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Permanence and Global Attractivity of a Discrete Semi-Ratio-Dependent Predator-Prey System with Holling IV Type Functional Response

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Abstract In this paper, we investigate a discrete semi-ratio dependent predator-prey system with Holling IV type functional response. For general nonautonomous case, sufficient conditions which ensure the permanence and the global stability of the system are obtained. Meanwhile, we discuss the existence of the positive periodic solution and global stability of the system.

Keywords discrete; periodic solution; permanence; global stability.

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1. Introduction

The aim of this paper is to investigate the dynamic behavior of the following discrete form of (1):

$$\begin{aligned} x(k+1) =& x(k)\exp\Big\{a(k)-b(k)x(k)-\frac{m(k)x(k)y(k)}{(A(k)+x(k))(B(k)+x(k))}\Big\},\\ y(k+1) =& y(k)\exp\Big\{d(k)-e(k)\frac{y(k)}{x(k)}\Big\}, \end{aligned} \tag{1}$$

where $x(k)$ is the density of prey species at kth generation and $y(k)$ is the density of predator species at kth generation. In (1), it has been assumed that the prey grows logistically with growth rate $a(k)$ and carrying capacity $\frac{a(k)}{b(k)}$ in the absence of predation. The predator consumes the prey according to the functional response $\frac{m(k)x^2(k)}{(A(k)+x(k))(B(k)+x(k))}$, and grows logistically with growth rate $d(k)$ and carrying capacity $\frac{x(k)}{e(k)}$ proportional to the population size of prey (or prey abundance). The parameter $e(k)$ is a measure of the food quality that the prey provides for conversion into predator birth.

Recently, many scholars paid attention to the non-autonomous discrete population models.

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Such kind of model could be more appropriate than the continuous one when there are nonoverlapping generations in the population [1–3].

As was pointed out by Berryman [4], the dynamic relationship between predators and their preys has long been and will keep on to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. Several scholars have done their work on the permanence and existence of positive periodic solution about several species discrete predator-prey system, see [5–7] and the references cited therein. However, so far as we know, to this day, still no scholar has done the work on the so-called semi-ratio dependent predator-prey system.

Fan [8] studied the following semi-ratio-dependent predator-prey system:

$$x' = x[a(t) - b(t)x] - c(t,x)y, \quad y' = y[d(t) - e(t)\frac{y}{x}]. \tag{2}$$

For the functional response $c(t,x) = \frac{m(t)x^2}{(A(t)+x)(B(t)+x)}$, we have the following model of R.M. May, also known as the so-called Holling-Tanner predator-prey model [9, 10], which takes the form of

$$x' = x[a(t) - b(t)x] - \frac{m(t)x^2 y}{(A(t)+x)(B(t)+x)}, \quad y' = y[d(t) - e(t)\frac{y}{x}]. \tag{3}$$

Obviously, system (1) is the counterpart of continuous semi-ratio-dependent predator-prey system (3). As far as we know, this is the first discrete semi-ratio-dependent predator-prey system ever considered. The aim of this paper is to investigate the persistence and global stability property of the system (1) by developing the analysis technique of Huo and Li [11] and Chen and Zhou [12].

We say that system (1) is permanent if there are positive constants M and m such that for each positive solution $(x(k), y(k))$ of system (1) there hold

$$m \le \lim_{k\to\infty} \inf x(k) \le \lim_{k\to\infty} \sup x(k) \le M,$$
$$m \le \lim_{k\to\infty} \inf y(k) \le \lim_{k\to\infty} \sup y(k) \le M.$$

Throughout this paper, we assume that $a(k)$, $b(k)$, $m(k)$, $A(k)$, $d(k)$, $e(k)$ are all bounded nonnegative sequences, and use the following notations for any bounded sequence $\{x(n)\}$,

$$x^u = \sup_{n\in N} x(n), \quad x^l = \inf_{n\in N} x(n).$$

For biological reasons, we only consider solution $(x(k), y(k))$ with $x(0) > 0, y(0) > 0$.

