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Existence of Weak Solutions to a Class of Semiconductor Equations with Fast Diffusion Term

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Abstract In this paper, we consider the transient drift-diffusion model with fast diffusion term. This problem is not only degenerate but also singular. We present the existence result for the Neumann boundary value problem with general nonlinear diffusivities.

Keywords semiconductor equations; fast diffusion term.

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1. Introduction

The scaled semiconductor drift-diffusion model reads

$$-\nabla\cdot(\nabla\psi)=p-n+C(x), \tag{1.1}$$

$$n_t-\nabla\cdot J_n=r(n,p)(1-np)+g,\quad J_n=(\nabla(n^{\gamma_n})-n\nabla\psi), \tag{1.2}$$

$$p_t+\nabla\cdot J_p=r(n,p)(1-np)+g,\quad -J_p=(\nabla(p^{\gamma_p})+p\nabla\psi), \tag{1.3}$$

with $x\in\Omega\subset R^N$, which denotes the bounded domain occupied by semiconductor crystal. Here the unknowns ψ, n and p denote the electrostatic potential, the electron density, and the hole density, respectively. The J_n represents the electron current, and J_p is the analogously defined physical quantity of the positively charged holes. Additionally the given function $C(x)$ denotes the doping profile (fixed charged background ions) characterizing the semiconductor under consideration, $R(n,p)=r(n,p)(1-np)$ the net recombination-generation rate, and g the laser density. The constants $\gamma_n,\ \gamma_p>0$ are the adiabatic or isothermal (if $\gamma_n=\gamma_p=1$) exponents. The regime $\gamma_n>1$ (or $\gamma_p>1$) describes a slow diffusion process in the electron (hole) density, whereas $0<\gamma_n<1$ (or $0<\gamma_p<1$) is related to fast diffusion.

We consider an insulated semiconductor modeled by the initial-boundary value problem for (1.1)–(1.3) subject to the initial boundary conditions

$$J_n\cdot\eta=J_p\cdot\eta=\nabla\psi\cdot\eta=0,\quad x\in\partial\Omega\times(0,T), \tag{1.4}$$

$$n(\cdot,0)=n_0,\ p(\cdot,0)=p_0,\quad x\in\Omega. \tag{1.5}$$

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A necessary solvability condition for the Poisson equation (1.1) subject to the Neumann boundary conditions for the field in (1.4) is global charge neutrality,

$$\int_\Omega (p - n + C(x))\mathrm{d}x = 0.$$

Since the total numbers of electrons and holes are conserved, it suffices to require the corresponding condition for the initial data,

$$\int_\Omega (p_0 - n_0 + C(x))\mathrm{d}x = 0. \tag{1.6}$$

The standard drift-diffusion model corresponding to $\gamma_n = \gamma_p = 1$ has been mathematically and numerically investigated in many papers [1–4]. Existence and uniqueness of weak solutions have been shown. The standard model can be derived from Boltzmann's equation once assumed that the semiconductor device is in the low injection regime, i.e., for small absolute values of the applied voltage. Recently, the existence analysis of the bipolar drift-diffusion model in the adiabatic case has been studied by many authors [5–9]. But as far as we know, few works are concerned with the existence of solutions for the fast diffusion model. Additionally, in [10] the quasineutral limit (zero-Debye-Length limit) for the fast diffusion case is performed by using the compactness argument and so-called entropy functional. But they did not give the existence results of the problem (1.1)–(1.5). So the existence result in this paper can be regarded as an important supplement to [10].

We make the following assumptions.

(H1) $\Omega \subset R^N (N = 1, 2, 3)$ is bounded and $\partial\Omega \in C^{0,1}$, whose outward normal vector is η;

(H2) $C(x), g(x) \in L^\infty(\Omega)$ and $g(x) \geq 0$ for a.e., $x \in \Omega$;

(H3) $r(n, p)$ is a locally Lipschitz continuous function defined for (n, p) and $0 \leq r(n, p) \leq \bar{r} < \infty$;

(H4) $n_0, p_0 \in L^\infty(\Omega)$ and $n_0, p_0 \geq 0$ a.e., in Ω.

Definition 1 *(ψ, n, p) is called the weak solution to the problem (1.1)–(1.5) if $(\psi, n, p) \in L^\infty(Q_T) \cap L^2(0, T; H^1(\Omega))$, $n_t, p_t \in L^2(0, T; H^{-1}(\Omega)), n|_{t=0} = n_0, p|_{t=0} = p_0$, and there hold*

$$\int_\Omega \nabla\psi \cdot \nabla\phi \mathrm{d}x = \int_\Omega (p - n + C)\phi \mathrm{d}x, \quad \forall t \in (0, T),\ \forall \phi \in H^1(\Omega), \tag{1.7}$$

$$\begin{aligned} &\langle n_t, v\rangle + \int_0^T \int_\Omega (\nabla(n^{\gamma_n}) - n\nabla\psi) \cdot \nabla v \mathrm{d}x\mathrm{d}t \\ &\quad = \int_0^T \int_\Omega [r(n, p)(1 - np) + g]v \mathrm{d}x\mathrm{d}t, \quad \forall v \in L^2(0, T; H^1(\Omega)), \end{aligned} \tag{1.8}$$

$$\begin{aligned} &\langle p_t, v\rangle + \int_0^T \int_\Omega (\nabla(p^{\gamma_p}) + p\nabla\psi) \cdot \nabla v \mathrm{d}x\mathrm{d}t \\ &\quad = \int_0^T \int_\Omega [r(n, p)(1 - np) + g]v \mathrm{d}x\mathrm{d}t, \quad \forall v \in L^2(0, T; H^1(\Omega)). \end{aligned} \tag{1.9}$$

Now we state our main result.

