exploiting the Hölder inequality and Young inequality. According to (h2)–(h3), we have

$$(\eta,\eta_s)_{\mu,V}=\frac{1}{2}\int_0^\infty\mu(s)\frac{\mathrm{d}}{\mathrm{d}s}|\triangle\eta^t(s)|^2\mathrm{d}s=-\frac{1}{2}\int_0^\infty\mu'(s)|\triangle\eta^t(s)|^2\mathrm{d}s\geq\frac{\alpha}{2}|\eta|^2_{\mu,V}, \tag{12}$$

and

$$\begin{aligned}\sigma(\eta,u)_{\mu,V}&=\sigma\int_0^\infty\mu(s)(\triangle\eta(s),\triangle u)\mathrm{d}s\\ &\geq-\sigma\Big(\int_0^\infty\mu(s)|\triangle\eta^t(s)|^2\mathrm{d}s\Big)^{\frac{1}{2}}\Big(\int_0^\infty\mu(s)|\triangle u|^2\mathrm{d}s\Big)^{\frac{1}{2}}\\ &\geq-\frac{\alpha}{4}\int_0^\infty\mu(s)|\triangle\eta^t(s)|^2\mathrm{d}s-\frac{\sigma^2}{\alpha}\int_0^\infty\mu(s)|\triangle u|^2\mathrm{d}s\\ &\geq-\frac{\alpha}{4}|\eta|^2_{\mu,V}-\frac{M\sigma^2}{\alpha}\|u\|^2.\end{aligned} \tag{13}$$

Integrating with (12)–(13), from (11) entails

$$(\eta,v)_{\mu,V}\geq\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}|\eta|^2_{\mu,V}+\frac{\alpha}{4}|\eta|^2_{\mu,V}-\frac{M\sigma^2}{\alpha}\|u\|^2. \tag{14}$$

In view of (g1), (g3), there exist constants $K_1,\ K_2 > 0$ such that

$$\phi(u(t)) + \frac{1}{8}\|u\|^2 \geq -K_1, \quad \forall u \in V, \tag{15}$$

$$(u, g(u)) - C_1\phi(u) + \frac{1}{4}\|u\|^2 \geq -K_2, \quad \forall u \in V. \tag{16}$$

Hence,

$$(g(u), v) = \frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega G(u)\mathrm{d}x + \sigma\int_\Omega g(u)u\mathrm{d}x \geq \frac{\mathrm{d}}{\mathrm{d}t}\phi(u) + \sigma(C_1\phi(u) - \frac{1}{4}\|u\|^2 - K_2). \tag{17}$$

Collecting with (14), (17), we obtain from (10) that

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\|u\|^2+|v|^2 + 2\phi(u) + |\eta|^2_{\mu,V}) + (\delta-\sigma)|v|^2 - \sigma(\delta-\sigma)(u,v)+$$
$$\sigma(\frac{3}{4} - \frac{M\sigma}{\alpha})\|u\|^2 + \frac{\alpha}{4}|\eta|^2_{\mu,V} + \sigma C_1\phi(u) - \sigma K_2 \leq \frac{|h|^2}{\delta} + \frac{\delta}{4}|v|^2.$$

Take σ small enough, such that

$$\frac{3\delta}{4} - \sigma > \frac{\delta}{2}, \quad \frac{3}{4} - \frac{M\sigma}{\alpha} > \frac{1}{2}.$$

Thus in line with (9), we have

$$(\delta-\sigma)|v|^2 - \sigma(\delta-\sigma)(u,v) + \sigma(\frac{3}{4} - \frac{M\sigma}{\alpha})\|u\|^2 \geq \frac{\delta}{2}|v|^2 + \frac{\sigma}{2}\|u\|^2. \tag{18}$$

Let $\sigma_0 = \min\{\sigma, \frac{\delta}{2}, \frac{\alpha}{2}, \sigma C_1\}$. We conclude

$$\frac{\mathrm{d}}{\mathrm{d}t}(\|u\|^2 + |v|^2 + 2\phi(u) + |\eta|^2_{\mu,V}) + \sigma_0(\|u\|^2 + |v|^2 + 2\phi(u) + |\eta|^2_{\mu,V}) \leq \frac{2|h|^2}{\delta} + 2\sigma K_2.$$