Then system (1) has a positive solution $(x(k), y(k))$ passing through $(x(0), y(0))$.

This paper is organized as follows: In Section 2, we give sufficient conditions which guarantee the permanence of the system (1). In Section 3, we obtain sufficient conditions which guarantee the global stability of the positive solution of system (1). As a consequence, for periodic case, we obtain sufficient conditions which ensure the existence of a globally stable positive solution of system (1).

2. Permanence

In this section, we establish a permanence result for system (1).

Lemma 1 *For every solution $(x(k), y(k))$ of system (1), we have*

$$\lim_{k\to+\infty} \sup x(k) \le p, \tag{4}$$

where $p = \frac{1}{b^l}\exp(a^u - 1)$.

Proof To prove (4), we first assume that there exists an $l_0 \in N$ such that $x(l_0+1) \ge x(l_0)$. Then

$$a(l_0) - b(l_0)x(l_0) - \frac{m(l_0)x(l_0)y(l_0)}{(A(l_0)+x(l_0))(B(l_0)+x(l_0))} \ge 0.$$

Hence

$$x(l_0) \le \frac{a(l_0)}{b(l_0)} \le \frac{a^u}{b^l}. \tag{5}$$

By applying the fact $\frac{\exp(x-1)}{x} \ge 1$, it immediately follows that

$$\frac{a^u}{b^l} \le \frac{1}{b^l}\exp(a^u - 1).$$

It follow from (5) that

$$\begin{aligned} x(l_0+1) &= x(l_0)\exp\left\{a(l_0) - b(l_0)x(l_0) - \frac{m(l_0)x(l_0)y(l_0)}{(A(l_0)+x(l_0))(B(l_0)+x(l_0))})\right\} \\ &\le x(l_0)\exp\{a^u - b^l x(l_0)\} \le \frac{1}{b^l}\exp(a^u - 1) = p, \end{aligned}$$

where we have used the fact $\max_{x\in R} x\exp(b - ax) = \frac{\exp(b-1)}{a}$ for $a, b > 0$.

We claim that

$$x(k) \le p \quad \text{for} \quad k \ge l_0.$$

Assume that there exists a $q_0 > l_0$ such that $x(q_0) > p$. Then $q_0 \ge l_0 + 2$. Let $\widetilde{q_0} \ge l_0 + 2$ be the smallest such that $x(\widetilde{q_0}) > p$. Then $x(\widetilde{q_0} - 1) < x(\widetilde{q_0})$. The above argument leads to $x(\widetilde{q_0}) < p$, a contradiction. This proves the claim. □

Now we assume that $x(k+1) < x(k)$ for all $k \in N$. In particular, $\lim_{k\to+\infty} x(k)$ exists, denoted by $\overline{x}$. We claim that $\overline{x} \le \frac{a^u}{b^l}$. Assume $\overline{x} > \frac{a^u}{b^l}$. Taking limit in the first equation in system gives

$$\lim_{k\to\infty}\left(a(k) - b(k)x(k) - \frac{m(k)x(k)y(k)}{(A(k)+x(k))(B(k)+x(k))}\right) = 0,$$

which is a contradiction since:

$$a(k) - b(k)x(k) - \frac{m(k)x(k)y(k)}{(A(k)+x(k))(B(k)+x(k))} \le a(k) - b(k)x(k) \le a^u - b^l\overline{x} < 0$$

for $k \in N$ large enough. This proves the claim.

Lemma 2 *For every solution $(x(k), y(k))$ of system (1), we have*

$$\lim_{k\to\infty} \sup y(k) \le q, \tag{6}$$

where $q = \frac{p}{e^l}\exp\{d^u - 1\}$.

