Journal of Mathematical Research & Exposition
Aug., 2008, Vol. 28, No. 3, pp. 713–719
DOI:10.3770/j.issn:1000-341X.2008.03.036
Http://jmre.dlut.edu.cn

Delay-Dependent Robust Stability Criteria for Neutral Lurie Control Systems

TIAN Jun Kang[1,2], *ZHONG Shou Ming*[1], *XIONG Liang Lin*[1]
(1. College of Science, Southwest Petroleum University, Sichuan 637100, China;
2. School of Applied Mathematics, University of Electronic Science and Technology of China, Sichuan 610054, China)
(E-mail: tianjunkang1980@163.com)

Abstract This paper deals with delay-dependent robust stability of neutral Lurie control systems with multiple nonlinearities and time-varying structured uncertainties. The Lyapunov functional method is used. By adding some appropriate zero terms to the deviation of V and constructing some linear matrix inequalities, some sufficient conditions for the delay-dependent absolute stability and robust stability are derived. Finally, a numerical example is presented to illustrate the effectiveness of the method.

Keywords Lurie control systems; absolute stability; linear matrix inequalities; delay-dependent.

Document code A
MR(2000) Subject Classification 34K35
Chinese Library Classification O175.1

1. Introduction

Lurie control system with time-delay is an important nonlinear control system. The problem of the absolute stability of Lurie control system has been widely studied for several decades[2−6]. However, a few number of studies have been done on the stability analysis of neutral Lurie control system and less research was focused on the robust absolute stability of this system[10]. Li[10] studied the robust absolute stability of this kind of system and derived some stability criteria, but the criteria are all delay-independent which do not include the information on delay. Generally, abandonment of information on the delay causes conservativeness of the criteria especially when the delay is not considered. The Lyapunov-Krasovskii functional method is used in this paper. By adding some appropriate zero terms to the derivation of V and constructing some linear matrix inequalities, the delay-dependent stability sufficient conditions are derived to ensure the neutral Lurie control system's absolute stability and robust absolute stability. Finally, a numerical example is presented to illustrate the effctiveness of the method.

2. Problem formulation and preliminaries

Received date: 2006-05-07; **Accepted date**: 2007-03-23

Consider the neutral Lurie control system with time-delay

$$\begin{cases} \dot{x}(t) - D\dot{x}(t-\tau) = \tilde{A}x(t) + \tilde{B}x(t-\tau) + \tilde{E}f(\sigma(t)) + \tilde{F}f(\sigma(t-\tau)), \\ \sigma(t) = C^{\mathrm{T}}x(t), \\ x(\theta) = \phi(\theta), \theta \in [-\tau, 0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $\tau > 0$ is time delay, $\tilde{A} = A + \Delta A(t)$, $\tilde{B} = B + \Delta B(t)$, $\tilde{D} = D + \Delta D(t)$, $\tilde{E} = E + \Delta E(t)$, $\tilde{F} = F + \Delta F(t)$, real matrices $A, B, D \in R^{n\times n}$, $C, E, F \in R^{n\times m}$, $C = (C_1, C_2, \ldots, C_m)$, $\sigma(t) = (\sigma_1(t), \sigma_2(t), \ldots, \sigma_m(t))^{\mathrm{T}}$, $f(\sigma(t)) = (f_1(\sigma(t)), f_2(\sigma(t)), \ldots, f_m(\sigma(t)))^{\mathrm{T}}$ and $\phi(\theta)$ is a continuous vector valued initial function. Let $\Gamma(x_t) = x(t) - Dx(t-\tau)$ denote the difference operator and $K = \mathrm{diag}(k_1, k_2, \ldots, k_m)(k_i > 0, i = 1, 2, \ldots, m)$ the diagonal matrix.

$$f_j(\cdot) \in k_j[0, k_j] = \{f_j(\sigma_j) \,|f_j\,(0) = 0; 0 \le \sigma_j f_j(\sigma_j) \le k_j\sigma_j^2,\, \sigma_j \ne 0\}\,, j = 1, 2, \ldots, m. \tag{2}$$