Due to (15), write

$$W(t) = \|u\|^2 + |v|^2 + 2\phi(u) + |\eta|^2_{\mu,V} + 2K_1 > 0.$$

Then we have

$$\frac{\mathrm{d}}{\mathrm{d}t}W(t) + \sigma_0 W(t) \leq C, \quad C = \frac{2|h|^2}{\delta} + 2\sigma K_2 + 2\sigma K_1.$$

By the Gronwall Lemma, we end up with

$$W(t) \leq W(0)\exp(-\sigma_0 t) + \frac{C}{\sigma_0}(1 - \exp(-\sigma_0 t)),\ \forall\, t \geq 0.$$

Thus, we have the following theorem:

Theorem 3 *Assume that (g1)–(g3) and (h1)–(h3) hold. Then the ball of $\mathcal{H}_0$, $B_0 = B_{\mathcal{H}_0}(\mathbf{0}, \mu_0)$, centered at $\mathbf{0}$ with radius $\mu_0 = \sqrt{\frac{C}{\sigma_0}}$, is a bounded absorbing set in $\mathcal{H}_0$ for the semigroup $\{S(t)\}_{t\geq 0}$.*

Secondly, taking the scalar product in H of the first equation of system (7) with $Av = Au_t + \sigma Au$, and combining with the second equation, we find

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(|Au|^2+\|v\|^2 + \|\eta\|^2_{\mu,D(A)}) + \sigma|Au|^2 + (\delta-\sigma)\|v\|^2 - \sigma(\delta-\sigma)(Au,v)+$$
$$(\eta,\eta_s)_{\mu,D(A)} + \sigma(\eta,u)_{\mu,D(A)} + (g(u), Av) = (h, Av). \tag{19}$$

Analogous to (12)–(13), we have

$$(\eta,\eta_s)_{\mu,D(A)}+\sigma(\eta,u)_{\mu,D(A)}\ge\frac{\alpha}{4}|\eta|^2_{\mu,D(A)}-\frac{M\sigma^2}{\alpha}|Au|^2. \tag{20}$$

According to (g2), for any $\varepsilon>0$, there exists a constant $C_\varepsilon>0$ such that

$$|g'(s)|\le\varepsilon|s|^\gamma+C_\varepsilon,\quad \forall\, 0\le\gamma<\infty.$$

By Theorem 3 and Sobolev embedding theorem, $g(u)$ and $g'(u)$ are uniformly bounded in L^∞, that is, there exists a constant $K_3>0$ such that

$$|g(u)|_{L^\infty}\le K_3,\quad |g'(u)|_{L^\infty}\le K_3. \tag{21}$$

Thanks to (21), we get

$$\begin{aligned}(g(u),Av)&=\frac{\mathrm{d}}{\mathrm{d}t}(g(u),Au)-(g'(u)u_t,Au)+\sigma(g(u),Au)\\&\ge\frac{\mathrm{d}}{\mathrm{d}t}(g(u),Au)+\sigma(g(u),Au)-\int_\Omega|g'(u)|\cdot|u_t|\cdot|Au|\mathrm{d}x\\&\ge\frac{\mathrm{d}}{\mathrm{d}t}(g(u),Au)+\sigma(g(u),Au)-K_3|u_t|\cdot|Au|\\&\ge\frac{\mathrm{d}}{\mathrm{d}t}(g(u),Au)+\sigma(g(u),Au)-K_3\mu_0|Au|\\&\ge\frac{\mathrm{d}}{\mathrm{d}t}(g(u),Au)+\sigma(g(u),Au)-\frac{\sigma^2}{8}|Au|^2-\frac{2K_3^2\mu_0^2}{\sigma^2}.\end{aligned} \tag{22}$$

In addition

$$(h,Av)=(h,Au_t)+\sigma(h,Au)\le\frac{\mathrm{d}}{\mathrm{d}t}(h,Au)+\frac{\sigma^2}{4}|Au|^2+|h|^2. \tag{23}$$

