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On the Hochschild Cohomology and Homology of Endomorphism Algebras of Exceptional Sequences over Hereditary Algebras

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Abstract In this paper, let A be a finite dimensional associative algebra over an algebraically closed field k, mod$A$ be the category of finite dimensional left A-module and $X_1, X_2, \dots, X_n$ in mod$A$ be a complete exceptional sequence, and let E be the endomorphism algebra of $X_1, X_2, \dots, X_n$. We study the global dimension of E, and calculate the Hochschild cohomology and homology groups of E.

Keywords Hochschild cohomology groups; exceptional sequences, endomorphism algebras; global dimensions.

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1. Introduction

In studying vector bundles in $\mathbb{P}^n$ Moscow school discovered exceptional sequences and the action of braid groups[1−4]. The natural setting is in the context of triangulated categories. At the 1992 Canadian Mathematical Society Annual Seminar, A. N. Rudakov lectured on exceptional sequences of vector bundles for $\mathbb{P}^n$. In 1993, W. Crawley-Boevey[5] introduced the concept of exceptional sequences into the categories of the representations of quivers. He proved that the action of the braid group with $(n-1)$ generators on the set of complete exceptional sequences in categories of representations of quivers with n simple representations is transitive. C. M. Ringel[6] generalized this result to the module categories of general hereditary Artin algebras, and gave the relation between complete exceptional sequences and tilting sequences. Since then, some problems on exceptional sequences[7−10] have been studied.

Let A be a finite dimensional associative algebra with identity over a field. The Hochschild cohomology groups $H^i(A, X)$ of A with coefficients in a finitely generated A-A-bimodule X were defined by G.Hochschild in 1945[1]. We write $H^i(A)$ instead of $H^i(A, A)$ in case of $X = A$, and we call $H^i(A)$ the i^{th}-Hochschild cohomology group of A. The lower degree groups ($i \leq 2$) have

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a very concrete interpretation of classical algebraic structures such as derivations and extensions. It was observed by Gerstenhaber[13] that there are connections to algebraic geometry. In fact, $H^2(A,A)$ controls the deformation theory of A, and it was shown that the algebra satisfying $H^2(A)=0$ is rigid. In Ref. [14], P. Gabriel gave the relation between $H^2(A)$ and the structure of A. But, in general, it is not easy to compute the Hochschild cohomology groups of a given algebra. Computations for semi-commutative Schurian algebras and algebras arising from narrow quivers have been provided in Refs. [12] and [15] respectively. The cases of monomial and truncated algebras have been studied in Refs. [16,17]. However, the actual calculations of Hochschild cohomology groups have been fairly limited.

Hochschild homology groups of algebras were not introduced by G. Hochschild himself. It seems that it appeared firstly in the book 'Homological algebra' written by H. Cartan & S.Eilenberg[18]. It plays an important role in studying cyclic homology groups. Hochschild Homology groups are not the duality of Hochschild cohomology group in the sense of categories. Perhaps it is easy to calculate the Hochschild homology groups of some algebras, but maybe it is difficult to calculate their Hochschild cohomology groups. The converse probably happens, too. Thus, we might say, Hochschild cohomology and homology groups give expressions of properties of different aspects for an algebra. Readers who hope to find the results on Hochschild homology groups may refer to the references $[19-24]$.

The aim of this paper is to investigate the Hochschild (Co)homology groups and the global dimension of the endomorphism algebras of exceptional sequences over hereditary algebras.

2. Preliminaries

Throughout this paper let k be an algebraically closed field, A be a finite-dimensional hereditary algebra with identity over k, and mod$A$ be the category of finitely generated left A-modules. An indecomposable module X is said to be exceptional if $\mathrm{Ext}_A^1(X,X)=0$. Note that $\mathrm{End}(X)$ is the field k in this case. A set of exceptional modules $X_1,X_2,\ldots,X_r$ is said to be an exceptional sequence if $\mathrm{Hom}_A(X_j,X_i)=0$ and $\mathrm{Ext}_A^1(X_j,X_i)=0$ for $j>i$. The set $\{X_1,X_2,\ldots,X_r\}$ is said to be a complete exceptional sequence if $r=n$ is the number of isomorphic classes of simple modules in mod$A$. $\mathrm{End}_A(X_1\oplus X_1\oplus\cdots\oplus X_n)=\mathrm{Hom}_A(X_1\oplus\cdots\oplus X_n,X_1\oplus\cdots\oplus X_n)$ is said to be the endomorphism algebra of the complete exceptional sequence $\{X_1,X_2,\ldots,X_n\}$.

