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On Extension of 1-Lipschitz Mappings between Two Unit Spheres of $l^p(\Gamma)$ Type Spaces $(1 < p < \infty)$

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Abstract Let T be a mapping from the unit sphere $S[l^p(\Gamma)]$ into $S[l^p(\Delta)]$ of two atomic AL^p-spaces. We prove that if T is a 1-Lipschitz mapping such that $-T[S[l^p(\Gamma)]] \subset T[S[l^p(\Gamma)]]$, then T can be linearly isometrically extended to the whole space for $p > 2$; if T is injective and the inverse mapping T^{-1} is a 1-Lipschitz mapping, then T can be extended to be a linear isometry from $l^p(\Gamma)$ into $l^p(\triangle)$ for $1 < p \leq 2$.

Keywords 1-Lipschitz mapping; $l^p(\Gamma)$ type space; isometric extension.

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1. Introduction

In [1] the author found "a condition under which an into-isometry T between unit spheres can be linearly isometrically extended" (for atomic AL^p-spaces, $0 < p < \infty$, $p \neq 2$). Under the same assumption as in [1], [2] presented a short and simple proof for the isometric extension theorem of atomic AL^p-spaces, i.e., $l^p(\Gamma)$-type space on index set Γ, $l^p(\Gamma) = \{x = (x_\gamma) | x : \Gamma \longrightarrow \mathbb{R}, \| x \|^p = \sum_{\gamma \in \Gamma} |x_\gamma|^p\}$ $(p > 1, p \neq 2)$. Ding[3] discussed the extension problem on Hilbert spaces E and F, who proved that for any 1-Lipschitz mapping T from $S(E)$ into $S(F)$, if $-T[S(E)] \subset T[S(E)]$, then T can be extended to a real linear isometry of the whole space E. In particular, every isometric mapping from $S(E)$ into $S(F)$ can be extended to a real linear isometric mapping from E into F.

In this paper, we study the linear isometric extension of mappings between two unit spheres of atomic AL^p-spaces $(1 < p < \infty)$. We obtain that for any 1-Lipschitz mapping T from the unit sphere $S[l^p(\Gamma)]$ of $l^p(\Gamma)$ into the unit sphere of $l^p(\Delta)$ $(2 < p < \infty)$, if $-T[S[l^p(\Gamma)]] \subset T[S[l^p(\Gamma)]]$, then T can be extended to be a real linear isometry from $l^p(\Gamma)$ into $l^p(\triangle)$; for any mapping T from $S[l^p(\Gamma)]$ into $S[l^p(\triangle)]$ $(1 < p \leq 2)$, if T is injective and the inverse mapping T^{-1} is a 1-Lipschitz mapping, then T can be extended to real linear isometry on the whole $l^p(\Gamma)$. In particular, any isometric mapping from $S[l^p(\Gamma)]$ into $S[l^p(\triangle)]$ $(1 < p < \infty, p \neq 2)$ can be extended to be a real linear isometry from $l^p(\Gamma)$ into $l^p(\triangle)$.The main-idea of the proofs in this paper is from the paper

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[3] and [5].

2. Some important lemmas

Definition 2.1 *Let E and F be two normed spaces, the set $U \subset E$ and $V \subset F$. A mapping $T : U \longrightarrow V$ is said to be a Lipschitz mapping for the constant $k > 0$, if $\| T(x) - T(y) \| \leq k \| x - y \|$ for all $x, y \in U$. When $k = 1$, the Lipshitz mapping is called a 1-Lipshitz mapping.*

We first recall the following well-known inequalities.

Lemma 2.1[4] *Suppose that ξ and η be two real numbers. Then*

1) $|\xi + \eta|^p + |\xi - \eta|^p \geq 2\ (|\xi|^p + |\eta|^p)$ for every real number p satisfying that $p > 2$,

2) $|\xi + \eta|^p + |\xi - \eta|^p \leq 2\ (|\xi|^p + |\eta|^p)$ for every real number p satisfying that $1 \leq p < 2$.

Moreover, equality of signs in the above inequalities can only hold if $\xi = 0$ or $\eta = 0$.

