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Left Multiplication Mappings on Operator Spaces

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Abstract Let X be a separable infinite dimensional Banach space and $B(X)$ denote its operator algebra, the algebra of all bounded linear operators $T : X \to X$. Define a left multiplication mapping $L_T : B(X) \to B(X)$ by $L_T(V) = TV,\ V \in B(X)$. We investigate the connections between hypercyclic and chaotic behaviors of the left multiplication mapping L_T on $B(X)$ and that of operator T on X. We obtain that L_T is SOT-hypercyclic if and only if T satisfies the Hypercyclicity Criterion. If we define chaos on $B(X)$ as SOT-hypercyclicity plus SOT-dense subset of periodic points, we also get that L_T is chaotic if and only if T is chaotic in the sense of Devaney.

Keywords Hypercyclicity Criterion; topological transitivity; operator space; chaos.

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1. Introduction

Throughout this paper, X will denote a separable infinite dimensional Banach space, and $B(X)$ will denote its operator algebra, the algebra of all bounded linear operators $T : X \to X$. By $\mathbb{N}$, $\mathbb{Z}^+$ and $\mathbb{C}$ we will refer to the sets of non-negative integers, positive integers, and the complex scalar fields respectively. Naturally there are several useful topologies on $B(X)$, but in this paper we use only two, namely, the topology induced by the operator norm and the strong operator topology. To distinguish the two, we use the convention that when a topological term is used for $B(X)$, it always refers to the topology induced by the operator norm. Otherwise we add the prefix 'SOT' in front of the term with reference to the strong operator topology.

In [1], Chan and Taylor exhibited a countable set of operators on a separable Banach space X that is dense in $B(X)$ relative to the strong operator topology. This means that $B(X)$ is SOT-separable. Thus, we may also investigate the hypercyclicity of operators on $B(X)$.

The organization of this paper is as follows. In Section 2, we recall some defintions and results necessary for the remainder. The main results are established in Section 3. We study the connections between hypercyclic and chaotic behaviors of the left multiplication mapping L_T on $B(X)$ and that of operator T on X.

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2. Preliminaries

For the convenience of the reader, we will give in this section all definitions and results needed for the subsequent section. Recall that the set $\mathrm{Orb}(T,x) = \{T^n x : n \in \mathbb{N}\}$ is called the orbit of x under T.

Definition 2.1 *A continuous linear operator $T : X \to X$ is said to be hypercyclic provided there is a vector $x \in X$ such that the orbit $\mathrm{Orb}(T,x)$ is dense in X.*

Since X is a separable infinite dimensional Banach space, by the Baire Category Theorem we can conclude that T is hypercyclic if and only if T is topologically transitive, i.e., for any two non-empty sets $U, V \subset X$ there exists $n \in \mathbb{N}$ such that $T^n U \cap V \neq \emptyset$. It was shown [2, 3] that any separable Banach space supports such an operator. Kitai [4] and Gethner and Shapiro [5] provided a useful sufficient condition for an operator to be hypercyclic, namely, the Hypercyclicity Criterion.

Definition 2.2 *We say that T satisfies the Hypercyclicity Criterion, provided there exists a dense subset D of X, a sequence (n_k) of non-negative integers, and (not necessarily continuous) mappings $S_{n_k} : D \to X$ such that $T^{n_k} S_{n_k} =$ Identity, $\lim_{n\to\infty} \|T^{n_k} x\| = 0$ and $\lim_{n\to\infty} \|S_{n_k} x\| = 0$ for every vector $x \in D$.*

Definition 2.3 *We say T is to be weakly mixing if $T \oplus T$ is hypercyclic on $X \oplus X$, and mixing if for every pair of non-empty open sets $U, V \subset X$ there exists some $N \in \mathbb{N}$ such that $T^n U \cap V \neq \emptyset$ for all $n > N$.*

Remark 2.4 It was shown by Bès and Peris [6] that T is weakly mixing if and only if T satisfies the Hypercyclicity Criterion. Moreover, T is mixing if and only if T satisfies the Hypercyclicity Criterion with respect to the sequence of all natural numbers. This can follow essentially straightforward from the definition.