Let $\mathcal{C}$ be a set of modules in mod$A$. Then

$$\mathcal{C}^{\perp}=\{M\mid \mathrm{Hom}_A(X,M)=0,\mathrm{Ext}_A^1(X,M)=0,M\in \mathrm{mod}A,X\in\mathcal{C}\}$$

and

$$^{\perp}\mathcal{C}=\{M\mid \mathrm{Hom}_A(M,X)=0,\mathrm{Ext}_A^1(M,X)=0,M\in \mathrm{mod}A,X\in\mathcal{C}\}$$

are respectively called the right and left perpendicular categories determined by $\mathcal{C}$. When $\mathcal{C}$ contains only one module X, we write $^{\perp}\mathcal{C}={}^{\perp}X$, $\mathcal{C}^{\perp}=X^{\perp}$.

The following lemma is from Ref. [25].

Lemma 2.1 *Let $H=k\vec{\Delta}$, where Δ is a quiver with n vertices, and $\mathcal{E}=(X_1,X_2,\ldots,X_r)$ be* *an exceptional sequence in* mod$H$. *Then* $\mathcal{E}^{\perp}$ *and* ${}^{\perp}\mathcal{E}$ *are respectively equivalent to* mod$kQ(\mathcal{E}^{\perp})$ *and* mod$kQ({}^{\perp}\mathcal{E})$, *where* $Q(\mathcal{E}^{\perp})$ *and* $Q({}^{\perp}\mathcal{E})$ *are quivers containing* $n-r$ *vertices without oriented cycles. Especially, if* X *is an exceptional module in* mod$H$, *then* $X^{\perp} = \mathrm{mod}H_r$ *and* ${}^{\perp}X = \mathrm{mod}H_l$, *where* $H_r = k\vec{\Delta}_r$ *and* $H_l = k\vec{\Delta}_l$, *with* $\vec{\Delta}_r$ *and* $\vec{\Delta}_l$ *each having one vertex less than* $\vec{\Delta}$.

Lemma 2.2 *Let* $\{X_1, X_2\}$ *be an exceptional sequence in* mod$A$. *If* $\mathrm{Hom}_A(X_1, X_2) \neq 0$, *then* $\mathrm{Ext}^1_A(X_1, X_2) = 0$.

Proof Since $\mathrm{Ext}^1_A(X_2, X_1) = 0$, the nonzero map $f \in \mathrm{Hom}(X_1, X_2)$ is either injective or surjective. If f is surjective, then f induces a surjective $\mathrm{Ext}^1_A(X_1, X_1) \longrightarrow \mathrm{Ext}^1_A(X_1, X_2)$. Thus, $\mathrm{Ext}^1_A(X_1, X_1) = 0$ implies $\mathrm{Ext}^1_A(X_1, X_2) = 0$. If f is injective, then f induces a surjective $\mathrm{Ext}^1_A(X_2, X_2) \longrightarrow \mathrm{Ext}^1_A(X_1, X_2)$. Thus, $\mathrm{Ext}^1_A(X_2, X_2) = 0$ implies $\mathrm{Ext}^1_A(X_1, X_2) = 0$. This completes the proof. □

Now, let A be a basic, connected finite-dimensional algebra over the algebraically closed field k and ${}_AX_A$ be a finite-dimensional A-bimodule. We define the Hochschild complex $C^* = (C^i, d^i)_{i\in\mathbb{Z}}$ associated with this data as follows:

$C^i = 0,\ d^i = 0$ for $i < 0$, $C^0 = {}_AX_A$, $C^i = \mathrm{Hom}_k(A^{\otimes i}, X)$ for $i > 0$, where $A^{\otimes i}$ denotes the i-fold tensor product over k of A with itself, $d^0 : X \longrightarrow \mathrm{Hom}_k(A, X)$ with $(d^0x)(a) = ax - xa$ for $x \in X$ and $a \in A$, $d^i : C^i \longrightarrow C^{i+1}$ with $(d^if)(a_1 \otimes \cdots \otimes a_{i+1}) = a_1f(a_2 \otimes \cdots \otimes a_{i+1}) + \sum_{j=1}^{i}(-1)^j f(a_1 \otimes \cdots \otimes a_ja_{j+1} \otimes \cdots \otimes a_{i+1}) + (-1)^{i+1}f(a_1 \otimes \cdots \otimes a_i)a_{i+1}$ for $f \in C^i$ and $a_1, \ldots, a_{i+1} \in A$.

Thus, we define $H^i(A, X) = H^i(C^*) = \ker d^i/\mathrm{im}d^{i-1}$, and we call it the i^{th} cohomology group of A with coefficients in the bi-modules X.

Of particular interest to us is the case of ${}_AX_A = {}_AA_A$. In this case $H^i(A, A)$ is denoted by $H^i(A)$. We call it the i^{th} Hochschild cohomology group of A.

Clearly, $H^0(A, X) = X^A = \{x \in X | ax = xa \text{ for any } a \in A\}$. In particular, $H^0(A)$ coincides with the center of A. Let $\mathrm{Der}(A, X) = \{\delta \in \mathrm{Hom}_k(A, X) | \delta(ab) = a\delta(b) + \delta(a)b\}$ be the k-vector space of derivations of A on X. We denote by $\mathrm{Der}^0(A, X)$ the subspace of inner derivations. Thus, $\mathrm{Der}^0(A, X) = \{\delta_x : A \longrightarrow X \,|\, \delta_x(a) = ax - xa ,\, x \in X\}$. It follows immediately from the definition that $H^1(A, X) = \mathrm{Der}(A, X)/\mathrm{Der}^0(A, X)$.

Let B be a finite dimensional k-algebra and M be a left B-module. By definition, the one-point extension $A = B[M]$ of B by M is the finite dimensional k-algebra $\begin{pmatrix} k & 0 \\ M & B \end{pmatrix}$ with multiplication $\begin{pmatrix} a & 0 \\ m & b \end{pmatrix}\begin{pmatrix} a' & 0 \\ m' & b' \end{pmatrix} = \begin{pmatrix} aa' & 0 \\ ma' + bm' & bb' \end{pmatrix}$, where $a,\ a' \in k,\ m,\ m' \in M$ and $b, b' \in B$.

Lemma 2.3[12] *Let* $A = B[M]$ *be a one-point extension algebra of* B *by* M. *Then there exists the following long exact sequence connecting the Hochschild cohomology groups of* A *and* B
$0 \longrightarrow H^0(A) \longrightarrow H^0(B) \longrightarrow \mathrm{End}_B(M)/k \longrightarrow H^1(A) \longrightarrow H^1(B) \longrightarrow \mathrm{Ext}^1_B(M, M) \longrightarrow \cdots \longrightarrow \mathrm{Ext}^i_B(M, M) \longrightarrow H^{i+1}(A) \longrightarrow H^{i+1}(B) \longrightarrow \mathrm{Ext}^{i+1}_B(M, M) \longrightarrow \cdots$.

Again let A be a finitely dimensional associative algebra with identity over a field k and $A^e = A \bigotimes_k A^{op}$ be the enveloping algebra of A. Denote by $A^{\otimes i}$ the i-fold tensor product k of A with itself, and by $(a_1, a_2, \ldots, a_i)$ the element $a_1 \otimes a_2 \otimes \cdots \otimes a_i$ in $A^{\otimes i}$. The Hochschild complex (A^{*+1}, b) of A is defined via $b: \ A^{\otimes(n+1)} \longrightarrow A^{\otimes n}$ with

$$b(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} (-1)^{i-1}(a_1, \ldots, a_i a_{i+1}, \ldots, a_{n+1}) + (-1)^n (a_{n+1}a_1, \ldots, a_n)$$

and $H_*(A) = H(A^{*+1}, b)$ is called the $*$-th Hochschild homology group of A. Then we have $H_i(A) \cong \mathrm{Tor}_i^{A^e}(A, A) = D\mathrm{Ext}^i_{A^e}(A, D(A))$ for $i \geq 0$[18], where $D = \mathrm{Hom}_k(-, k)$ is the standard dual functor.