Corollary 2.1 *If $x, y, \in l^p(\Gamma)$, then*

$$\| x + y \|^p + \| x - y \|^p \leq 2\ (\| x \|^p + \| y \|^p), \quad 1 \leq p < 2;$$

$$\| x + y \|^p + \| x - y \|^p \geq 2\ (\| x \|^p + \| y \|^p), \quad 2 < p < \infty.$$

Proof Let $x = (x_\gamma)$ and $y = (y_\gamma)$. When $1 \leq p < 2$, by Lemma 2.1(2), we have

$$\begin{aligned} \| x + y \|^p + \| x - y \|^p &= \sum_\gamma |x_\gamma + y_\gamma|^p + \sum_\gamma |x_\gamma - y_\gamma|^p \\ &= \sum_\gamma (|x_\gamma + y_\gamma|^p + |x_\gamma - y_\gamma|^p) \\ &\leq \sum_\gamma 2(|x_\gamma|^p + |y_\gamma|^p) \\ &= 2(\| x \|^p + \| y \|^p). \end{aligned}$$

When $2 < p < \infty$, by Lemma 2.1(1), we have $\| x + y \|^p + \| x - y \|^p \geq 2\ (\| x \|^p + \| y \|^p)$.

Corollary 2.2 *If $x, y, \in l^p(\Gamma)$ $(1 \leq p < \infty, p \neq 2)$, then*

$$(\operatorname{supp} x) \bigcap (\operatorname{supp} y) = \varnothing \Longleftrightarrow \| x + y \|^p + \| x - y \|^p = 2\ (\| x \|^p + \| y \|^p),$$

(here, $\operatorname{supp} x = \{\gamma | x(\gamma) \neq 0, \gamma \in \Gamma\}$).

Lemma 2.2 *Let T be an injection from $S(E)$ into $S(F)$ (E and F are two normed spaces) such that the inverse mapping T^{-1} is a 1-Lipshitz mapping, i.e., let T satisfy $\| x - y \| \leq \| Tx - Ty \|$ for all $x, y \in S(E)$. If F is strictly convex, then $T(-x) = -T(x)$, $\forall x \in S(E)$.*

Proof For any $x \in S(F)$, we have

$$2 = \| 2x \| = \| x - (-x) \| \leq \| T(x) - T(-x) \| \leq 2.$$

So, $\| T(x) - T(-x) \| = 2$. Since F is strictly convex, $T(-x) = -T(x)$.

Lemma 2.3 *Let T be a mapping from the unit spheres $S[l^p(\Gamma)]$ into $S[l^p(\triangle)]$ (Γ, Δ are two* *index sets). If $\| x-y \| \leq \| Tx-Ty \|$ for all x,y $\in S[l^p(\Gamma)]$, then for any $x,y\in S[l^p(\Gamma)]$,*

$$(\operatorname{supp} x)\bigcap(\operatorname{supp} y)=\varnothing \Longrightarrow (\operatorname{supp} T(x))\bigcap(\operatorname{supp} T(y))=\varnothing,$$

where $1<p<2$.

Proof By the hypothesis of the T, we have $\|x-y\|\leq\|T(x)-T(y)\|$ for all $x,y\in S[l^p(\Gamma)]$. Since $l^p(\triangle)$ $(p>1)$ is strictly convex, by Lemma 2.2, $T(-y)=-T(y)$ and so $\|x+y\|=\|x-(-y)\|\leq \|T(x)-T(-y)\|=\|T(x)+T(y)\|$. Therefore, for any $x,y\in S[l^p(\Gamma)]$ with $(\operatorname{supp} x)\bigcap(\operatorname{supp} y)=\varnothing$, using Corollaries 2.1 and 2.2, we have

$$\begin{aligned}2(\| x \|^p+\| y \|^p)=\| x+y \|^p+\| x-y \|^p&\leq \|T(x)+T(y)\|^p+\|T(x)-T(y)\|^p\\ &\leq 2(\| T(x) \|^p+\| T(y) \|^p)=2(\| x \|^p+\| y \|^p).\end{aligned}$$

Which implies that $\|T(x)+T(y)\|^p+\|T(x)-T(y)\|^p=2(\| T(x) \|^p+\| T(y) \|^p)$.

