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System of Generalized Symmetric Vector Quasi-Equilibrium Problems for Set-Valued Mappings

YAN Yong Juan, LI Sheng Jie
(College of Mathematics and Science, Chongqing University, Chongqing 400044, China)
(E-mail: yjyan0511@163.com; lisj@cqu.edu.cn)

Abstract In this paper, a system of generalized symmetric vector quasi-equilibrium problems for set-valued mappings is introduced. By using a scalarization method and a fixed-point theorem, the existence result for its solution is established. The main result extends the corresponding results in Fu (J. Math. Anal. Appl. 285, 708–713, 2003) and Zhang, Chen and Li (OR Transactions 10, 24–32, 2006).

Keywords system of generalized symmetric vector quasi-equilibrium problems; nonlinear scalarization function; existence.

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1. Introduction

The system of vector quasi-equilibrium problems is a family of quasi-equilibrium problem for vector-valued bifunction defined on a product set. It provides a unified model for several important problems, such as the system of vector equilibrium problems, the system of variational inequality problems, the system of vector variational inequality problems, the system of vector optimization problems and the Nash equilibrium problem for vector-valued functions. Recently, much attention has been attracted to the system of vector quasi-equilibrium models. Ansari et al.[3] investigated the models and then discussed the Debreu type equilibrium problems for vector-valued functions. Lin et al.[4] introduced the constrained Nash-type equilibrium problem for multimap. Fu[1] introduced the symmetric vector quasi-equilibrium problem in real locally convex Hausdorff topological vector spaces. Zhang et al.[2] obtained existence results for generalized symmetric vector quasi-equilibrium problems for set-valued mappings by using the Kakutani-Fan-Glicksberg fixed point theorem and a nonlinear scalarization function.

As generalizations of the above models, we introduce the system of generalized symmetric vector quasi-equilibrium problems (for short, SGSVQEP), and establish the existence result for its solutions in locally convex Hausdorff topological vector space. Then, we explain that our results are generalizations of the corresponding results in [1] and [2].

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2. The preliminaries

Let $\{X_i\}_{i\in I}$ and $\{Y_j\}_{j\in J}$ be two families of locally convex Hausdorff topological vector spaces with I and J being finite index sets. For each $i\in I$ and $j\in J$, let Z_i and Z'_j be two real Hausdorff topological vector spaces, C_i and D_j be two nonempty subsets of X_i and Y_j, respectively, and $C=\prod_{i\in I}C_i$, $D=\prod_{j\in J}D_j$. Let $M_i:C\to 2^{Z_i}$ and $N'_j:D\to 2^{Z'_j}$ be two set-valued mappings such that for any $x\in C$, $y\in D$, $M_i(x)$ and $N'_j(y)$ are closed, convex and pointed cones with $\operatorname{int}M(x)\neq\emptyset$ and $\operatorname{int}N'(y)\neq\emptyset$.

The (SGSVQEP) is to find $(\bar{x},\bar{y})\in C\times D$ such that for each $i\in I$ and $j\in J$,

$$\bar{x_i}\in S_i(\bar{x},\bar{y}),\ F_i(x_i,\bar{y})-F_i(\bar{x_i},\bar{y})\nsubseteq -\operatorname{int}M_i(\bar{x})\quad \forall x_i\in S_i(\bar{x},\bar{y}),$$

and

$$\bar{y_j}\in T_j(\bar{x},\bar{y}),\ G_j(\bar{x},y_j)-G_j(\bar{x},\bar{y_j})\nsubseteq -\operatorname{int}N'_j(\bar{y})\quad \forall y_j\in T_j(\bar{x},\bar{y}),$$

where $S_i:C\times D\to 2^{C_i}$, $T_j:C\times D\to 2^{D_j}$, $F_i:C_i\times D\to 2^{Z_i}$ and $G_j:C\times D_j\to 2^{Z'_j}$ are four set-valued mappings.

In the following, we recall some notations and results which will be used in the sequel.