Let H be a finitely dimensional hereditary algebra over k and T_H be a tilting module in mod$H$. Then $B = \mathrm{End}(T_A)$ is said to be a tilted algebra.

3. The global dimension and Hochschild cohomology groups of endomorphism algebras of exceptional sequences

Let A be a basic, connected finitely dimensional hereditary algebra over an algebraically closed field k. Let $\{X_1, X_2, \ldots, X_n\}$ be a complete exceptional sequence in mod$A$, and $E = \mathrm{End}(X_1 \oplus \cdots \oplus X_n)$ be its endomorphism algebra. Firstly we will study the global dimension of E in this section. The following lemma is due to Ref.[26].

Lemma 3.1 *Suppose T and U are artin algebras over R with T semi-simple where R is a commutative artin ring with identity. Let ${}_U M_T$ be a nonzero bimodule where R acts centrally (i.e. $rm = mr$ for any $r \in R$ and $m \in M$) and M is finitely generated over R. If $\Lambda = \begin{pmatrix} T & 0 \\ {}_U M_T & U \end{pmatrix}$, then* $\mathrm{g.l.dim}\Lambda = \max\{\mathrm{gl.dim}U, p\mathrm{dim}_U M + 1\}$.

So we immediately get the following corollary.

Corollary 3.2 *Let $A = B[M]$ be one-point extension of B by M where B is a finitely dimensional associative k-algebra with identity and M be a left B-module. Then* $\mathrm{gl.dim}A \leq \mathrm{gl.dim}B + 1$.

Theorem 3.3 $\mathrm{gl.dim}E \leq n$.

Proof By induction on n.

The case of $n = 1$ is trivial, so we begin our proof from $n = 2$. In this case, $E = \mathrm{End}(X_1 \oplus X_2)$. If $\mathrm{Hom}(X_1, X_2) = 0$, then $E = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$, and it is easy to see that the theorem is true. If $\mathrm{Hom}(X_1, X_2) \neq 0$, then $X_1 \oplus X_2$ is a tilting module by Lemma 2.2, and E is a tilted algebra. Thus, $gl.\mathrm{dim}E \leq 2$ from the well-known result that the global dimension of a tilted algebra is less than or equal to 2.

Assume inductively that the theorem is true for the case in $n(\geq 2)$. Now, we consider the case in $(n + 1)$.

Firstly,

$$E=\begin{pmatrix} \mathrm{Hom}_A(X_1,X_1) & 0 & \cdots & 0 \\ \mathrm{Hom}_A(X_1,X_2) & \mathrm{Hom}_A(X_2,X_2) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \mathrm{Hom}_A(X_1,X_{n+1}) & \mathrm{Hom}_A(X_2,X_{n+1}) & \cdots & \mathrm{Hom}_A(X_{n+1},X_{n+1}) \end{pmatrix}.$$

Now that $\mathrm{End}(X_i)=\mathrm{Hom}_A(X_i,X_i)=k$ for $i=1,2,\ldots,n+1$.

Let

$$C=\begin{pmatrix} k & 0 & \cdots & 0 \\ \mathrm{Hom}(X_2,X_3) & k & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \mathrm{Hom}(X_2,X_{n+1}) & \mathrm{Hom}(X_3,X_{n+1}) & \cdots & k \end{pmatrix}$$

and

$$M=(\mathrm{Hom}_A(X_1,X_2),\ldots,\mathrm{Hom}_A(X_1,X_{n+1}))^{\mathrm{T}}.$$

Then we have $E=\begin{pmatrix} k & 0 \\ M & C \end{pmatrix}$. It is easy to see that M is a left C-module. Thus, E is the one-point extension of C by M.

According to Lemma 2.1, we have that $\{X_2,X_3,\ldots,X_{n+1}\}$ can be regarded as a complete exceptional sequence over some hereditary algebra with n isomorphic classes of simple modules.