So, using Corollary 2.2 again, we have $(\operatorname{supp} T(x))\bigcap(\operatorname{supp} T(y))=\varnothing$.

Lemma 2.4[3] *Let T be a 1-Lipschitz mapping from the unit sphere $S(E)$ into $S(F)$ (E and F are two normed spaces). If E is strcictly convex and if $-T[S(E)]\subset T[S(E)]$, then $T(-x)=-T(x)$, $\forall x\in S(E)$.*

Lemma 2.5 *Let T be a 1-Lipschitz mapping from the unit sphere $S[l^p(\Gamma)]$ into $S[l^p(\Delta)]$ $(2<p<\infty)$. If $-T[S[l^p(\Gamma)]]\subset T[S[l^p(\Gamma)]]$, then, for any $x,y\in S[l^p(\Gamma)]$,*

$$(\operatorname{supp} x)\bigcap(\operatorname{supp} y)=\varnothing \Longrightarrow (\operatorname{supp} T(x))\bigcap(\operatorname{supp} T(y))=\varnothing.$$

Proof For any $x,y\in S[l^p(\Gamma)]$, we have $\| T(x)-T(y) \|\leq\| x-y \|$, by Lemma 2.4, $\| T(x)+T(y) \|=\| T(x)-T(-y) \|\leq\| x-(-y) \|=\| x+y \|$ (notice that $l^p(\Gamma)$ $(p>1)$ is strictly convex).

Hence, if $(\operatorname{supp} x)\bigcap(\operatorname{supp} y)=\varnothing$, by Corollaries 2.1 and 2.2,

$$\begin{aligned}&2(\| T(x) \|^p+\| T(y) \|^p)\leq\| T(x)+T(y) \|^p+\| T(x)-T(y) \|^p\leq \|x+y\|^p+\|x-y\|^p\\ &=2(\| x \|^p+\| y \|^p)=2(\| T(x) \|^p+\| T(y) \|^p),\end{aligned}$$

which implies that

$$\| T(x)+T(y) \|^p+\| T(x)-T(y) \|^p=2(\| T(x) \|^p+\| T(y) \|^p).$$

So, using Corollary 2.2 again, we obtain $(\operatorname{supp} T(x))\bigcap(\operatorname{supp} T(y))=\varnothing$.

3. The main results

Now we derive the isometric extension theorem for the 1-Lipschitz mapping between two unit spheres of atomic AL^p-spaces $(1<p<\infty)$.

Theorem 3.1 *Let T be a mapping from the unit spheres $S[l^p(\Gamma)]$ into $S[l^p(\triangle)]$ $(1<p<2)$. If $\| x-y \|\leq\| Tx-Ty \|$ for all x,y $\in S[l^p(\Gamma)]$, then we have*

$$T(x)=\sum_{\gamma\in\Gamma}\xi_\gamma T(e_\gamma),\quad \forall x=\sum_{\gamma\in\Gamma}\xi_\gamma e_\gamma\in S[l^p(\Gamma)].$$

Furthermore, the map

$$V(x) = \sum_{\gamma\in\Gamma} \lambda_\gamma T(e_\gamma), \quad \forall x = \sum_{\gamma\in\Gamma} \lambda_\gamma e_\gamma \in l^p(\Gamma),$$

is a real linear isometric extension of T *(here,* $e_\gamma = \{\xi_{\gamma'} | \xi_\gamma = 1, \xi_{\gamma'} = 0, \gamma' \neq \gamma, \gamma' \in \Gamma\}$*).*

Proof For any $x = \sum_{\gamma\in\Gamma} \xi_\gamma e_\gamma \in S[l^p(\Gamma)]$, when $\xi_{\gamma_1} \neq 0$, we have

$$\begin{aligned}
&\| x + \frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|} e_{\gamma_1} \|^p = \| \sum_{\gamma\in\Gamma} \xi_\gamma e_\gamma + \frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|} e_{\gamma_1} \|^p \\
&= \| \sum_{\gamma\neq\gamma_1} \xi_\gamma e_\gamma + (\xi_{\gamma_1} + \frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|}) e_{\gamma_1} \|^p = \sum_{\gamma\neq\gamma_1} |\xi_\gamma|^p + |\xi_{\gamma_1} + \frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|}|^p \\
&= 1 - |\xi_{\gamma_1}|^p + (1 + |\xi_{\gamma_1}|)^p. \qquad (1)
\end{aligned}$$