Hypercyclicity is the main step to obtain chaos in linear systems. It is well known that linear chaos can only occur on infinite dimensional spaces.

Definition 2.5 *Suppose that $T : X \to X$ is a contionuous linear operator. Then T is chaotic in the sense of Devaney if T is transitive and the periodic points for T are dense in X.*

A point $x \in X$ is periodic for T if there is a positive integer n such that $T^n x = x$. The least such n is called the period of x. In fact, the Devaney's definition of chaos has another condition, i.e., T has sensitive dependence on initial conditions. Recall that T has sensitive dependence on initial conditions if there exists $\delta > 0$ such that for any $x \in X$ and for any neighborhood $B(x)$ of x, there exist $y \in B(x)$ and $n \geq 1$ such that $d(T^n(x), T^n(y)) > \delta$. But it was shown by Banks et.al. [7] that if T has a dense set of periodic points and is transitive, then T must have sensitive dependence on initial conditions.

Next we will give some similar definitions on operator space $B(X)$. It is well known that operator space $B(X)$ with norm topology is not separable, but in [1], Chan and Taylor showed that $B(X)$ is SOT-separable. This opens the door to that we investigate the hypercyclicity on operator space $B(X)$. To do this, we first give some definitions relative to hypercyclicity on $B(X)$.

Definition 2.6 *Let $L : B(X) \to B(X)$ be a continuous linear operator on $B(X)$. The operator L is called SOT-hypercyclic if there exists $P \in B(X)$ such that orbit $\mathrm{Orb}(L, P) = \{P, L(P), L^2(P), \ldots\}$ is SOT-dense in $B(X)$. Such an element P is called an SOT-hypercyclic vector of L.*

Remark 2.7 By the above definition it easily follows that SOT-hypercyclicity of L implies its transitivity, i.e., for any two non-empty SOT-open sets $U, V \subset B(X)$ there exists $n \in \mathbb{N}$ such that $L^n U \cap V \neq \emptyset$.

The following theorem (we will call it SOT-Hypercyclicity Criterion) gives a sufficient condition for a bounded linear mapping $L : B(X) \longrightarrow B(X)$ to be SOT-hypercyclic. This condition is analogous to the Hypercyclicity Criterion that we mentioned above. We omit the proof here because we can adopt the same proof as in Chan [8, Theorem 2.1], which in turn is based on some techniques used by Kitai [4, Theorem 1.4].

Theorem 2.8 *Let $L : B(X) \to B(X)$ be a continuous linear operator on $B(X)$. If there exist a countable SOT-dense set $D \subset B(X)$, a sequence (n_k) of non-negative integers and a sequence S_{n_k} of maps $S_{n_k} : D \to B(X)$ such that for all $P \in D$:*

$$\lim_{k\to\infty} \|L^{n_k} P\| = 0, \quad \lim_{k\to\infty} \|S_{n_k} P\| = 0, \quad \text{and} \quad L^{n_k} S_{n_k} P = P,$$

then L is SOT-hypercyclic on $B(X)$.

Definition 2.9 *An operator L is said to be SOT-weakly mixing if for any four non-empty SOT-open sets $U_1, U_2, V_1, V_2 \subset B(X)$ there exists $n \in \mathbb{N}$ such that $L^n U_1 \cap V_1 \neq \emptyset$ and $L^n U_2 \cap V_2 \neq \emptyset$. L is said to be SOT-mixing if for any two non-empty SOT-open sets $U, V \subset B(X)$ there exists $N \in \mathbb{N}$ such that $L^n U \cap V \neq \emptyset$ for all $n > N$.*

Remark 2.10 Since the operator space $B(X)$ with strong operator topology is not a Baire space, we cannot generalize some results about hypercyclicity on X to the operator space $B(X)$ with strong operator topology. For example, we don't know how the relations are between SOT-Hypercyclicity Criterion and SOT-weak mixing, although we know that there exists the equivalent relation between Hypercyclicity Criterion and weak mixing on Banach X.