The uncertainties are assumed to be of the following form: $(\Delta A(t), \Delta B(t), \Delta E(t), \Delta F(t)) = MF(t)(E_a, E_b, E_e, E_f)$, where M, E_a, E_b, E_e, E_f are constant matrices with appropriate dimensions, and $F(t)$ is continuous time-varying matrix with Lebesque measurable element satisfying $\sigma_{\max}(F(t)) \le 1$, where $\sigma_{\max}(.)$ is the maximal singular value of the matrix

Definition 1 *The system (1) is said to be absolutely stable in the sector bounded by* $K = \mathrm{diag}(k_1, k_2, \ldots, k_m)$ *if for any* $f_j(\cdot) \in k_j[0, k_j]$, *the trivial solution of system(1) is globally asymptotically stable.*

Definition 2 *The system (1) is said to be robustly absolutely stable in the sector bounded by* $K = \mathrm{diag}(k_1, k_2, \ldots, k_m)$ *if for all uncertainties, the system (1) is absolutely stable in the sector bounded by* $K = \mathrm{diag}(k_1, k_2, \ldots, k_m)$.

Lemma 1[14] *Given matrices* $Q = Q^{\mathrm{T}}, H, E$ *and* $R = R^{\mathrm{T}} > 0$ *of appropriate dimensions. Then* $Q + HFE + E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}} < 0$ *holds for all* F *satisfying* $F^{\mathrm{T}}F \le R$, *if and only if there exists a positive number* $\varepsilon > 0$, *such that* $Q + \varepsilon^{-1}HH^{\mathrm{T}} + \varepsilon E^{\mathrm{T}}RE < 0$.

3. Main results

Consider the absolute stability of system (1). First, we consider the nominal system of system (1):

$$\begin{cases} \dot{x}(t) - D\dot{x}(t-\tau) = Ax(t) + Bx(t-\tau) + Ef(\sigma(t)) + Ff(\sigma(t-\tau)), \\ \sigma(t) = C^{\mathrm{T}}x(t), \\ x(\theta) = \phi(\theta), \theta \in [-\tau, 0]. \end{cases} \tag{3}$$

Theorem 1 *System (3) is absolutely stable in the sector bounded by* $K = \mathrm{diag}(k_1, k_2, \ldots, k_m)$, *if the spectral radius of* D *satisfies* $\rho(D) < 1$, *and there exist* $P = P^{\mathrm{T}} > 0$, $Q = Q^{\mathrm{T}} > 0$, $R = R^{\mathrm{T}} > 0$, $N = N^{\mathrm{T}} > 0$, $\alpha_i \ge 0$, $t_i \ge 0$, $\lambda_i \ge 0$, $i = 1, 2, \ldots, m$, $\alpha = \mathrm{diag}(\alpha_1, \alpha_2, \ldots, \alpha_m) > 0$, $\mathrm{T} = \mathrm{diag}(t_1, t_2, \ldots, t_m) > 0$, $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_m) > 0$, *and any matrices* S_i, $i = 1, \ldots, 5$,

X_{ij}, $i = 1, \ldots, 5$; $j = 1, \ldots, 5$ *with appropriate dimensions, such that the following LMIs hold:*

$$\Xi = \begin{pmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & A^{\mathrm{T}}S \\ \Phi_{12}^{\mathrm{T}} & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} + CK\mathrm{T} & B^{\mathrm{T}}S \\ \Phi_{13}^{\mathrm{T}} & \Phi_{23}^{\mathrm{T}} & \Phi_{33} & \Phi_{34} & \Phi_{35} & D^{\mathrm{T}}S \\ \Phi_{14}^{\mathrm{T}} & \Phi_{24}^{\mathrm{T}} & \Phi_{34}^{\mathrm{T}} & \Phi_{44} & \Phi_{45} & E^{\mathrm{T}}S \\ \Phi_{15}^{\mathrm{T}} & \Phi_{25}^{\mathrm{T}} + \mathrm{T}KC^{\mathrm{T}} & \Phi_{35}^{\mathrm{T}} & \Phi_{45}^{\mathrm{T}} & \Phi_{55} - 2\mathrm{T} & F^{\mathrm{T}}S \\ SA & SB & SD & SE & SF & -S \end{pmatrix} < 0, \tag{4}$$