By Lemmas 2.2 and 2.3, $T(-e_\gamma) = -T(e_\gamma)$, $\forall \gamma \in \Gamma$, $(\operatorname{supp} T(e_\gamma)) \bigcap (\operatorname{supp} T(e_{\gamma'})) = \varnothing$, $\gamma \neq \gamma'$, γ, $\gamma' \in \Gamma$. Hence, we can put

$$T(x) = \Sigma_{\gamma\in\Gamma} T(x)|_{\operatorname{supp} T(e_\gamma)} + y \text{ (here } y = T(x)|_{\Delta \setminus \bigcup_{\gamma\in\Gamma} \operatorname{supp} T(e_\gamma)}). \qquad (2)$$

Thus, we have

$$\begin{aligned}
&\| T(x) + T(\frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|} e_{\gamma_1}) \|^p = 1 - \| T(x)|_{\operatorname{supp} T(e_{\gamma_1})} \|^p + \| T(x) |_{\operatorname{supp} T(e_{\gamma_1})} + T(\frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|} e_{\gamma_1}) \|^p \\
&\leq 1 - \| T(x)|_{\operatorname{supp} T(e_{\gamma_1})} \|^p + (\| T(x)|_{\operatorname{supp} T(e_{\gamma_1})} \| + \| T(\frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|} e_{\gamma_1}) \|)^p \\
&= 1 - \| T(x) |_{\operatorname{supp} T(e_{\gamma_1})} \|^p + (1 + \| T(x) |_{\operatorname{supp} T(e_{\gamma_1})} \|)^p. \qquad (3)
\end{aligned}$$

Noticing the assumption of T and Lemma 2.2, we obtain

$$1 - |\xi_{\gamma_1}|^p + (1 + |\xi_{\gamma_1}|)^p \leq 1 - \| T(x) |_{\operatorname{supp} T(e_{\gamma_1})} \|^p + (1 + \| T(x) |_{\operatorname{supp} T(e_{\gamma_1})} \|)^p \qquad (4)$$

i.e.,

$$(1 + |\xi_{\gamma_1}|)^p - |\xi_{\gamma_1}|^p \leq (1 + \| T(x) |_{\operatorname{supp} T(e_{\gamma_1})} \|)^p - \| T(x) |_{\operatorname{supp} T(e_{\gamma_1})} \|^p . \qquad (5)$$

Recall that the function $f(t) = (1+t)^p - t^p$ is strictly increasing in R^+ when $p > 1$. Therefore, by (5), we get

$$|\xi_{\gamma_1}| \leq \| T(x) |_{\operatorname{supp} T(e_{\gamma_1})} \|, \quad \forall \gamma_1 \in \operatorname{supp} x.$$

Moreover, it follows from

$$\begin{aligned}
\sum_{\gamma\in\Gamma} |\xi_\gamma|^p &= \| x \|^p = \| T(x) \|^p = \| \sum_{\gamma\in\Gamma} T(x)|_{\operatorname{supp} T(e_\gamma)} + y \|^p \\
&= \sum_{\gamma\in\Gamma} \| T(x)|_{\operatorname{supp} T(e_\gamma)} \|^p + \| y \|^p
\end{aligned}$$

that $y = 0$ and

$$|\xi_\gamma| = \| T(x) |_{\operatorname{supp} T(e_\gamma)} \|, \quad \forall \gamma \in \Gamma. \qquad (6)$$

Hence, (4) is an equality. By (1), (3) and (4), (3) is also an equality. The equality (3) implies that

$$\| T(x) |_{\operatorname{supp} T(e_{\gamma_1})} + T(\frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|} e_{\gamma_1}) \| = \| T(x) |_{\operatorname{supp} T(e_{\gamma_1})} \| + \| T(\frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|} e_{\gamma_1}) \| .$$