Definition 2.1[5] *Let C be a nonempty subset of X and $H:C\to 2^Z$ be a set-valued mapping.*

(i) H is said to be P-convex on C if for any $x_1,x_2\in C$ and $\lambda\in[0,1]$,

$$H(\lambda x_1+(1-\lambda)x_2)\subseteq \lambda H(x_1)+(1-\lambda)H(x_2)-P.$$

(ii) H is said to be P-properly quasiconvex on C if for every $x_1,x_2\in C,\lambda\in[0,1]$ and $z\in H(\lambda x_1+(1-\lambda)x_2)$, there exist $z_1\in H(x_1),z_2\in H(x_2)$ such that $z\in z_1-P$ or $z\in z_2-P$.

(iii) H is said to be P-naturally quasiconvex on C if for every $x_1,x_2\in C,\lambda\in[0,1]$,

$$H(\lambda x_1+(1-\lambda)x_2)\subseteq \operatorname{co}\{H(x_1)\bigcup H(x_2)\}-P,$$

where $\operatorname{co}A$ denotes the convex hull of A.

Definition 2.2[6] *Let X, Y be two topological spaces, $T:X\to 2^Y$ be a set-valued mapping.*

(i) T is said to be upper semi-continuous (u.s.c.) at $x_0\in X$ if for any open set $V\subset X$ with $T(x_0)\subset V$, there is a neighborhood $U(x_0)$ of x_0 such that for each $x\in U(x_0)$, $T(x)\subseteq V$; It is said to be u.s.c. on X if it is u.s.c. at every point $x\in X$;

(ii) T is said to be lower semi-continuous (l.s.c.) at $x_0\in X$ if for any $y_0\in T(x_0)$ and any net $\{x_\alpha\}$ with $x_\alpha\to x_0$, there exists $y_\alpha\in T(x_\alpha)$ such that $y_\alpha\to y_0$; It is said to be l.s.c. on X if it is l.s.c. at every point $x\in X$;

(iii) T is said to be continuous at $x_0\in X$ if it is both u.s.c. and l.s.c. at x_0; It is said to be continuous on X if it is continuous at every point $x\in X$.

Definition 2.3[7] *Let X and Y be two locally convex Hausdorff topological vector spaces, and $C:X\to 2^Y$ be a set-valued mapping such that for any $x\in X$, $C(x)$ is a proper, closed and convex cone in Y with $\operatorname{int}C(x)\neq\emptyset$. Let $e:X\to Y$ be a vector-valued mapping, and for any $x\in X$, $e(x)\in\operatorname{int}C(x)$. The nonlinear scalarization function $\xi_e:X\times Y\to R$ is defined as* *follows:*

$$\xi_e(x,y) = \inf\{\lambda \in R : y \in \lambda e(x) - C(x)\}, \quad \forall (x,y) \in X \times Y.$$

If $e(x) = k^0$ for all $x \in X$, then the nonlinear scalarization function ξ_e reduces to the nonlinear scalarization function ξ_{k^0} introduced by Chen and Yang in [8].

For each $i \in I$, let $P_i \subset Z_i$ be a closed, convex and pointed cone with $\operatorname{int} P_i \neq \emptyset$ and $e_i \in \operatorname{int} P_i$. Suppose that $S_i : C \times D \to 2^{C_i}$ and $F_i : C_i \times D \to 2^{Z_i}$ are continuous set-valued mappings with compact values. Set $\xi_{e_i}(F_i(x,y)) = \bigcup_{u_i \in F_i(x,y)} \xi_{e_i}(u_i)$. We introduce a group of set-valued mappings $A_i : C \times D \to 2^{C_i}$ defined by

$$A_i(x,y) = \{u_i \in S_i(x,y) | \max \xi_{e_i}(F_i(u_i,y)) = \min \bigcup_{\theta_i \in S_i(x,y)} \max \xi_{e_i}(F_i(\theta_i,y))\}.$$

Remark 2.1 $A_i(\cdot,\cdot)$ is well defined. In fact, it follows from the continuity of $\xi_{e_i}(\cdot)$ and Proposition 6 in [6, p10] and Proposition 21 in [6, p119] that $\max \xi_{e_i}(F_i(\cdot,y))$ is upper semicontinuous for each fixed $y \in D$. By the compactness of $S_i(\cdot,\cdot)$ and Proposition 11 in [6, p112], the set $\bigcup_{\theta_i \in S_i(x,y)} \max \xi_{e_i}\ (F_i(\theta_i,y))\}$ is compact. Thus, $A_i(x,y)$ is nonempty for any $(x,y) \in C \times D$.