Proof For any $\varepsilon > 0$, according to Lemma 1, there exists a $k_1 \in N, x(k) \le p + \varepsilon$ for all $k \ge k_1$. Thus, by using the second equation in system (1), for $k \ge k_1$, one has

$$y(k+1) \le y(k)\exp\Big\{d(k) - e(k)\frac{y(k)}{p+\varepsilon}\Big\}. \tag{7}$$

To prove (6), we first assume that there exists an $l_0 > k_1$, such that $y(l_0+1) \ge y(l_0)$, then it follows from (7) that

$$d(l_0) - e(l_0)\frac{y(l_0)}{x(l_0)} \ge 0.$$

And so,

$$y(l_0) \le \frac{d(l_0)}{e(l_0)}x(l_0) \le \frac{d(l_0)}{e(l_0)}(p+\varepsilon) \le \frac{d^u}{e^l}(p+\varepsilon). \tag{8}$$

By using (8), one has

$$\begin{aligned} y(l_0+1) &\le y(l_0)\exp\Big\{d(l_0) - e(l_0)\frac{y(l_0)}{x(l_0)}\Big\} \le y(l_0)\exp\Big\{d(l_0) - e(l_0)\frac{y(l_0)}{p+\varepsilon}\Big\} \\ &\le \frac{p+\varepsilon}{e^l}\exp\{d^u - 1\}. \end{aligned}$$

Let

$$q_\varepsilon = \frac{p+\varepsilon}{e^l}\exp\{d^u - 1\}.$$

We claim that $y(k) \le q_\varepsilon$ for $k \ge l_0$. Assume that there exists a $q_1 > l_0$ such that $y(q_1) > q_\varepsilon$. Then $q_1 \ge l_0 + 2$. Let $\widetilde{q_1} \ge l_0 + 2$ be the smallest such that $y(\widetilde{q_1}) > q_\varepsilon$. Then $y(\widetilde{q_1} - 1) < y(\widetilde{q_1})$. The above argument produces that $y(\widetilde{q_1}) < q_\varepsilon$, This proves the claim.

Now assume $y(k+1) < y(k)$ for all $k \ge k_1$. In particular, $\lim_{k\to\infty} y(k)$ exists, denoted by $\overline{y}$. We claim that

$$\overline{y} \le \frac{d^u}{e^l}(p+\varepsilon).$$

Assume

$$\overline{y} > \frac{d^u}{e^l}(p+\varepsilon).$$

Taking limit in the second equation in system (1) gives

$$\lim_{k\to\infty}\Big(d(k) - e(k)\frac{y(k)}{x(k)}\Big) = 0,$$

which is a contradiction, since for $k \ge k_1$,

$$d(k) - e(k)\frac{y(k)}{x(k)} \le d^u - e^l\frac{y(k)}{p+\varepsilon} \le d^u - e^l\frac{\overline{y}}{p+\varepsilon} < 0.$$

This proves the claim. Noting the fact that $\frac{d^u}{e^l}(p+\varepsilon) \le \frac{p+\varepsilon}{e^l}\exp\{d^u - 1\}$ and $\lim_{\varepsilon\to 0} q_\varepsilon = q$. It follows that (6) holds. This completes the proof of Lemma 2. $\square$

Lemma 3 *Assume that*

$$a^l - \frac{m^u q}{(\sqrt{A^l} + \sqrt{B^l})^2} > 0 \tag{9}$$

holds, where q is defined by (6). Then for every solution $(x(k), y(k))$ of system (1), we have

$$\lim_{k\to\infty} \inf x(k) \geq \alpha, \tag{10}$$

where

$$\alpha = \frac{a^l - \frac{m^u q}{(\sqrt{A^l}+\sqrt{B^l})^2}}{b^u} \exp\Big\{a^l - b^u p - \frac{m^u q}{(\sqrt{A^l}+\sqrt{B^l})^2}\Big\}.$$

Proof Condition (9) implies that there exists positive number $\varepsilon > 0$, such that

$$\frac{a^l - \frac{m^u(q+\varepsilon)}{(\sqrt{A^l}+\sqrt{B^l})^2}}{b^u} > 0.$$

For above ε, from Lemmas 1 and 2 we know that there exists k_2 such that

$$x(k) \leq p+\varepsilon, \quad y(k) \leq q+\varepsilon \quad \text{for all} \quad k \geq k_2. \tag{11}$$