$$X = \begin{pmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} \\ X_{12}^{\mathrm{T}} & X_{22} & X_{23} & X_{24} & X_{25} \\ X_{13}^{\mathrm{T}} & X_{23}^{\mathrm{T}} & X_{33} & X_{34} & X_{35} \\ X_{14}^{\mathrm{T}} & X_{24}^{\mathrm{T}} & X_{34}^{\mathrm{T}} & X_{44} & X_{45} \\ X_{15}^{\mathrm{T}} & X_{25}^{\mathrm{T}} & X_{35}^{\mathrm{T}} & X_{45}^{\mathrm{T}} & X_{55} \end{pmatrix} \geq 0, \tag{5}$$

$$\Pi = \begin{pmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & S_1 \\ X_{12}^{\mathrm{T}} & X_{22} & X_{23} & X_{24} & X_{25} & S_2 \\ X_{13}^{\mathrm{T}} & X_{23}^{\mathrm{T}} & X_{33} & X_{34} & X_{35} & S_3 \\ X_{14}^{\mathrm{T}} & X_{24}^{\mathrm{T}} & X_{34}^{\mathrm{T}} & X_{44} & X_{45} & S_4 \\ X_{15}^{\mathrm{T}} & X_{25}^{\mathrm{T}} & X_{35}^{\mathrm{T}} & X_{45}^{\mathrm{T}} & X_{55} & S_5 \\ S_1^{\mathrm{T}} & S_2^{\mathrm{T}} & S_3^{\mathrm{T}} & S_4^{\mathrm{T}} & S_5^{\mathrm{T}} & N \end{pmatrix} \geq 0, \tag{6}$$

where

$$\Phi_{11} = A^{\mathrm{T}}P + PA + Q + S_1 + S_1^{\mathrm{T}} + \tau X_{11}, \quad \Phi_{12} = PB - A^{\mathrm{T}}PD - S_1 + S_2^{\mathrm{T}} + \tau X,$$

$$\Phi_{13} = S_3^{\mathrm{T}} + \tau X, \quad \Phi_{14} = PE + A^{\mathrm{T}}C\Lambda + S_4^{\mathrm{T}} + \tau X_{14},$$

$$\Phi_{15} = PF + S_5^{\mathrm{T}} + \tau X_{15}, \quad \Phi_{22} = -Q - B^{\mathrm{T}}PD - D^{\mathrm{T}}PB + S_2 + S_2^{\mathrm{T}} + \tau X_{22},$$

$$\Phi_{23} = -S_3^{\mathrm{T}} + \tau X_{23}, \quad \Phi_{24} = -D^{\mathrm{T}}PE + B^{\mathrm{T}}C\Lambda - S_4^{\mathrm{T}} + \tau X_{24},$$

$$\Phi_{25} = -D^{\mathrm{T}}PF - S_5^{\mathrm{T}} + \tau X_{25}, \quad \Phi_{33} = -R + \tau X_{33},$$

$$\Phi_{35} = \tau X_{35}, \quad \Phi_{44} = \alpha + \Lambda C^{\mathrm{T}}E + E^{\mathrm{T}}C\Lambda + \tau X_{44},$$

$$\Phi_{45} = \Lambda C^{\mathrm{T}}F + \tau X_{45}, \quad \Phi_{55} = -\alpha + \tau X_{55}, \quad S = R + \tau N.$$

Proof Choose a new class of Lyapunov-Krasovskii functional candidate as follows:

$$\begin{aligned} V(x_t) =& V_1(x_t) + V_2(x_t), \\ V_1(x_t) =& \Gamma^{\mathrm{T}}(x_t)P\Gamma(x_t) + \int_{t-\tau}^{t} x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s + \int_{t-\tau}^{t} \dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\mathrm{d}s + \\ & \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)N\dot{x}(s)\mathrm{d}s\mathrm{d}\theta, \\ V_2(x_t) =& \sum_{i=1}^{m} \alpha_i \int_{t-\tau}^{t} f_i^2(\sigma_i(s))\mathrm{d}s + 2\sum_{i=1}^{m} \lambda_j \int_{0}^{\sigma_j} f_j(\sigma_j)\mathrm{d}\sigma_j. \end{aligned}$$