Theorem 2.1[9] *Let E be a nonempty compact convex subset of a locally convex Hausdorff topological space X. If $G : E \to 2^E$ is upper semicontinuous, and for each $x \in E$, $G(x)$ is a nonempty, closed and convex subset, then there exists an $\bar{x} \in E$ such that $\bar{x} \in G(\bar{x})$.*

3. Main results

Throughout this section, let $\{X_i\}_{i \in I}$ and $\{Y_j\}_{j \in J}$ be families of locally convex Hausdorff topological vector spaces with I and J being finite index sets, for each $i \in I$ and $j \in J$, Z_i and Z'_j be real Hausdorff topological vector space, C_i and D_j be two nonempty, compact convex subsets of X_i and Y_j, respectively, and $C = \prod_{i \in I} C_i$ and $D = \prod_{j \in J} D_j$. Let $M_i : C \to 2^{Z_i}$ and $N'_j : D \to 2^{Z'_j}$ be two set-valued mappings such that for $\forall\ x \in C$, $\forall y \in D$, $M_i(x)$ and $N'_j(y)$ are closed convex and pointed cones with $\operatorname{int} M(x) \neq \emptyset$ and $\operatorname{int} N(y)' \neq \emptyset$.

As Lemmas 3.1 and 3.2 in [2], we can similarly prove the following two lemmas.

Lemma 3.1 *Suppose that the set-valued mappings $F_i : C_i \times D \to 2^{Z_i}$ and $S_i : C_i \times D \to 2^{C_i}$ are continuous with compact values on $C \times D$ for each $i \in I$. Then, $A_i(\cdot,\cdot)$ is closed on $C \times D$.*

Lemma 3.2 *For each $i \in I$, suppose that:*

(i) $F_i : C_i \times D \to 2^{Z_i}$ is upper semicontinuous with compact values on $C \times D$;

(ii) $S_i : C_i \times D \to 2^{C_i}$ is continuous with compact convex values on $C \times D$;

(iii) $F_i(\cdot,y)$ is P_i-naturally quasi-convex for each $y \in D$.

Then, $A_i(x,y)$ is a convex set for every $(x,y) \in C \times D$.

In the sequel, we state our main result.

Theorem 3.1 *Let $P_i = \bigcup_{x \in C} M_i(x)$ and $P'_j = \bigcup_{y \in D} N'_j(y)$ be closed convex and pointed cones for each $i \in I$ and $j \in J$, and $e_i \in \operatorname{int} P_i$, $e'_j \in \operatorname{int} P'_j$. Suppose that the following conditions hold:*

(i) $S_i : C \times D \to 2^{C_i}$ *and* $T_j : C \times D \to 2^{D_j}$ *are continuous with compact convex values on* $C \times D$ *for each* $i \in I$ *and* $j \in J$*;*

(ii) $F_i : C_i \times D \to 2^{Z_i}$ *and* $G_j : C \times D_j \to 2^{Z'_j}$ *are continuous with compact values on* $C \times D$ *for each* $i \in I$ *and* $j \in J$*;*

(iii) $F_i(\cdot, y)$ *is* P_i*-naturally quasi-convex for each* $y \in D$ *and* $G_j(x, \cdot)$ *is* P'_j*-naturally quasi-convex for each fixed* $x \in C$*.*

Then, there exists an $(\bar{x}, \bar{y}) \in C \times D$ *with* $\bar{x} = (\bar{x_i})_{i \in I} \in C$, $\bar{y} = (\bar{y_j})_{j \in J} \in D$ *such that*

$$\bar{x_i} \in S_i(\bar{x}, \bar{y}),\ F_i(x_i, \bar{y}) - F_i(\bar{x_i}, \bar{y}) \not\subseteq -\mathrm{int}\, M_i(\bar{x}), \quad \forall x_i \in S_i(\bar{x}, \bar{y}),\ \forall i \in I,$$

$$\bar{y_j} \in T_j(\bar{x}, \bar{y}),\ G_j(\bar{x}, y_j) - G_j(\bar{x}, \bar{y_j}) \not\subseteq -\mathrm{int}\, N'_j(\bar{y}), \quad \forall y_j \in T_j(\bar{x}, \bar{y}),\ \forall j \in J.$$