To prove (10), We first assume that there exists an $l_0 > k_2$ such that $x(l_0+1) \leq x(l_0)$, which derives

$$a(l_0) - b(l_0)x(l_0) - \frac{m(l_0)x(l_0)y(l_0)}{(A(l_0)+x(l_0))(B(l_0)+x(l_0))} \leq 0.$$

Then, from (11) it follows that

$$x(l_0) \geq \frac{a(l_0) - \frac{m(l_0)x(l_0)(q+\varepsilon)}{(A(l_0)+x(l_0))(B(l_0)+x(l_0))}}{b(l_0)} \geq \frac{a^l - \frac{m^u(q+\varepsilon)}{(\sqrt{A^l}+\sqrt{B^l})^2}}{b^u}. \tag{12}$$

It follows from (12) that

$$\begin{aligned} x(l_0+1) &= x(l_0)\exp\Big\{a(l_0) - b(l_0)x(l_0) - \frac{m(l_0)x(l_0)y(l_0)}{(A(l_0)+x(l_0))(B(l_0)+x(l_0))}\Big\} \\ &\geq x(l_0)\exp\Big\{a^l - b^u x(l_0) - \frac{m^u x(l_0)y(l_0)}{(A(l_0)+x(l_0))(B(l_0)+x(l_0))}\Big\} \\ &\geq \frac{a^l - \frac{m^u(q+\varepsilon)}{(\sqrt{A^l}+\sqrt{B^l})^2}}{b^u} \exp\Big\{a^l - b^u(p+\varepsilon) - \frac{m^u(q+\varepsilon)}{(\sqrt{A^l}+\sqrt{B^l})^2}\Big\} = \alpha_\varepsilon. \end{aligned}$$

We claim that $x(k) \geq \alpha_\varepsilon$ for $k \geq l_0$. Assume that there exists a $q_2 > l_0$, such that $x(q_2) < \alpha_\varepsilon$, then $q_2 \geq l_0+2$. Let $\widetilde{q}_2 \geq l_0+2$ be the smallest integer such that $x(\widetilde{q}_2) < \alpha_\varepsilon$. Then $x(\widetilde{q}_2 - 1) > x(\widetilde{q}_2)$. The above argument produces $x(\widetilde{q}_2) \geq \alpha_\varepsilon$.

Now we assume that $x(k+1) > x(k)$, for all $k \geq k_2$. In particular, $\lim_{k\to\infty} x(k)$ exists, denoted by $\underline{x}$. We claim that

$$\underline{x} \geq \frac{a^l - \dfrac{m^u(q+\varepsilon)}{(\sqrt{A^l}+\sqrt{B^l})^2}}{b^u}.$$

Assume

$$\underline{x} < \frac{a^l - \dfrac{m^u(q+\varepsilon)}{(\sqrt{A^l}+\sqrt{B^l})^2}}{b^u}.$$

Taking limit in the first equation in system (1) gives

$$\lim_{k\to\infty}\Big\{a(k) - b(k)x(k) - \frac{m(k)x(k)y(k)}{(A(k)+x(k))(B(k)+x(k))}\Big\} = 0,$$

which results in a contradiction since

$$\begin{aligned}
&\lim_{k\to\infty}\inf\left\{a(k)-b(k)x(k)-\frac{m(k)x(k)y(k)}{(A(k)+x(k))(B(k)+x(k))}\right\}\\
&\geq a^l-b^u\underline{x}-\frac{m^u(q+\varepsilon)}{(\sqrt{A^l}+\sqrt{B^l})^2}>0.
\end{aligned}$$

This proves the claim. Note that $\frac{a^u}{b^l}\leq p$ implies that

$$\frac{a^l-\frac{m^u(q+\varepsilon)}{(\sqrt{A^l}+\sqrt{B^l})^2}}{b^u}\geq\alpha_\varepsilon.$$

The fact $\lim_{\varepsilon\to0}\alpha_\varepsilon=\alpha$ implies that (10) holds. This completes the proof of Lemma 3. □