3. Main results

In this section we state and prove our main results. To this end, we first give a key lemma, which will be frequently used in the following.

Lemma 3.1 *Let X be a Banach space and $x_1, \ldots, x_n, x'_1, \ldots, x'_n \in X$. If $x_1, \ldots, x_n$ are linearly independent, then there exists a finite rank operator $S \in B(X)$ such that $Sx_i = x'_i$ for all $1 \leq i \leq n$.*

Proof Let Y denote the subspace spanned by $x_1, \ldots, x_n$. Then Y is a finite dimensional subspace of X, espacially, Y is closed in X. Since $x_1, \ldots, x_n$ are linearly independent, we can find bounded linear functionals $f_1, f_2, \ldots, f_n$ on Y such that

$$f_i(x_j) = \delta_{ij}, \quad 1 \leq i, j \leq n,$$

where $\delta_{ij} = 1$ if $i = j$; otherwise, 0. By Hahn Banach Extension Theorem $f_1, f_2, \ldots, f_n$ can be extended to bounded linear functionals $\widetilde{f}_1, \widetilde{f}_2, \ldots, \widetilde{f}_n$ on X and satisfy $\widetilde{f}_i(x_j) = \delta_{ij}$, $1 \leq i, j \leq n$. Define an operator S on X as follows:

$$Sx = \widetilde{f}_1(x)x'_1 + \cdots + \widetilde{f}_n(x)x'_n.$$

It is easy to check that S is a bounded linear operator with finite rank and satisfies $Sx_i = x'_i$, $1 \leq i \leq n$. $\Box$

Next theorem simplifies the proof of Theorem 2 in [1], which shows that $B(X)$ with strong operator topology is separable by exhibiting a specific countable set of finite rank operators.

Theorem 3.2 *If E is a dense subset of X, then there exists a countable SOT-dense subset $D(E)$ of $B(X)$ consisting of only finite rank operators whose range is contained in the span of E.*

Proof Let $U \subset B(X)$ be a non-empty SOT-open set. Then there exists an SOT-basic open set $B(A : x_1, \ldots, x_k; \varepsilon) = \{T \in B(X) : \|(T - A)x_i\| < \varepsilon,\ i = 1, \ldots, k\} \subset U$, where $A \in U$ and $x_1, \ldots, x_k \in X$. Moreover, we may assume that $x_1, \ldots, x_k$ are linearly independent by the definition of SOT-basic open set. Since E is dense in X, we can take a countable subset $E' \subset E$ that is dense in X. Let $D(E')$ denote the set of finite rank operators whose range is contained in the span of E'. It is clear that $D(E') \subset D(E)$. Next we will prove there exists a finite rank operator $S \in D(E')$ such that $S \in B(A : x_1, \ldots, x_k; \varepsilon)$.

Since $E' \subset E$ is dense in X, we can choose $x'_1, x'_2, \ldots, x'_k \in E'$ such that

$$\|x'_i - Ax_i\| < \varepsilon, \tag{1}$$

where ε is as in $B(A : x_1, \ldots, x_k; \varepsilon)$. By Lemma 3.1 we can construct a finite rank operator $S \in B(X)$ such that $Sx_i = x'_i$ for $i = 1, \ldots, k$. Thus, by (1) we have

$$\|(S - A)x_i\| = \|x'_i - Ax_i\| < \varepsilon,$$

for all $i = 1, \ldots, k$. This means that $S \in B(A : x_1, \ldots, x_k; \varepsilon)$. Moreover, since E' is countable, the collection of all finite linear independent subsets of E' is a countable collection. It follows that $D(E')$ consisting of only finite rank operators whose range is contained in the span of E' is countable. Thus, we complete the whole proof. $\Box$

Definition 3.3 *Corresponding to any operator $T \in B(X)$, we define a left multiplication mapping $L_T : B(X) \to B(X)$ by $L_T(P) = TP$, $P \in B(X)$.*

Chan and Taylor obtained the result that if T satisfies the Hypercyclicity Criterion, then L_T is SOT-hypercyclic ([1, Corollary 6]. In fact, we can prove that the converse of this result is also correct.