Collecting with (20), (22) and (23), from (19) we have

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\Big(|Au|^2+\|v\|^2-2(h,Au)+|\eta|^2_{\mu,D(A)}+2(g(u),Au)\Big)+\sigma(1-\frac{M\sigma}{\alpha}-\frac{3\sigma}{8})|Au|^2+\\&\quad(\delta-\sigma)\|v\|^2-\sigma(\delta-\sigma)(Au,v)+\sigma(g(u),Au)+\frac{\alpha}{4}|\eta|^2_{\mu,D(A)}\le\frac{2K_3^2\mu_0^2}{\sigma}+|h|^2,\ \forall\, t\ge t_0(B).\end{aligned}$$

Choose σ small enough such that

$$\frac{3\delta}{4}-\sigma>\frac{\delta}{4},\quad 1-\frac{\sigma\delta}{\lambda_1^2}-\frac{M\sigma}{\alpha}-\frac{3\sigma}{8}>\frac{1}{2}.$$

Therefore, by the above inequality

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\Big(|Au+g(u)-h|^2+\|v\|^2+|\eta|^2_{\mu,D(A)})\Big)-2\int_\Omega g'(u)u_t\mathrm{d}x+2(g'(u)u_t,\ h)+\\&\qquad\sigma|Au|^2+\frac{\delta}{2}\|v\|^2+\frac{\alpha}{2}|\eta|^2_{\mu,D(A)}+2\sigma(g(u),Au)\\&\quad\le 2(|h|^2+\frac{2K_3^2\mu_0^2}{\sigma^2}),\quad t\ge t_0(B).\end{aligned}$$

Combining with (21), Theorem 3 and Sobolev embedding theorem we have

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\Big(|Au+g(u)-h|^2+\|v\|^2+|\eta|^2_{\mu,D(A)}\Big)+\sigma|Au+g(u)-h|^2+\frac{\delta}{2}\|v\|^2+\frac{\alpha}{2}|\eta|^2_{\mu,D(A)}\\
&\quad\le 2\int_\Omega|g'(u)|\cdot|u_t|\mathrm{d}x+2\int_\Omega|g'(u)|\cdot|u_t|\cdot|h|\mathrm{d}x+\sigma|g(u)|^2+2(1+\sigma)|h|^2+\frac{4K_3^2\mu_0^2}{\sigma^2}\\
&\quad\le C,\quad t\ge t_0(B),
\end{aligned}$$

where C is a constant relying on $\mu_0,\ |h|,\ K_3,\ \sigma$.

Let $\sigma_0=\min\{\sigma,\frac{\delta}{2},\frac{\alpha}{2}\}$. The following inequality is obtained immediately

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\Big(|Au+g(u)-h|^2+\|v\|^2+|\eta|^2_{\mu,D(A)}\Big)+\sigma_0\Big(|Au+g(u)-h|^2+\|v\|^2+|\eta|^2_{\mu,D(A)}\Big)\\
&\quad\le C,\ t\ge t_0(B).
\end{aligned}$$

By the Gronwall lemma, we have

$$\begin{aligned}
&|Au(t)+g(u(t))-h|^2+\|v(t)\|^2+|\eta^t(s)|^2_{\mu,D(A)}\\
&\quad\le\Big(|Au(t_0)+g(u(t_0))-h|^2+\|v(t_0)\|^2+|\eta^{t_0}(s)|^2_{\mu,D(A)}\Big)\exp(-\sigma_0(t-t_0))+\frac{C}{\sigma_0},\ \forall\ t\ge t_0(B).
\end{aligned}$$

Now, in the light of (21), if $B\subset B_{\mathcal{H}_1}(p_0,\ \rho)$ ($B_{\mathcal{H}_1}$ is the ball , centered at p_0 with radius ρ in $\mathcal{H}_1$), then B is also bounded in $\mathcal{H}_0$, and integrating with Theorem 3, there exists a constant $R_1>0$, such that

$$\sup_{(u(t_0),u_t(t_0),\eta^{t_0}(s))\in B}\{|Au(t_0)+g(u(t_0))-h|^2+\|v(t_0)\|^2+|\eta^{t_0}(s)|^2_{\mu,D(A)}\le R_1^2.$$

Take t_1 to satisfy $t_1-t_0\ge\frac{1}{\sigma_0}\log R_1^2$, then we have

$$|Au(t)+g(u(t))-h|^2+\|v(t)\|^2+|\eta^t(s)|^2_{\mu,D(A)}\le\rho_1^2,\quad t\ge t_1,$$

where $\rho_1^2=1+\frac{C}{\sigma_0}$.