Since $l^p(\Delta)$ $(p > 1)$ is strictly convex,

$$T(x)\,|_{\operatorname{supp} T(e_{\gamma_1})} = |\xi_{\gamma_1}| T(\frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|} e_{\gamma_1}) = \xi_{\gamma_1} T(e_{\gamma_1}), \quad \forall \gamma_1 \in \operatorname{supp} x. \tag{7}$$

From (2), (6) and (7), we find

$$T(x) = \sum_{\gamma \in \Gamma} \xi_\gamma T(e_\gamma), \quad \forall x = \sum_{\gamma \in \Gamma} \xi_\gamma e_\gamma \in S[l^p(\Gamma)].$$

Thus we immediately obtain that the real linear isometric extension of T as follows:

$$V(x) = \sum_{\gamma \in \Gamma} \lambda_\gamma T(e_\gamma), \forall x = \sum_{\gamma \in \Gamma} \lambda_\gamma e_\gamma \in l^p(\Gamma). \qquad \square$$

Theorem 3.2 *Let T be a mapping from the unit spheres $S(E)$ into $S(F)$ (E and F are two Hilbert spaces). If $\| x - y \| \leq \| Tx - Ty \|$ for all $x, y \in S(E)$, then T can be extended to a real linear isometry from E into F.*

Proof Since E and F are Hilbert spaces, by the "parallelogram law", we have

$$\| x + y \|^2 + \| x - y \|^2 = 2(\| x \|^2 + \| y \|^2)$$

and $\| T(x) + T(y) \|^2 + \| T(x) - T(y) \|^2 = 2(\| T(x) \|^2 + \| T(y) \|^2)$, for any $x, y \in S(E)$. By the assumption of T and Lemma 2.2, we have $\| x - y \| \leq \| T(x) - T(y) \|$ and $\| x + y \| \leq \| T(x) + T(y) \|$. So,

$$\begin{aligned} 2(\| x \|^2 + \| y \|^2) &= \| x + y \|^2 + \| x - y \|^2 \leq \|T(x) + T(y)\|^2 + \|T(x) - T(y)\|^2 \\ &= 2(\| T(x) \|^2 + \| T(y) \|^2) = 2(\| x \|^2 + \| y \|^2), \end{aligned}$$

which means that $\| x - y \| = \| T(x) - T(y) \|$, $\forall x, y \in S(E)$, i.e., T is an isometry from $S(E)$ into $S(F)$. Thus, by [3], T can be extended to a real linear isometry from E into F.

Theorem 3.3 *Let T be a 1-Lipschitz mapping from the unit sphere $S[l^p(\Gamma)]$ into $S[l^p(\Delta)]$ $(2 < p < \infty)$. If $-T[S(l^p(\Gamma))] \subset T[S(l^p(\Gamma))]$, then we have $T(x) = \sum_{\gamma \in \Gamma} \xi_\gamma T(e_\gamma)$, $\forall x = \sum_{\gamma \in \Gamma} \xi_\gamma e_\gamma \in S[l^p(\Gamma)]$. Furthermore, the map $V(x) = \sum_{\gamma \in \Gamma} \lambda_\gamma T(e_\gamma), \forall x = \sum_{\gamma \in \Gamma} \lambda_\gamma e_\gamma \in l^p(\Gamma)$ is a real linear isometric extension of T.*

Proof For each $x = \sum_{\gamma \in \Gamma} \xi_\gamma e_\gamma \in S[l^p(\Gamma)]$, from the above assumption and Lemmas 2.4 and 2.5, we can put

$$T(x) = \Sigma_{\gamma \in \Gamma} T(x)|_{\operatorname{supp} T(e_\gamma)} + y \, (y = T(x)|_{\Delta \setminus \bigcup_{\gamma \in \Gamma} \operatorname{supp} T(e_\gamma)}). \tag{8}$$