Then the time derivative of $V(x_t)$ along the trajectory of system (3) is given by

$$\begin{aligned}\dot{V}(x_t) =&(\dot{\Gamma}(x_t))^{\mathrm{T}}P\Gamma(x_t)+\Gamma^{\mathrm{T}}(x_t)P\dot{\Gamma}(x_t)+x^{\mathrm{T}}(t)Qx(t)-x^{\mathrm{T}}(t-\tau)Qx(t-\tau)+\dot{x}^{\mathrm{T}}(t)R\dot{x}(t)-\\&\dot{x}^{\mathrm{T}}(t-\tau)R\dot{x}(t-\tau)+\tau\dot{x}^{\mathrm{T}}(t)N\dot{x}(t)-\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)N\dot{x}(s)\mathrm{d}s+\\&\sum_{i=1}^{m}\alpha_i[f_i^2(\sigma_i(t))-f_i^2(\sigma_i(t-\tau))]+2\sum_{i=1}^{m}\lambda_i f_i(\sigma_i)\dot{\sigma}_j(t)\\=&[Ax(t)+Bx(t-\tau)+Ef(\sigma(t))+Ff(\sigma(t-\tau))]^{\mathrm{T}}P[x(t)-Dx(t-\tau)]+\\&[x(t)-Dx(t-\tau)]^{\mathrm{T}}P[Ax(t)+Bx(t-\tau)+Ef(\sigma(t))+Ff(\sigma(t-\tau))]+\\&x^{\mathrm{T}}(t)Qx(t)-x^{\mathrm{T}}(t-\tau)Qx(t-\tau)+\dot{x}^{\mathrm{T}}(t)R\dot{x}(t)-\dot{x}^{\mathrm{T}}(t-\tau)R\dot{x}(t-\tau)+\\&\tau\dot{x}^{\mathrm{T}}(t)N\dot{x}(t)+f^{\mathrm{T}}(\sigma(t))\alpha f(\sigma(t))-f^{\mathrm{T}}(\sigma(t-\tau))\alpha f(\sigma(t-\tau))+\\&2f^{\mathrm{T}}(\sigma(t))\Lambda C^{\mathrm{T}}[Ax(t)+Bx(t-\tau)+\dot{D}x(t-\tau)+Ef(\sigma(t))+Ff(\sigma(t-\tau))]-\\&\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)N\dot{x}(s)\mathrm{d}s.\end{aligned}$$

Using the Leibniz-Newton formula yields: $x(t)-x(t-\tau)=\int_{t-\tau}^{t}\dot{x}(s)\mathrm{d}s$. Then, for any constant matrices S_i, $i=1,2,\ldots,m$, with appropriate dimensions, the following is true.

$$\begin{aligned}&2[x^{\mathrm{T}}(t)S_1+x^{\mathrm{T}}(t)S_2+\dot{x}^{\mathrm{T}}(t-\tau)S_3+f^{\mathrm{T}}(\sigma(t))S_4+f^{\mathrm{T}}(\sigma(t-\tau))S_5]\cdot\\&\quad[x(t)-x(t-\tau)-\int_{t-\tau}^{t}\dot{x}(s)\mathrm{d}s]=0.\end{aligned}\tag{7}$$

On the other hand, for any constant matrices X_{ij}, $i=1,\ldots,5$; $j=1,\ldots,5$ with appropriate dimensions, the following is also true.

$$\begin{pmatrix}x(t)\\x(t-\tau)\\\dot{x}(t-\tau)\\f(\sigma(t))\\f(\sigma(t-\tau))\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\Theta_{11}&\Theta_{12}&\Theta_{13}&\Theta_{14}&\Theta_{15}\\\Theta_{12}^{\mathrm{T}}&\Theta_{22}&\Theta_{23}&\Theta_{24}&\Theta_{25}\\\Theta_{13}^{\mathrm{T}}&\Theta_{23}^{\mathrm{T}}&\Theta_{33}&\Theta_{34}&\Theta_{35}\\\Theta_{14}^{\mathrm{T}}&\Theta_{24}^{\mathrm{T}}&\Theta_{34}^{\mathrm{T}}&\Theta_{44}&\Theta_{45}\\\Theta_{15}^{\mathrm{T}}&\Theta_{25}^{\mathrm{T}}&\Theta_{35}^{\mathrm{T}}&\Theta_{45}^{\mathrm{T}}&\Theta_{55}\end{pmatrix}\begin{pmatrix}x(t)\\x(t-\tau)\\\dot{x}(t-\tau)\\f(\sigma(t))\\f(\sigma(t-\tau))\end{pmatrix}=0,\tag{8}$$