According to (21), we conclude that

$$\begin{aligned}
&|Au(t)|^2+\|v(t)\|^2+|\eta^t|^2_{\mu,D(A)}\\
&\quad\le|g(u)|^2+2(h,Au)+|h|^2+2|g(u)|\cdot|Au|+2(g(u),h)+\rho_1^2\\
&\quad\le K_3^2+|h|^2+2K_3|Au|+2|h||Au|+2K_3|h|+\rho_1^2\\
&\quad\le\sigma|Au|^2+K_3^2+|h|^2+2K_3|h|+\rho_1^2+\frac{2K_3^2}{\sigma}+\frac{2|h|^2}{\sigma},
\end{aligned}$$

namely,

$$(1-\sigma)|Au(t)|^2+\|v(t)\|^2+|\eta^t|^2_{\mu,D(A)}\le C,$$

where $C=(3+\frac{2}{\sigma})K_3^2+(\frac{3}{2}+\frac{2}{\sigma})|h|^2+\rho_1^2$.

Thus, we have the following theorem:

Theorem 4 *Assume that (g1)–(g3) and (h1)–(h3) hold. Then the ball of* $\mathcal{H}_1$, $B_1 = B_{\mathcal{H}_1}(\mathbf{0},\ \mu_1)$, *centered at* $(0,0,0)$ *with radius* μ_1*, is a bounded absorbing set in* $\mathcal{H}_1$.

4. Global attractor $\mathcal{A}$ in $\mathcal{H}_1$

In order to obtain our main results, we need the following compactness results.

Lemma 1 Assume that $g \in C^3(\mathbb{R},\mathbb{R})$ with (g2), $g(0)=0$, and $g:\ D(A) \to H_0^2(\Omega)$ be defined by

$$((g(u),v)) = \int_\Omega g'(u)\triangle u \triangle v \mathrm{d}x + \int_\Omega g''(u)(\nabla u)^2 \triangle v \mathrm{d}x,$$

$\forall\, u \in D(A),\ \ v \in H_0^2(\Omega)$. Then g is continuous compact.

Lemma 2 *Let* $f(u) = g'(u)u_t$*, and* $g \in C^2(\mathbb{R},\mathbb{R})$ *satisfy (g2),* $g(0)=0$*. Then* $f:\ D(A)\times V \longrightarrow H$ *is continuous compact.*

The above two lemmas are easy to verify, so we omit it. The reader can also see [5, 6].

5. Our main results

Theorem 5 *Suppose that* $h \in V$, Ω *is a bounded smooth domain in* $\mathbb{R}^2$*, and conditions (g1)–(g3) and (h1)–(h3) are hold. Then the solution semigroup* $\{S(t)\}_{t\geq 0}$ *associated with system (7)–(8) has a global attractor* $\mathcal{A}_1$ *in* $\mathcal{H}_1$*, and it attracts all bounded subsets of* $\mathcal{H}_1$ *in the norm of* $\mathcal{H}_1$.

Proof Applying theorem 2, we only have to prove that the condition (C) is hold in $\mathcal{H}_1$.

Let $\{\tilde{w}_i\}$ be an orthonormal basis of $D(A)$ which consists of eigenvectors of A. It is also an orthonormal basis of $V,\ H$, respectively. The corresponding eigenvalues are denoted by

$$0 < \tilde{\lambda}_1 < \tilde{\lambda}_2 \leq \tilde{\lambda}_3 \leq \cdots,\ \ \tilde{\lambda}_i \to \infty,\ \ \text{as}\ \ i \to \infty$$

with $A\tilde{\omega}_i = \tilde{\lambda}_i\tilde{\omega}_i$, $\forall i \in \mathbb{N}$. We write $V_m = \operatorname{span}\{\tilde{\omega}_1,\ldots,\tilde{\omega}_m\}$. For any $(u,u_t,\eta) \in \mathcal{H}_1$, we decompose that

$$(u,u_t,\eta) = (u_1,u_{1t},\eta_1) + (u_2,u_{2t},\eta_2),$$

where $(u_1,u_{1t},\eta_1) = (P_m u, P_m u_t, P_m\eta)$, and $P_m : V \to V_m$ is the orthogonal projector.