Proof For each $i \in I$ and $j \in J$, define $A_i : C \times D \to 2^{C_i}$ and $B_j : C \times D \to 2^{D_j}$ by

$$A_i(x, y) = \{u_i \in S_i(x, y) | \max \xi_{e_i}(F_i(u_i, y)) = \min \bigcup_{\theta_i \in S_i(x,y)} \max \xi_{e_i}(F_i(\theta_i, y))\},$$

$$B_j(x, y) = \{v_j \in T_j(x, y) | \max \xi_{e'_j}(G_j(x, v_j)) = \min \bigcup_{\eta_j \in T_j(x,y)} \max \xi_{e'_j}(G_j(x, \eta_j))\}$$

for $(x, y) \in C \times D$.

It follows from Remark 2.1, Lemmas 3.1 and 3.2 that $A_i(x, y)$ and $B_j(x, y)$ are nonempty, convex and closed subsets for every $(x, y) \in C \times D$. Now, we define $\Psi : C \times D \to 2^{C \times D}$ by

$$\Psi(x, y) = \Big(\prod_{i \in I} A_i(x, y)\Big) \times \Big(\prod_{j \in J} B_j(x, y)\Big), \quad \forall (x, y) \in C \times D.$$

Then, $\Psi(X, Y)$ is a nonempty, convex and closed subset of $C \times D$ for each $(x, y) \in C \times D$.

From the fact that $A_i(\cdot, \cdot)$ and $B_j(\cdot, \cdot)$ are closed, we get $\Psi(\cdot, \cdot)$ is closed. Since $\Psi(x, y) \subseteq C \times D$, $C \times D$ is compact, it follows from Corollary 9 in [6, p111] that Ψ is u.s.c.. Then, by Theorem 2.1, there exists a point $(\bar{x}, \bar{y}) \in C \times D$ such that $(\bar{x}, \bar{y}) \in \Psi(\bar{x}, \bar{y})$, that is

$$\bar{x} \in \prod_{i \in I} A_i(\bar{x}, \bar{y}), \quad \bar{y} \in \prod_{j \in J} B_j(\bar{x}, \bar{y}),$$

i.e., for each $i \in I$ and $j \in J$, $\bar{x_i} \in A_i(\bar{x}, \bar{y})$, $\bar{y_j} \in B_j(\bar{x}, \bar{y})$. By the definitions of A_i and B_j, we have

$$\bar{x_i} \in S_i(\bar{x}, \bar{y}),\ \max \xi_{e_i}(F_i(x_i, \bar{y})) \ge \max \xi_{e_i}(F_i(\bar{x_i}, \bar{y})), \quad \forall\ x_i \in S_i(\bar{x}, \bar{y}), \tag{1}$$

$$\bar{y_j} \in T_j(\bar{x}, \bar{y}),\ \max \xi_{e'_j}(G_j(\bar{x}, y_j)) \ge \max \xi_{e'_j}(G_j(\bar{x}, \bar{y_j})), \quad \forall\ y_j \in T_j(\bar{x}, \bar{y}). \tag{2}$$

From (1), we have $\max \xi_{e_i}(F_i(x_i, \bar{y})) \ge \xi_{e_i}(l_i)$ for $\forall\ l_i \in F_i(\bar{x_i}, \bar{y})$. By the compactness of $F_i(x_i, \bar{y})$ and the continuity of $\xi_{e_i}(\cdot)$, there exists an $l_{i_0} \in F_i(x_i, \bar{y})$, such that

$$\xi_{e_i}(l_{i_0}) = \max \xi_{e_i}(F_i(x_i, \bar{y})).$$

Thus, for all $l_i \in F_i(\bar{x_i}, \bar{y})$, there exists $l_{i_0} \in F_i(x_i, \bar{y})$ such that $\xi_{e_i}(l_i) \le \xi_{e_i}(l_{i_0})$. So, we can get $\xi_{e_i}(l_{i0} - l_i) \ge 0$ from the subadditivity of $\xi_{e_i}(\cdot)$. By Lemma 2.1 (v) in [5], we have