where $\Theta_{ij}=\tau(X_{ij}-X_{ij})$. Then, adding the terms on the left sides of (7) and (8) to $\dot{V}(x_t)$ yields

$$\dot{V}(x_t)=Z^{\mathrm{T}}(t)\Omega Z(t)-\int_{t-\tau}^{t}Z_1^{\mathrm{T}}(t,s)\Pi Z_1(t,s)\mathrm{d}s,\tag{9}$$

where

$$Z(t)=(x^{\mathrm{T}}(t),x^{\mathrm{T}}(t-\tau),\dot{x}^{\mathrm{T}}(t-\tau),f^{\mathrm{T}}(\sigma(t)),f^{\mathrm{T}}(\sigma(t-\tau)))^{\mathrm{T}},Z_1(t)=(Z^{\mathrm{T}}(t),\dot{x}^{\mathrm{T}}(s))^{\mathrm{T}},$$

$$\Omega=\begin{pmatrix}\Phi_{11}+A^{\mathrm{T}}SA&\Phi_{12}+A^{\mathrm{T}}SB&\Phi_{13}+A^{\mathrm{T}}SD&\Phi_{14}+A^{\mathrm{T}}SE&\Phi_{15}+A^{\mathrm{T}}SF\\\Phi_{12}^{\mathrm{T}}+B^{\mathrm{T}}SA&\Phi_{22}+B^{\mathrm{T}}SB&\Phi_{23}+B^{\mathrm{T}}SD&\Phi_{24}+B^{\mathrm{T}}SE&\Phi_{25}+B^{\mathrm{T}}SF\\\Phi_{13}^{\mathrm{T}}+D^{\mathrm{T}}SA&\Phi_{22}^{\mathrm{T}}+D^{\mathrm{T}}SB&\Phi_{23}+D^{\mathrm{T}}SE&\Phi_{24}+D^{\mathrm{T}}SE&\Phi_{25}+D^{\mathrm{T}}SF\\\Phi_{14}^{\mathrm{T}}+E^{\mathrm{T}}SA&\Phi_{24}^{\mathrm{T}}+E^{\mathrm{T}}SB&\Phi_{34}^{\mathrm{T}}+E^{\mathrm{T}}SD&\Phi_{44}+E^{\mathrm{T}}SF&\Phi_{45}+E^{\mathrm{T}}SF\\\Phi_{15}^{\mathrm{T}}+F^{\mathrm{T}}SA&\Phi_{25}^{\mathrm{T}}+F^{\mathrm{T}}SB&\Phi_{35}^{\mathrm{T}}+F^{\mathrm{T}}SD&\Phi_{45}^{\mathrm{T}}+F^{\mathrm{T}}SE&\Phi_{55}+F^{\mathrm{T}}SF\end{pmatrix}.\tag{10}$$

Using the conditions (2) gives:

$$f_j(\sigma_j(t-\tau))(f_j(\sigma_j(t-\tau)) - k_j C_j^{\mathrm{T}} x(t-\tau)) \le 0, j = 1, 2, \ldots, m. \tag{11}$$

If, after applying S-procedure, there exist $\mathrm{T} = \mathrm{diag}(t_1, t_2, \ldots, t_m) > 0$ and $t_i \ge 0$, $i = 1, 2, \ldots, m$ such that the following inequality holds:

$$\begin{aligned} Z^{\mathrm{T}}(t)\Omega Z(t) - \int_{t-\tau}^{t} Z_1^{\mathrm{T}}(t,s)\Pi Z_1(t,s)\mathrm{d}s - 2\sum_{i=1}^{m} t_i f_j(\sigma_j(t-\tau))\cdot \\ (f_j(\sigma_j(t-\tau)) - k_j C_j^{\mathrm{T}} x(t-\tau)) < 0, \end{aligned} \tag{12}$$

then, under condition (2), system(3) is absolutely stable in the sector bounded by $K = \mathrm{diag}(k_1, k_2, \ldots, k_m)$ for $\forall Z(t) \ne 0$. According to Schur Complement[13], (12) is equivalent to (4), (5) and (6).