Since $h \in V$, $g: D(A) \to V$ is compact by Lemma 1, for any $\varepsilon > 0$, there exists some m, such that

$$\|(I-P_m)h\|_V \leq \frac{\varepsilon}{4},$$

$$\|(I-P_m)g(u)\|_V \leq \frac{\varepsilon}{4},\quad \forall u \in B_{D(A)}(\mathbf{0},\mu_1). \tag{24}$$

Taking the scalar product of the first equation of (7) in H with $Av_2 = Au_{2t}+\sigma Au_2$ and combining with the second equation, we find

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(|Au_2|^2+\|v_2\|^2 + |\eta_2|^2_{\mu,D(A)}) + \sigma|Au_2|^2 + (\delta-\sigma)\|v_2\|^2\\ &\qquad \sigma(\delta-\sigma)(Au_2,v_2) + \frac{\alpha}{4}|\eta_2|^2_{\mu,D(A)} + ((g(u),v_2)) \leq (h,Av_2).\end{aligned}$$

Note

$$\sigma|Au_2|^2 + (\delta-\sigma)\|v_2\|^2 - \sigma(\delta-\sigma)(Au_2, v_2) \geq \sigma(1-\frac{\sigma\delta}{\lambda_1^2})|Au_2|^2 + (\frac{3\delta}{4}-\sigma)\|v_2\|^2 \tag{25}$$

and

$$(h, Av_2) \leq \frac{\mathrm{d}}{\mathrm{d}t}(h_2, Au_2) + \frac{\sigma^2}{4}|Au_2|^2 + \frac{\varepsilon^2}{16} \tag{26}$$

where $h_2 = (I-P_m)h$. Integrating (25)–(26), when σ small enough, we obtain

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}(|Au_2-h_2|^2 + \|v_2\|^2 + |\eta_2|^2_{\mu,D(A)}) + \sigma(|Au_2-h_2|^2 + \|v_2\|^2 + |\eta_2|^2_{\mu,D(A)})\\ &\quad \leq C\varepsilon^2, \quad C = \frac{1}{16}(1+\sigma+\frac{1}{\delta}).\end{aligned}$$

By the Gronwall lemma

$$\begin{aligned}&|Au_2(t)-h_2|^2 + \|v_2(t)\|^2 + |\eta_2^t(s)|^2_{\mu,D(A)}\\ &\quad \leq (|Au_2(t_1)-h_2|^2 + \|v_2(t_1)\|^2 + |\eta_2^{t_1}(s)|^2_{\mu,D(A)})\exp(-\sigma(t-t_1)) + \frac{C\varepsilon^2}{\sigma}\\ &\quad \leq \mu_1\exp(-\sigma(t-t_1)) + \frac{C\varepsilon^2}{\sigma}, \quad \forall t \geq t_1,\end{aligned}$$

where μ_1 is given by Theorem 4. Take t_2 large enough such that $t_2 - t_1 \geq \frac{1}{\sigma}\log\frac{\mu_1}{\varepsilon^2}$, so we conclude that

$$|Au_2(t)-h_2|^2 + \|v_2(t)\|^2 + |\eta_2^t(s)|^2_{\mu,D(A)} \leq (1+\frac{C}{\sigma})\varepsilon^2, \quad \text{for} \quad t \geq t_2.$$

Thus $\{S(t)\}_{t\geq 0}$ satisfies the condition (C).

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记忆型梁方程强解的全局吸引子

马 巧 珍

(西北师范大学数学与信息科学学院，甘肃 兰州 730070)

摘要: 记忆型梁方程出现于一般的 Kirchhoff 粘弹性梁模型中．本文在记忆核满足指数衰退的条件下证明了系统的能量也是指数衰退的．进一步，通过对条件 (C) 的验证获得了系统强解的全局吸引子.

关键词: 梁方程；线性记忆；全局吸引子.