Lemma 4 *In addition to (9), assume further that*

$$d^l-\frac{e^uq}{\alpha}<0 \tag{13}$$

holds, where q, α are defined by (6) and (10), respectively. Then for every solution $(x(k),y(k))$ of system (1), we have

$$\lim_{k\to\infty}y(k)\geq\beta,$$

where

$$\beta=\frac{\alpha d^l}{e^u}\exp\{d^u-\frac{e^uq}{\alpha}\}.$$

Proof From (13) one could take $\varepsilon>0$, such that

$$d^l-\frac{e^u(q+\varepsilon)}{(\alpha-\varepsilon)}<0.$$

For above $\varepsilon>0$, according to Lemmas 1–3, there exists a $k_3\geq k_2$ such that for all $k\geq k_3$,

$$x(k)\leq p+\varepsilon,\ y(k)\leq q+\varepsilon,\ x(k)\geq\alpha-\varepsilon.$$

The rest of the proof of Lemma 4 is similar to that of Lemma 3 and we omit the detail here. □

Theorem 1 *Assume that (9) and (13) hold. Then system (1) is permanent.*

It should be noticed that, from the proofs of Lemmas 1-4, we know that under the assumption of Theorem 1, the set $[\alpha,p]\times[\beta,q]$ is an invariant set of system (1).

3. Global stability

On the basis of permanence, we further investigate the stability of system (1) and provide the following sufficient conditions that guarantee the global stability of system (1).

Theorem 2 *Assume that (9) and (13) hold. Assume further that*

$$\begin{aligned}
\lambda=&\max\{|1-b^l\alpha|,|1-b^up|\}+\frac{A^uB^upq+p^3q}{(A^l+\alpha)^2(B^l+\beta)^2}+\\
&\frac{A^uB^upq+A^up^2q+B^up^2q+p^3q}{(A^l+\alpha)^2(B^l+\beta)^2}<1,
\end{aligned}$$

$$\delta = \max\left\{|1 - \frac{e^u q}{\alpha}|, |1 - \frac{e^l \beta}{p}|\right\} + \frac{e^u pq}{\alpha^2} < 1. \tag{14}$$

Then for any two positive solutions $(x(k), y(k))$ and $(\widetilde{x}(k), \widetilde{y}(k))$ of system (1), we have

$$\lim_{k\to\infty} (\tilde{x}(k) - x(k)) = 0, \quad \lim_{k\to\infty} (\tilde{y}(k) - y(k)) = 0. \tag{15}$$

Proof Let

$$x(k) = \tilde{x}(k) \exp u_1(k), \quad y(k) = \tilde{y}(k) \exp u_2(k). \tag{16}$$

The system (1) is equivalent to

$$\begin{aligned} u_1(k+1) =& u_1(k) - b(k)\widetilde{x}(k)(e^{u_1(k)} - 1) - \\ & \frac{(A(k)B(k)\widetilde{x}(k)y(k) - x(k)\widetilde{x}^2(k)\widetilde{y}(k))(e^{u_1(k)} - 1)}{(A(k) + x(k))(B(k) + x(k))(A(k) + \widetilde{x}(k))(B(k) + \widetilde{x}(k))} - \\ & \frac{(A(k)B(k)\widetilde{x}(k)\widetilde{y}(k) + A(k)x(k)\widetilde{x}(k)\widetilde{y}(k))(e^{u_2(k)} - 1)}{(A(k) + x(k))(B(k) + x(k))(A(k) + \widetilde{x}(k))(B(k) + \widetilde{x}(k))} - \\ & \frac{(B(k)\widetilde{x}(k)x(k)\widetilde{y}(k) + x(k)\widetilde{x}^2(k)\widetilde{y}(k))(e^{u_2(k)} - 1)}{(A(k) + x(k))(B(k) + x(k))(A(k) + \widetilde{x}(k))(B(k) + \widetilde{x}(k))}, \\ u_2(k+1) =& u_2(k) - \frac{e(k)}{\widetilde{x}(k)x(k)}[\widetilde{x}(k)\widetilde{y}(k)(e^{u_2(k)} - 1) - \widetilde{x}(k)\widetilde{y}(k)(e^{u_1(k)} - 1)]. \end{aligned} \tag{17}$$