$$l_{i_0} - l_i \notin -\mathrm{int}\, P_i,$$

i.e.,

$$l_{i_0} - l_i \notin -\text{int} \bigcup_{x \in C} M_i(x).$$

Furthermore, we have $l_{i0} - l_i \notin -\bigcup_{x\in C} \text{int}\, M_i(x)$. Then, for $\bar{x} \in C$, $l_{i_0} - l_i \notin -\text{int}\, M_i(\bar{x})$. Hence,

$$F_i(x_i, \bar{y}) - F_i(\bar{x_i}, \bar{y}) \nsubseteq -\text{int}\, M_i(\bar{x}).$$

Similarly, from (2), we can get

$$G_j(\bar{x}, y_j) - G_j(\bar{x}, \bar{y_j}) \nsubseteq -\text{int}\, N'_j(\bar{y}).$$

So $(\bar{x}, \bar{y})$ is a solution of (SGSVQEP) and this completes the proof. □

Remark 3.1 If for each $x \in C$, $y \in D$, $M_i(x) \equiv M_i$, $N'_j(y) \equiv N'_j$ in Theorem 3.1, then, it is easy to see that the result of Theorem 3.1 still holds, where M_i and N'_j are closed convex and pointed cones for each $i \in I$ and $j \in J$.

To illustrate Theorem 3.1, we give the following example.

Example 3.1 Let $\{X_i\}_{i\in I}$ and $\{Y_j\}_{j\in J}$ be families of vector spaces with I and J being finite index sets. Let $X_i = R$, $Y_j = R$, $C_i = [0, 1] \subset R$, $D_j = [0, 1] \subset R$, $Z_i = R^2$, $Z'_j = R^2$ for $\forall\ i \in I$, $\forall j \in J$, $C = \prod_{i\in I} C_i$, $D = \prod_{j\in J} D_j$.

$M_i(x)$ is a cone which consists of the rays of the form $\{\lambda(x_i, u)|0 \le u \le 1 - x_i, \lambda \ge 0\}$ and $N'_j(y)$ is a cone which consists of the rays of the form $\{\lambda(y_j, v)|0 \le v \le 1 - y_j, \lambda \ge 0\}$ for each $x \in C$, $y \in D$. Four set-valued mappings $S_i : C \times D \to 2^{C_i}$, $T_j : C \times D \to 2^{D_j}$ and $F_i : C_i \times D \to 2^{Z_i}$, $G_j : C \times D_j \to 2^{Z'_j}$ are defined as follows:

$$S_i(x, y) = [x_i, 1 - x_i], \quad T_j(x, y) = [y_j, 1 - y_j],$$

$$F_i(x_i, y) = \{(x_i m, n)|m^2 + n^2 \le i^2\}, \quad G_j(x, y_j) = \{(m, y_j n)|m^2 + n^2 \le j^2\}.$$

It follows from a direct computation that $\bar{x} = (\overbrace{0, 0, \ldots, 0}^{I})$, $\bar{y} = (\overbrace{0, 0, \ldots, 0}^{J})$, such that

$$\bar{x_i} \in S_i(\bar{x}, \bar{y}),\ F_i(x_i, \bar{y}) - F_i(\bar{x_i}, \bar{y}) \nsubseteq -\text{int}\, M_i(\bar{x}), \quad \forall x_i \in S_i(\bar{x}, \bar{y}),\ \forall i \in I,$$

$$\bar{y_j} \in T_j(\bar{x}, \bar{y}),\ G_j(\bar{x}, y_j) - G_j(\bar{x}, \bar{y_j}) \nsubseteq -\text{int}\, N'_j(\bar{y}), \quad \forall y_j \in T_j(\bar{x}, \bar{y}),\ \forall j \in J.$$

On the other hand, it is easy to verify that conditions (i)–(iii) of Theorem 3.1 hold. Then, the solution of (SGSVQEP) exists by Theorem 3.1.

From Theorem 3.1, we have the following Corollary, which is an extension of Corollary 3.1 in [2] from vector quasi-equilibrium to the system of vector quasi-equilibrium.