Similarly to the proof of Theorem 3.1, we have, for each $\gamma_1 \in \operatorname{supp} x$,

$$\begin{aligned} 1 - |\xi_{\gamma_1}|^p + (1 - |\xi_{\gamma_1}|)^p &= \| x - \frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|} e_{\gamma_1} \|^p \geq \| T(x) - T(\frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|} e_{\gamma_1}) \|^p \\ &= 1 - \| T(x)|_{\operatorname{supp} T(e_{\gamma_1})} \|^p + \| T(x)\,|_{\operatorname{supp} T(e_{\gamma_1})} - T(\frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|} e_{\gamma_1}) \|^p \\ &\geq 1 - \| T(x)\,|_{\operatorname{supp} T(e_{\gamma_1})} \|^p + (1 - \| T(x)\,|_{\operatorname{supp} T(e_{\gamma_1})} \|)^p. \end{aligned} \tag{9}$$

Notice that the function $g(t) = (1-t)^p - t^p$ is decreasing in [0,1], so that from the above inequality we can derive

$$|\xi_{\gamma_1}| \leq \| T(x) \,|_{\operatorname{supp} T(e_{\gamma_1})} \|, \quad \forall \gamma_1 \in \operatorname{supp} x.$$

It follows from

$$\sum_{\gamma \in \Gamma} |\xi_\gamma|^p = \| x \|^p = \| T(x) \|^p = \| \sum_{\gamma \in \Gamma} T(x)|_{\operatorname{supp} T(e_\gamma)} + y \|^p = \sum_{\gamma \in \Gamma} \| T(x)|_{\operatorname{supp} T(e_\gamma)} \|^p + \| y \|^p$$

that $y = 0$ and

$$|\xi_\gamma| = \| T(x) \,|_{\operatorname{supp} T(e_\gamma)} \|, \quad \forall \gamma \in \Gamma. \tag{10}$$

Hence, (9) is an equality and so

$$\| T(x) \,|_{\operatorname{supp} T(e_{\gamma_1})} - T(\frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|} e_{\gamma_1}) \| = \| T(\frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|} e_{\gamma_1}) \| - \| T(x) \,|_{\operatorname{supp} T(e_{\gamma_1})} \|, \quad \forall \gamma_1 \in \operatorname{supp} x.$$

Since $l^p(\Delta)$ $(p > 2)$ is strictly convex, we get

$$T(x) \,|_{\operatorname{supp} T(e_{\gamma_1})} = |\xi_{\gamma_1}| T(\frac{\xi_{\gamma_1}}{|\xi_{\gamma_1}|} e_{\gamma_1}) = \xi_{\gamma_1} T(e_{\gamma_1}), \quad \forall \gamma_1 \in \operatorname{supp} x.$$

Together with (8) and (10), we obtain

$$T(x) = \sum_{\gamma \in \Gamma} \xi_\gamma T(e_\gamma), \quad \forall x = \sum_{\gamma \in \Gamma} \xi_\gamma e_\gamma \in S[l^p(\Gamma)].$$

Thus, we immediately obtain the map $V(x) = \sum_{\gamma \in \Gamma} \lambda_\gamma T(e_\gamma)$, $\forall x = \sum_{\gamma \in \Gamma} \lambda_\gamma e_\gamma \in l^p(\Gamma)$ is a real linear isometry from $l^p(\Gamma)$ into $l^p(\Delta)$ that agrees with T on $S[l^p(\Gamma)]$. □

Corollary 3.1 *Let T be an isometry from the unit sphere $S[l^p(\Gamma)]$ into $S[l^p(\Delta)]$ $(p > 1, p \neq 2)$. Then T can be extended to a real linear isometry from $l^p(\Gamma)$ into $l^p(\Delta)$.*

Proof When $1 < p < 2$, the conclusion is obtained directly from Theorem 3.1.

If $p > 2$, noticing that $\| T(x) - T(-x) \| = \| 2x \| = \| T(x) \| + \| T(-x) \|$, $\forall x \in S[l^p(\Gamma)]$, we have $T(-x) = -T(x)$. So, $-T[S(l^p(\Gamma))] \subset T[S(l^p(\Gamma))]$. By Theorem 3.3, the conclusion is valid.

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