And so,

$$\begin{aligned} u_1(k+1) =& \big(1 - b(k)\widetilde{x}(k)e^{\theta_1(k)u_1(k)}\big)u_1(k) - \\ & \frac{(A(k)B(k)\widetilde{x}(k)y(k) - x(k)\widetilde{x}^2(k)\widetilde{y}(k))e^{\theta_1(k)u_1(k)}}{(A(k) + x(k))(B(k) + x(k))(A(k) + \widetilde{x}(k))(B(k) + \widetilde{x}(k))}u_1(k) - \\ & \frac{(A(k)B(k)\widetilde{x}(k)\widetilde{y}(k) + A(k)x(k)\widetilde{x}(k)\widetilde{y}(k))e^{\theta_2(k)u_2(k)}}{(A(k) + x(k))(B(k) + x(k))(A(k) + \widetilde{x}(k))(B(k) + \widetilde{x}(k))}u_2(k) - \\ & \frac{(B(k)\widetilde{x}(k)x(k)\widetilde{y}(k) + x(k)\widetilde{x}^2(k)\widetilde{y}(k))e^{\theta_2(k)u_2(k)}}{(A(k) + x(k))(B(k) + x(k))(A(k) + \widetilde{x}(k))(B(k) + \widetilde{x}(k))}u_2(k), \\ u_2(k+1) =& \Big(\frac{e(k)\widetilde{y}(k)e^{\theta_1(k)u_1(k)}\widetilde{x}(k)}{x(k)\widetilde{x}(k)}\Big)u_1(k) + \Big(1 - \frac{e(k)\widetilde{y}(k)e^{\theta_2(k)u_2(k)}}{x(k)}\Big)u_2(k), \end{aligned} \tag{18}$$

where $\theta_1(k), \theta_2(k) \in [0, 1]$. To complete the proof, it suffices to show

$$\lim_{k\to\infty} u_1(k) = 0, \quad \lim_{k\to\infty} u_2(k) = 0.$$

In view of (14), we can choose $\varepsilon > 0$ small enough such that

$$\begin{aligned} \lambda^\varepsilon =& \max\{|1 - b^l(\alpha - \varepsilon)|, |1 - b^u(p + \varepsilon)|\} + \frac{A^u B^u(p + \varepsilon)(q + \varepsilon) + (p + \varepsilon)^3(q + \varepsilon)}{(A^l + \alpha - \varepsilon)^2(B^l + \beta - \varepsilon)^2} + \\ & \frac{A^u B^u(p + \varepsilon)(q + \varepsilon) + A^u(p + \varepsilon)^2(q + \varepsilon) + B^u(p + \varepsilon)^2(q + \varepsilon) + (p + \varepsilon)^3(q + \varepsilon)}{(A^l + \alpha - \varepsilon)^2(B^l + \beta - \varepsilon)^2} < 1; \\ \delta^\varepsilon =& \max\{|1 - \frac{e^u(q + \varepsilon)}{(\alpha - \varepsilon)}|, |1 - \frac{e^l(\beta - \varepsilon)}{p + \varepsilon}|\} + \frac{e^u(p + \varepsilon)(q + \varepsilon)}{(\alpha - \varepsilon)^2} < 1. \end{aligned} \tag{19}$$