Theorem 2 *System (1) is robustly absolutely stable in the sector bounded by* $K = \mathrm{diag}(k_1, k_2, \ldots, k_m)$, *if the spectral radius of* D *satisfies* $\rho(D) < 1$, *and there exist* $\varepsilon > 0$, $P = P^{\mathrm{T}} > 0$, $Q = Q^{\mathrm{T}} > 0$, $R = R^{\mathrm{T}} > 0$, $N = N^{\mathrm{T}} > 0$, $\alpha_i \ge 0$, $t_i \ge 0$, $\lambda_i \ge 0$, $i = 1, 2, \ldots, m$, $\alpha = \mathrm{diag}(\alpha_1, \alpha_2, \ldots, \alpha_m) > 0$, $\mathrm{T} = \mathrm{diag}(t_1, t_2, \ldots, t_m) > 0$, $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_m) > 0$, *and any matrices* S_i, $i = 1, \ldots, 5$, X_{ij}, $i = 1, \ldots, 5$; $j = 1, \ldots, 5$ *with appropriate dimensions, such that (5), (6) and the following LMI holds:*

$$W = \begin{pmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & A^{\mathrm{T}}S & PM \\ \Psi_{12}^{\mathrm{T}} & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} & B^{\mathrm{T}}S & -D^{\mathrm{T}}PM \\ \Psi_{13}^{\mathrm{T}} & \Psi_{23}^{\mathrm{T}} & \Psi_{33} & \Psi_{34} & \Psi_{35} & D^{\mathrm{T}}S & 0 \\ \Psi_{14}^{\mathrm{T}} & \Psi_{24}^{\mathrm{T}} & \Psi_{34}^{\mathrm{T}} & \Psi_{44} & \Psi_{45} & E^{\mathrm{T}}S & \Lambda C^{\mathrm{T}}M \\ \Psi_{15}^{\mathrm{T}} & \Psi_{25}^{\mathrm{T}} & \Psi_{35}^{\mathrm{T}} & \Psi_{45}^{\mathrm{T}} & \Psi_{55} & F^{\mathrm{T}}S & 0 \\ SA & SB & SD & SE & SF & -S & SM \\ M^{\mathrm{T}}P & -M^{\mathrm{T}}PD & 0 & M^{\mathrm{T}}C\Lambda & 0 & M^{\mathrm{T}}S & -\varepsilon I \end{pmatrix} < 0, \tag{13}$$

where

$$\Psi_{11} = \Phi_{11} + \varepsilon E_a^{\mathrm{T}} E_a, \quad \Psi_{12} = \Phi_{12} + \varepsilon E_a^{\mathrm{T}} E_b, \quad \Psi_{13} = \Phi_{13}, \quad \Psi_{14} = \Phi_{14} + \varepsilon E_a^{\mathrm{T}} E_e,$$

$$\Psi_{15} = \Phi_{15} + \varepsilon E_a^{\mathrm{T}} E_f, \quad \Psi_{22} = \Phi_{22} + \varepsilon E_b^{\mathrm{T}} E_b, \quad \Psi_{23} = \Phi_{23}, \quad \Psi_{24} = \Phi_{24} + \varepsilon E_b^{\mathrm{T}} E_e,$$

$$\Psi_{25} = \Phi_{25} + \varepsilon E_b^{\mathrm{T}} E_f + CK\mathrm{T}, \quad \Psi_{3j} = \Phi_{3j}, j = 3, 4, 5, \quad \Psi_{44} = \Phi_{44} + \varepsilon E_e^{\mathrm{T}} E_e,$$

$$\Psi_{45} = \Phi_{45} + \varepsilon E_e^{\mathrm{T}} E_f, \quad \Psi_{55} = \Phi_{55} + \varepsilon E_f^{\mathrm{T}} E_f - 2\mathrm{T}.$$

Proof For system (1) with uncertainties, replace A, B, E, F, in (4) with $A + MF(t)E_a$, $B + MF(t)E_b$, $E + MF(t)E_e$, $F + MF(t)E_f$, respectively. We find (4) is equivalent to

$$\Xi + HF(t)U + U^{\mathrm{T}}F^{\mathrm{T}}(t)H^{\mathrm{T}} < 0, \tag{14}$$

where

$$H^{\mathrm{T}} = \left(M^{\mathrm{T}}P, -M^{\mathrm{T}}PD, 0, M^{\mathrm{T}}C\Lambda, 0, M^{\mathrm{T}}\mathrm{T}\right), \quad U = (E_a, E_b, 0, E_e, E_f, 0).$$

By Lemma 1, a sufficient condition for (14) is that there exists a positive number $\varepsilon > 0$, such that

$$\Xi + \varepsilon^{-1} H H^{\mathrm{T}} + \varepsilon U^{\mathrm{T}} U < 0. \tag{15}$$

According to Schur Complement[13], (13) is equivalent to (11). Then system(1) is robustly absolutely stable in the sector bounded by $K = \mathrm{diag}(k_1, k_2, \ldots, k_m)$.