Corollary 3.1 *Let $P_i = \bigcup_{x\in C} M_i(x) \subset Z_i$ and $P'_j = \bigcup_{y\in D} N'_j(y) \subset Z'_j$ be closed convex and pointed cones for each $i \in I$ and $j \in J$, $e_i \in \text{int}\, P_i$ and $e'_j \in \text{int}\, P'_j$, respectively. Suppose that:*

(i) $S_i : C \times D \to 2^{C_i}$ and $T_j : C \times D \to 2^{D_j}$ are continuous on $C \times D$, and $S_i(x, y)$ and $T_j(x, y)$ are nonempty, closed convex sets for each $(x, y) \in C \times D$;

(ii) $F_i : C_i \times D \to 2^{Z_i}$ and $G_j : C \times D_j \to 2^{Z'_j}$ are two continuous mappings with compact values on $C \times D$ for each $i \in I$ and $j \in J$;

(iii) $F_i(\cdot, y)$ *is* P_i*-properly quasiconvex for each* $y \in D$ *and* $G_j(x, \cdot)$ *is* P'_j*-properly quasiconvex for each* $x \in C$.

Then, (SGSVQEP) has a solution.

If the index sets I and J are singleton sets in Theorem 3.1, then we have the following result.

Corollary 3.2 *Let* X *and* Y *be real locally convex Hausdorff topological vector spaces,* Z *and* Z' *be real Hausdorff topological vector spaces,* C *and* D *be nonempty, compact convex subsets of* X *and* Y*, respectively. Let* $M : C \to 2^Z$ *and* $N : D \to 2^{Z'}$ *be closed set-valued mappings for each* $x \in C$ *and* $y \in D$*,* $M(x)$ *and* $N'(y)$ *be closed convex and pointed cones with* $\operatorname{int} M(x) \neq \emptyset$ *and* $\operatorname{int} N'(y) \neq \emptyset$*. Let* $P = \bigcup_{x \in C} M(x) \subset Z$ *and* $P' = \bigcup_{y \in D} N'(y) \subset Z'$ *be closed convex and pointed cones,* $e \in \operatorname{int} P$*,* $e' \in \operatorname{int} P'$*. Suppose that the following conditions hold:*

(i) $S : C \times D \to 2^C$ *and* $T : C \times D \to 2^D$ *are continuous with compact convex values on* $C \times D$*;*

(ii) $F, G : C \times D \to 2^Z$ *are continuous with compact values on* $C \times D$*;*

(iii) $F(\cdot, y)$ *is* P*-naturally quasi-convex for each fixed* $y \in D$ *and* $G(x, \cdot)$ *is* P*-naturally quasi-convex for each fixed* $x \in C$*.*

Then, there exists an $(\bar{x}, \bar{y}) \in C \times D$ *such that*

$$\bar{x} \in S(\bar{x}, \bar{y}), \ F(x, \bar{y}) - F(\bar{x}, \bar{y}) \nsubseteq -\operatorname{int} M(\bar{x}), \quad \forall\, x \in S(\bar{x}, \bar{y}),$$

$$\bar{y} \in T(\bar{x}, \bar{y}), \ G(\bar{x}, y) - G(\bar{x}, \bar{y}) \nsubseteq -\operatorname{int} N'(\bar{y}), \quad \forall\, y \in T(\bar{x}, \bar{y}).$$

Remark 3.2 If for each $x \in C$ and $y \in D$, $M(x) \equiv M$ and $N'(y) \equiv N'$, then Corollary 3.2 reduces to Theorem 3.1 in [2].

Corollary 3.3 *In Corollary 3.1, if condition (ii) is replaced by (ii′), then, (SGSVQEP) has a solution, where*

(ii′) $S_i : D \to 2^{C_i}$ *and* $T_j : C \to 2^{D_j}$ *are continuous for* $\forall\, i \in I$*,* $j \in J$*, and* $S_i(y)$*,* $T_j(x)$ *are nonempty closed convex subsets for* $\forall\, x \in C$*,* $y \in D$*.*

Corollary 3.3 is the extension of Corollary 1 in [1], and its proof is similar to that of Corollary 1 in [1].

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