For above $\varepsilon > 0$, according to Lemmas 1–4, there exists $k^* \in N$ such that

$$\alpha - \varepsilon \le x(k) \le p + \varepsilon,$$

$$\beta-\varepsilon \le y(k) \le q+\varepsilon$$

for all $k \ge k^*$. Note that $\theta_1(k), \theta_2(k) \in [0,1]$ implies that $\widetilde{x}(k)\exp\{\theta_1(k)u_1(k)\}$ lies between $x(k)$ and $\widetilde{x}(k)$, and $\widetilde{y}(k)\exp\{\theta_2(k)u_2(k)$ lies between $y(k)$ and $\widetilde{y}(k)$. From (18), we get

$$\begin{aligned}
|u_1(k+1)| =&\Big(\max\{|1-b^l(\alpha-\varepsilon)|, |1-b^u(p+\varepsilon)|\}+\\
&\frac{A^uB^u(p+\varepsilon)(q+\varepsilon)+(p+\varepsilon)^3(q+\varepsilon)}{(A^l+\alpha-\varepsilon)^2(B^l+\beta-\varepsilon)^2}\Big)|u_1(k)|+\\
&\frac{A^uB^u(p+\varepsilon)(q+\varepsilon)+A^u(p+\varepsilon)^2(q+\varepsilon)}{(A^l+\alpha-\varepsilon)^2(B^l+\beta-\varepsilon)^2}|u_2(k)|+\\
&\frac{B^u(p+\varepsilon)^2(q+\varepsilon)+(p+\varepsilon)^3(q+\varepsilon)}{(A^l+\alpha-\varepsilon)^2(B^l+\beta-\varepsilon)^2}|u_2(k)|,\\
|u_2(k+1)| =&\max\{|1-\frac{e^u(q+\varepsilon)}{(\alpha-\varepsilon)}|, |1-\frac{e^l(\beta-\varepsilon)}{p+\varepsilon}|\}|u_2(k)|+\\
&\frac{e^u(p+\varepsilon)(q+\varepsilon)}{(\alpha-\varepsilon)^2}|u_1(k)|. \qquad (20)
\end{aligned}$$

Let $\gamma=\max\{\lambda^\varepsilon, \delta^\varepsilon\}$. Then $\gamma<1$. In view of (20), for $k \ge k^*$, we get

$$\max\{|u_1(k+1)|, |u_2(k+1)|\} \le \gamma \max\{|u_1(k)|, |u_2(k)|\}.$$

This implies

$$\max\{|u_1(k)|, |u_2(k)|\} \le \gamma^{k-k^*} \max\{|u_1(k^*)|, |u_2(k^*)|\}.$$

Therefore, (15) holds and the proof is completed. □

4. Existence and stability of periodic solution

In this section, we further assume that the coefficients of system (1) satisfies (21).

There exists a positive integer ω such that for $k \in N$,

$$\begin{aligned}
&0<a(k+\omega)=a(k), \qquad 0<b(k+\omega)=b(k),\\
&0<m(k+\omega)=m(k), \quad 0<d(k+\omega)=d(k),\\
&0<e(k+\omega)=e(k), \qquad 0<A(k+\omega)=A(k). \qquad (21)
\end{aligned}$$

Our first result concerns the existence of positive periodic solution of system (1).

Theroem 3 *Assume that (9) and (13) hold. Then system (1) admits at least one positive ω-periodic solution which we denote by $(\widetilde{x}(k), \widetilde{y}(k))$.*

Proof As mentioned at the end of Section 2,

$$D^2=[\alpha,p]\times[\beta,q]$$

is an invariant set of system (1). Thus, we can define a mapping F on D^2 by

$$F(x(0), y(0))=(x(\omega), y(\omega)),$$

for $(x(0), y(0)) \in D^2$. Obviously, F depends continuously on $(x(0), y(0))$. Thus, F is continuous and maps the compact set D^2 into itself. Therefore, F has a fixed point. It is easy to see that the solution $(x(k), y(k))$ passing through this fixed point is an ω-periodic solution of the system (1). This completes the proof of Theorem 3. $\square$

Theorem 4 *Assume that (9), (13) and (14) hold. Then system (1) has a global stable positive ω-periodic solution.*

Proof Under the assumption of Theorem 4, it follows from Theorem 3 that system (1) admits at least one positive ω-periodic solution. Also, Theorem 3 ensures the positive solution to be globally stable. This completes the proof of Theorem 4. $\square$

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