So, $\mathrm{gl.dim}C\le n$ according to the induction hypothesis. Hence, we learn that $\mathrm{gl.dim}E\le \mathrm{gl.dim}C+1=n+1$, due to Corollary 3.2. This completes the proof. □

We need the following lemmas to prove the theorem about Hochschild cohomology of E.

Lemma 3.4[12] *Let $\vec{\Delta}$ be a finite quiver without oriented cycles and B be a finitely dimensional tilted algebra which is tiltable to $k\vec{\Delta}$. Then $H^0(B)=k$, $\dim H^1(B)=\dim H^1(k\vec{\Delta})$, and $H^i(B)=0$ for $i\ge 2$.*

Lemma 3.5 *Let $\vec{\Delta}$ be a finite quiver without oriented cycles and $\vec{\Delta}$ have two vertices. Let (X_1,X_2) be a complete exceptional sequence in $\mathrm{mod}k\vec{\Delta}$ and $E=\mathrm{End}(X_1\oplus X_2)$. Then $H^i(E)=0$ for $i\ge 2$.*

Proof We have $E\cong\begin{pmatrix} k & 0 \\ \mathrm{Hom}(X_1,X_2) & k \end{pmatrix}$. Now consider it in the following two cases.

(1) If $\mathrm{Hom}(X_1,X_2)=0$, then $E=k\oplus k$. It is easy to check the conclusion.

(2) If $\mathrm{Hom}(X_1,X_2)\ne 0$, then $\mathrm{Ext}^1_A(X_1,X_2)=0$, and thus, $X_1\oplus X_2$ is a tilting module in $\mathrm{mod}k\vec{\Delta}$ due to Lemma 2.2. So, E is a tilted algebra over $k\vec{\Delta}$. We learn from Lemma 3.4 that $H^0(E)=k$ and $H^i(E)=0$ for $i\ge 2$. This completes the proof. □

Theorem 3.6 *$H^i(E)=0$ for $i\ge n+1$.*

Proof $E=\begin{pmatrix} k & 0 \\ M & C \end{pmatrix}$, M and C are as the above in the proof of Theorem 3.3. Thus

$E = C[M]$ according to the Lemma 2.3. We have the following long exact sequence connecting the Hochschild cohomology groups of E and C,

$$0 \to H^0(E) \to H^0(C) \to \mathrm{End}_C(M)/k \to H^1(E) \to H^1(C) \to \mathrm{Ext}^1_C(M, M) \to \cdots$$
$$\to \mathrm{Ext}^i_C(M, M) \to H^{i+1}(E) \to H^{i+1}(C) \to \mathrm{Ext}^{i+1}_C(M, M) \to \cdots . \qquad (*)$$

Now we prove the theorem by induction on n.

For $n = 1$, the theorem is trivial.

For $n = 2$, the theorem is true according to Lemma 3.5.

We assume inductively that the theorem is true for the case of $n = l$ for $l \geq 1$. Then we consider the case of $n = l + 1$. According to Theorem 3.3 and Lemma 2.2 we know gl.dim$C \leq l$. Thus, we have $\mathrm{Ext}^i_C(M, M) = 0$ for $i \geq l + 1$, so we have $H^i(E) \cong H^i(C)$ for $i \geq l + 2$ due to the long exact sequence (*). But $H^i(C) = 0$ for $i \geq l + 1$. Therefore $H^i(E) = 0$ for $i \geq l + 2 = (l + 1) + 1 = n + 1$. This completes the proof. □

4. The Hochschild homology groups of the endomorphism algebras of exceptional sequences

Let A be a finitely dimensional k-algebra with identity, and $e_1, e_2, \ldots, e_n$ be a complete set of primitive orthogonal idempotents in A. By $P(i)$ we denote the indecomposable projective A-module Ae_i, and $S(i) = \mathrm{top}P(i)$ be the corresponding simple A-module. Then $\{e_i \otimes e_j\}$ $(1 \leq i, j \leq n)$ is a complete set of primitive orthogonal idempotents of A^e. So, $\{P(i, j) = A^e e_i \otimes_k e_j \cong Ae_i \otimes_k e_j A | for\ 1 \leq i, j \leq n\}$ is a complete set of representatives from the isomorphism classes of indecomposable projective A^e-modules. By $S(i, j') = \mathrm{top}P(i, j')$ we denote the corresponding simple A^e-module. Observe that $S(i, j') = \mathrm{top}P(i, j') \cong \mathrm{Hom}_k(S(i), S(j))$.