Proposition 3.4 *Let X be a separable infinite dimensional Banach space and $T \in B(X)$. If left multiplication operator L_T is SOT-hypercyclic, then T satisfies Hypercyclicity Criterion.*

Proof Take any four non-empty sets $U_i \subset X, i = 1, 2, 3, 4$. Then there exist $x_i \in U_i$ and $\delta > 0$ such that $B(x_i, \delta) = \{x \in X : \|x - x_i\| < \delta\} \subset U_i$, $i = 1, 2, 3, 4$. To prove that T satisfies the Hypercyclicity Criterion, by Remark 2.4 it suffices to prove there exists $n \in \mathbb{N}$ such that

$$T^n B(x_1, \delta) \cap B(x_3, \delta) \neq \emptyset, \quad T^n B(x_2, \delta) \cap B(x_4, \delta) \neq \emptyset.$$

To this end, we consider the following two cases.

Case 1 Suppose that x_1, x_2 are linearly independent. By Lemma 3.1 there exists an operator $S \in B(X)$ satisfying

$$Sx_1 = x_3, \quad Sx_2 = x_4. \tag{2}$$

Consider the SOT-basic open sets

$$B(I : x_1, x_2; \delta) = \{T \in B(X) : \|(T - I)x_i\| < \delta, i = 1, 2\},$$

$$B(S : x_1, x_2; \delta) = \{T \in B(X) : \|(T - S)x_i\| < \delta, i = 1, 2\}.$$

Since L_T is SOT-hypercyclic, by Remark 2.7 there exists $n \in \mathbb{N}$ such that

$$L_T^n B(I : x_1, x_2; \delta) \cap B(S : x_1, x_2; \delta) \neq \emptyset.$$

This means there exists $S' \in B(I : x_1, x_2; \delta)$ such that $L_T^n S' \in B(S : x_1, x_2; \delta)$. So we have

$$\|L_T^n S' x_i - Sx_i\| < \delta, \quad \text{for } i = 1, 2. \tag{3}$$

By $S' \in B(I : x_1, x_2; \delta)$, it follows $\|S' x_i - x_i\| < \delta$ for $i = 1, 2$. This means

$$S' x_i \in B(x_i, \delta), \quad \text{for } i = 1, 2. \tag{4}$$

Thus by (2), (3) and (4), we obtain

$$T^n B(x_1, \delta) \cap B(x_3, \delta) \neq \emptyset, \quad T^n B(x_2, \delta) \cap B(x_4, \delta) \neq \emptyset.$$

Case 2 Suppose that x_1, x_2 are linearly dependent. Take $x_2' \in B(x_2, \delta/2)$ such that x_1, x_2' are linearly independent. Using x_1, x_2' we can construct an operator $S \in B(X)$ as in Case 1 with $S(x_1) = x_3$ and $S(x_2') = x_4$. Consider the SOT-basic open sets $B(I : x_1, x_2'; \delta/2)$ and $B(S : x_1, x_2'; \delta/2)$. Since L_T is hypercyclic, there exists $n \in \mathbb{N}$ such that $L_T^n B(I : x_1, x_2'; \delta/2) \cap B(S : x_1, x_2'; \delta/2) \neq \emptyset$. This means there exists $S' \in B(I : x_1, x_2'; \delta/2)$ such that $L_T^n S' \in B(S : x_1, x_2'; \delta/2)$. Thus,

$$\|L_T^n S' x_1 - Sx_1\| < \delta/2, \quad \|L_T^n S' x_2' - Sx_2'\| < \delta/2. \tag{5}$$