4. Numerical example

Consider the system (3), where:

$$A = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} -0.5 & -0.1 & 0.2 & 0.3 \\ 0.1 & -0.5 & 0.1 & 0.2 \\ -0.2 & 0.2 & -0.5 & 0.1 \\ -0.3 & 0.1 & -0.1 & -0.5 \end{pmatrix},$$

$$D = \begin{pmatrix} 0.2 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.2 \end{pmatrix}, \quad E = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & -0.1 \end{pmatrix},$$

$$F = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & -0.1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad K = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.5 \end{pmatrix}.$$

Using the Matlab LMI Toolbox,we can obtain $\tau_{\max} = 1.2922$. So the system (3) is absolutely stable for $\tau \leq 1.2922$.

References

[1] ZHANG Xianming, HE Yong, WU Min. *Delay-depenent stability for linear neutral type systems with delay* [J]. Acta Automat. Sinica, 2004, **30**(4): 621–628. (in Chinese)

[2] HE Yong. *Absolute stability for Lurie control systems with time-delay stationary components* [J]. Math. Theory Appl. (Changsha), 2002, **22**(2): 55–58. (in Chinese)

[3] Liu Zurun, NIAN Xiaohong. *Absolute stability of control systems with time-delay in both state and control inpute* [J]. J. Syst. Eng., 2000, **15**(1): 103–106. (in Chinese)

[4] He Yong, WU Min. *Absolute sta bility for Lurie control systems with time-varying delays in both state and control input* [J]. Control Theory Appl., 2004, **21**(4): 595–598. (in Chinese)

[5] GAN Zuoxin, GE Weigao. *Absolute stability of multiple nonlinear Lurie control systems with delay* [J]. Math. Appl. (Wuhan), 2000, **13**(4): 1–4. (in Chinese)

[6] XU Bingji, LIAO Xiaoxin. *Criteria for delay-dependent absolute stability of Lurie control systems* [J]. Acta Automat. Sinica, 2002, **28**(2): 317–320. (in Chinese)

[7] NIAN Xiaohong, HUANG Limin. *Absolute stability for Lurie control systems with time-delay feedback controllers* [J]. Inform. and Control (Shenyang), 2001, **30**(2): 155–158. (in Chinese)

[8] XU Bingji, LIU Xinzhi. *Absolute stability analysis of neutral type Lurie control ystems* [J]. J. Shandong Univ. Technol. Nat. Sci., 2005, **19**(1): 8–16. (in Chinese)

[9] XU Bingji, SHEN Yi, LIAO Xiaoxin. *LMI approach for absolute stability of neutral type Lurie control systems with time delay in control input* [J]. J. Central China Normal Univ. Natur. Sci., 2005, **39**(2): 198–204. (in Chinese)

[10] LI Hongfei, LUO Xuebo. *Robust absolute stability of a neutral Lurie control system* [J]. Pure Appl. Math. (Xi'an), 2004, **20**(2): 166–172. (in Chinese)

[11] HE Yong, WU Min, SHE Jinhua. et al. *Delay-dependent robust stability criteria for uncertain neutral systems with mixed delays* [J]. Systems Control Lett., 2004, **51**(1): 57–65.

[12] NIAN Xiaohong, LU Zhensu. *Absolute stability of neutral type Lurie indirect control systems* [J]. J. Lanzhou Univ. Nat. Sci., 1999, **35**(2): 64–68.

[13] BOYD S, ELGHAOUI L, FERON E. et al. *Linear Matrix Inequalities in Systems and Conyrol* [M]. SIMA, Philadelphia, 1994.

[14] XIE Lihua. *Output feedback H_∞ control of systems with parameter uncertainty* [J]. Internat. J. Control, 1996, **63**(4): 741–750.