Lemma 4.1[12] *Let* $\cdots \to R_m \to R_{m-1} \cdots \to R_1 \to R_0 \to A \to 0$ *be a minimal projective resolution of* A *over* A^e*. Then*

$$R_m = \bigoplus_{i,j} P(i, j)^{\dim_k \mathrm{Ext}^m_A(S_i, S_j)}.$$

Proof Let $R_m = \bigoplus_{i,j} P(i, j)^{r_{ij}}$. Then by definition we have that

$$r_{ij} = \dim\mathrm{Ext}^n_{A^e}(A, S(i, j')) = \dim\mathrm{Ext}^n_{A^e}(A, \mathrm{Hom}_k(S(i), S(j)))$$
$$= \dim H^n(A, \mathrm{Hom}_k(S(i), S(j))) = \dim\mathrm{Ext}^n_A(S(i), S(j)).$$

The last equality follows from Corollary 4.4, P.170 in Ref. [18].

Lemma 4.2[22,28] *If* $A = k\vec{\Delta}/I$*, where* $\vec{\Delta} = (\Delta_0, \Delta_1)$*, has no oriented cycles, then*

$$H_i(A) = \begin{cases} k^{|\Delta_0|}, & if\ i = 0, \\ 0, & if\ i > 0. \end{cases}$$

Proof Applying $A \otimes_{A^e}$-to the minimal projective resolution given in Lemma 4.1, we have

$$A \otimes_{A^e} P(i, j) = A \otimes_{A^e} A^e(e_i \otimes e_j) \simeq e_j A e_i.$$

But $\mathrm{Ext}_A^m(S_i, S_j) \neq 0$ for some $m \geq 1$ implies that there exists a path in $\vec{\Delta}$ from j to i. As Q has no oriented cycles, we have $e_j A e_i = 0$. Therefore $A \otimes_{A^e} R_m = 0$ for $m \geq 1$. This completes the proof. □

Proposition 4.3 *Let A be a finite-dimensional hereditary k-algebra. Then there is no oriented cycle in any exceptional sequence over A.*

Proof Let $(X_1, X_2, \ldots, X_t)$ be any exceptional sequence over A. Since $\mathrm{Ext}_A^1(X_j, X_i) = 0$ for $j > i$, any nonzero map $\varphi : X_i \longrightarrow X_j$ is either a monomorphism or an epimorphism. Thus, for any X_i, X_j and X_s with $i < j < s$, if we have $X_i \xrightarrow{\varphi} X_j \xrightarrow{\psi} X_s$, then φ and ψ are both injective or both surjective, or φ is injective and ψ is surjective.

If there is an oriented cycle $X_1 \xrightarrow{\varphi_1} X_2 \xrightarrow{\varphi_2} \cdots \longrightarrow X_m \xrightarrow{\varphi_m} X_1$, then the maps φ_i are all injective or all surjective. Thus, they are all isomorphism. But this is impossible. This completes the proof. □

Theorem 4.4 *Let $(X_1, X_2, \ldots, X_n)$ be a complete exceptional sequence over a finitely dimensional hereditary k-algebra A, and $E = \mathrm{End}(X_1 \oplus \cdots \oplus X_n)$. Then $H_0(E) = k^n$ and $H_i(E) = 0$ for $i \geq 1$.*

Proof $E = \begin{pmatrix} k & 0 \\ M & C \end{pmatrix}$ is a one-point extension algebra of C by M, where M and C are as in the proof of Theorem 3.3. According to Lemma 2.1, C can be regarded as an endomorphism algebra of a complete exceptional sequence over a finite-dimensional hereditary k-algebra with $(n-1)$ isomorphic classes of simple modules and so, C is also a one-point extension algebra. Thus, E is a triangular algebra, whose ordinary quiver has no oriented cycle. So, it follows immediately from Lemma 4.2 that the theorem is true.

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