By $S' \in B(I : x_1, x_2'; \delta/2)$, we have $S' x_1 \in B(x_1, \delta/2)$ and $S' x_2' \in B(x_2', \delta/2)$. By $S' x_2' \in B(x_2', \delta/2)$ and $x_2' \in B(x_2, \delta/2)$ we get $\|S' x_2' - x_2\| \leq \|S' x_2' - x_2'\| + \|x_2' - x_2\| \leq \delta$. So we have

$$S' x_1 \in B(x_1, \delta), \quad S' x_2' \in B(x_2, \delta). \tag{6}$$

Therefore, by (5),(6) and $S(x_1) = x_3$ and $S(x_2') = x_4$ we obtain

$$T^n B(x_1, \delta) \cap B(x_3, \delta) \neq \emptyset, \quad T^n B(x_2, \delta) \cap B(x_4, \delta) \neq \emptyset.$$

Summing up Case 1 and Case 2, we complete the proof. □

By Corollary 6 in [1] and the Proposition 3.3 above, one can easily get the following result.

Theorem 3.5 *L_T is SOT-hypercyclic if and only if T satisfies the Hypercyclicity Criterion.*

Next let us consider the relations between chaotic behavior occurring in operator space $B(X)$ and that occurring in its original space X. We first build the connection between the set of periodic points of L_T and that of T.

Proposition 3.6 *L_T has SOT-dense subset of periodic points in $B(X)$ if and only if T has dense subset of periodic points in X.*

Proof "⇒" Suppose that L_T has SOT-dense subset of periodic points in $B(X)$. Take any point $x_0 \in X$ and $\varepsilon > 0$. To prove that T has dense subset of periodic points in X, it is enough to prove there exists a periodic point for T in the open ball $B(x_0, \varepsilon) = \{x \in X : \|x - x_0\| < \varepsilon\}$.

For the point x_0 and ε given above, take the strong operator topology basic open set

$$B(I : x_0; \varepsilon) = \{S \in B(X) : \|(S - I)x_0\| < \varepsilon\}.$$

Since L_T has dense subset of periodic points in $B(X)$, there exist $A \in B(I : x_0; \varepsilon)$ and $N \in \mathbb{N}$ such that $L_T^N A = A$. This means that $L_T^N Ax = Ax$ for all $x \in X$, especially for x_0, i.e., $L_T^N Ax_0 = Ax_0$. Thus, we have

$$T^N Ax_0 = Ax_0. \tag{7}$$

Since $A \in B(I : x_0; \varepsilon)$ it follows that $Ax_0 \in B(x_0, \varepsilon)$. By this, together with equation (7), we conclude that there is a periodic point Ax_0 for T in $B(x_0, \varepsilon)$.

"⇐" Suppose that T has dense subset of periodic points in X. Let $U \subset B(X)$ be a non-empty SOT-open set. Then there exists an SOT-basic open set $B(A : x_1, \ldots, x_k; \varepsilon) = \{S \in B(X) : \|(S - A)x_i\| < \varepsilon, \ i = 1, \ldots, k\} \subset U$, where $A \in U$, $\varepsilon > 0$ and $x_1, \ldots, x_k \in X$. By the definition of SOT-basic open set of $B(X)$, we may assume that $x_1, \ldots, x_k$ are linearly independent. To prove that L_T has SOT-dense subset of periodic points in $B(X)$, it suffices to prove there exist a bounded linear operator $S \in B(A : x_1, \ldots, x_k; \varepsilon)$ and an $N \in \mathbb{N}$ such that $L_T^N S = S$.

With above $x_1, \ldots, x_k$ and ε, let $B(Ax_i, \varepsilon) = \{x \in X : \|x - Ax_i\| < \varepsilon\}$ $(i = 1, \ldots, k)$ denote the open ball centered at Ax_i with radius ε in X. Since T has dense subset of periodic points in X, there exist $x_i' \in B(Ax_i, \varepsilon)$ and $n_i \in \mathbb{N}$ such that $T^{n_i} x_i' = x_i'$ for $i = 1, \ldots, k$. Let N denote the least common multiple of $n_1, \ldots, n_k$. Then we have

$$T^N x_i' = x_i', \quad \text{for all} \ \ i = 1, \ldots, k. \tag{8}$$

Since $x_1, \ldots, x_k \in X$ are linearly independent, using Lemma 3.1 we can obtain an operator $S \in B(X)$ satisfying the following condition:

$$Sx_i = x_i', \ \ \text{for all } i = 1, \ldots, k. \tag{9}$$

By (8) and (9) we have $T^N Sx_i = Sx_i$ for all $i = 1, \dots, k$. By the construction of S it easily follows that $T^N Sx = Sx$ for all $x \in X$, i.e., $L_T^N S = S$. Moreover, by $x_i' \in B(Ax_i, \varepsilon)$ we have $Sx_i \in B(Ax_i, \varepsilon)$, that is, $\|(S - A)x_i\| < \varepsilon$ for all $i = 1, \dots, k$. This means that $S \in B(A : x_1, \dots, x_k; \varepsilon)$. Thus, we complete the proof of sufficient condition. □

In [6], Bès and Peris showed the following result.

Proposition 3.7 *Every chaotic operator on a Banach space X satisfies the Hypercyclicity Criterion.*

If we define chaos on $B(X)$ as SOT-hypercyclicity plus dense subset of periodic points, then by Theorem 3.5 and Proposition 3.6, together with Proposition 3.7, we immediately obtain the following theorem.

Theorem 3.8 *Let $T \in B(X)$. Then T is chaotic in the sense of Devaney if and only if L_T is chaotic.*

Proposition 3.9 *Suppose that L_T is SOT-mixing. Then $T \in B(X)$ is mixing.*

Proof Suppose that L_T is mixing. Take any two points $x_1, x_2 \in X$ and let $B(x_1, \delta), B(x_2, \delta)$ denote the open balls centered at x_1, x_2, respectively, with radius $\delta > 0$. To prove that T is mixing, it suffices to prove there exists $N \in \mathbb{N}$ such that $T^n B(x_1, \delta) \cap B(x_2, \delta) \neq \emptyset$ for all $n > N$.

For the points x_1 and x_2 given above, by Lemma 3.1 we can construct a bounded linear operator $S : X \to X$ such that $S(x_1) = x_2$. Let I be the identity operator of $B(X)$. Using I and S we can obtain two strong operator topology basic open sets

$$B(I : x_1; \delta) = \{T \in B(X) : \|(T - I)x_1\| < \delta\}$$

and

$$B(S : x_1; \delta) = \{T \in B(X) : \|(T - S)x_1\| < \delta\},$$

where $\delta > 0$ is as above. Since L_T is mixing, it follows that there exists $N \in \mathbb{N}$ such that

$$L_T^n B(I : x_1; \delta) \cap B(S : x_1; \delta) \neq \emptyset \tag{10}$$

for all $n > N$. Equation (10) implies there exists a sequence $\{S_j\}_{j\in\mathbb{N}} \subset B(I : x_1; \delta)$ satisfying $L_T^{N+j} S_j \in B(S : x_1; \delta)$ for all $j \in \mathbb{N}$. This means the following equation holds:

$$\|T^{N+j} S_j x_1 - Sx_1\| < \delta \tag{11}$$

for all $j \in \mathbb{N}$. Furthermore, by $\{S_j\}_{j\in\mathbb{N}} \subset B(I : x_1; \delta)$ it follows that $\|(S_j - I)x_1\| < \delta$, i.e.,

$$S_j x_1 \in B(x_1, \delta), \quad \text{for all } j \in \mathbb{N}. \tag{12}$$

By (12), together with (11) and $Sx_1 = x_2$, we conclude that

$$T^{N+j} B(x_1, \delta) \cap B(x_2, \delta) \neq \emptyset$$

for all $j \in \mathbb{N}$. Therefore, T is mixing. □

By Remark 2.4 we know that T is mixing if and only if T satisfies the Hypercyclicity Criterion with respect to the sequence of all natural numbers. But we do not know whether there are some connections between the SOT-Hypercyclicity Criterion and SOT-mixing on operator space $B(X)$. So we cannot judge whether or not the converse of the Theorem 3.9 is correct by using the SOT-Hypercyclicity Criterion. We end this paper with the following question:

Are there some relations between the SOT-Hypercyclicity Criterion and SOT-weak mixing (or SOT-mixing)?

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