Theorem 1.1 *Under hypotheses (H1)-(H4), there exists at least one weak solution of the* *problem (1.1)–(1.5).*

2. The proof of main result

This section is devoted to the proof of existence of weak solution to the problem (1.1)–(1.5). Our main difficulty in the proof is that the problem (1.1)–(1.5) is degenerate at points where $n, p = +\infty$, and is singular at points where n, $p = 0$. This difficulty leads us to consider a auxiliary regularized problem.

Let

$$s_k = \min\{k, \max\{0, s\}\} \tag{2.1}$$

and $0 < \varepsilon < 1$.

Let us consider first the following auxiliary problem:

$$-\nabla \cdot (\nabla\psi) = p_k - n_k + C(x), \tag{2.2}$$

$$n_t - \nabla \cdot (a(\varepsilon, n, \gamma_n)\nabla n - n\nabla\psi) = r(n_k, p_k)(1 - np_k) + g, \tag{2.3}$$

$$p_t - \nabla \cdot (a(\varepsilon, p, \gamma_p)\nabla p + p\nabla\psi) = r(n_k, p_k)(1 - n_k p) + g, \tag{2.4}$$

subject to initial conditions (1.5) and the following boundary conditions

$$(a(\varepsilon, n, \gamma_n)\nabla n - n\nabla\psi) \cdot \eta = 0, \tag{2.5}$$

$$(a(\varepsilon, p, \gamma_p)\nabla p + p\nabla\psi) \cdot \eta = 0, \tag{2.6}$$

$$\nabla\psi \cdot \eta = 0, \quad x \in \Omega. \tag{2.7}$$

Here k stands for a positive parameter to be chosen later and the function a is defined as

$$a(\varepsilon, u, \gamma) = \gamma(\varepsilon^2 + (u^2)_k)^{\frac{\gamma-1}{2}}.$$

With the use of the Schauder's fixed point theorem [11] we can solve this regularized problem (see [9, Lemma 3.1] for details), and obtain at least one weak solution $(\psi_\varepsilon, n_\varepsilon, p_\varepsilon)$ such that

$$0 \le n_\varepsilon, p_\epsilon \le C(\varepsilon) < \infty, \ \text{ a.e., in } \Omega.$$

Therefore what we need to do is to prove that the limit of $(\psi_\varepsilon, n_\varepsilon, p_\varepsilon)$ is a solution of (1.1)–(1.5). To this end, we need some priori estimates for $(\psi_\varepsilon, n_\varepsilon, p_\varepsilon)$ which are uniform with respect to ε and k. From now on, we denote by C a positive constant depending on T, $|\Omega|$, γ_n, γ_p and the known data but independent of ε and k. For notational simplicity, we omit the index ε at ψ_ε, n_ε, p_ε.

Lemma 2.1 *The solutions of problem (1.5), (2.2)–(2.7) satisfy the estimate*

$$\|n\|_{L^\infty(Q_T)} + \|p\|_{L^\infty(Q_T)} + \|\psi\|_{L^\infty(Q_T)} \le K(T) \tag{2.8}$$

with a constant $K(T)$ not depending on ε and k.

Proof Multiplying (2.3) by n^{2m-1} and integrating the resulting equation over Ω, we have, by integration by parts, that

$$\begin{aligned}
&\frac{1}{2m}\frac{\mathrm{d}}{\mathrm{d}t}\|n(\cdot,t)\|^{2m}_{L^{2m}(\Omega)}+(2m-1)\int_\Omega a(\varepsilon,n,\gamma_n)n^{2m-2}|\nabla n|^2\mathrm{d}x\\
&=\frac{2m-1}{2m}\int_\Omega \nabla\psi\cdot\nabla(n^{2m})+\int_\Omega[r(n_k,p_k)(1-np_k)+g]n^{2m-1}\mathrm{d}x\\
&\le\int_\Omega(p_k-n_k+C(x))n^{2m}\mathrm{d}x+\int_\Omega(\overline{r}+g)n^{2m-1}\mathrm{d}x.
\end{aligned}\tag{2.9}$$

Similarly, we have

$$\begin{aligned}
&\frac{1}{2m}\frac{\mathrm{d}}{\mathrm{d}t}\|p(\cdot,t)\|^{2m}_{L^{2m}(\Omega)}+(2m-1)\int_\Omega a(\varepsilon,p,\gamma_p)p^{2m-2}|\nabla p|^2\mathrm{d}x\\
&\le-\int_\Omega(p_k-n_k+C(x))p^{2m}\mathrm{d}x+\int_\Omega(\overline{r}+g)p^{2m-1}\mathrm{d}x.
\end{aligned}\tag{2.10}$$

Thus, we have

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\|n(\cdot,t)\|^{2m}_{L^{2m}(\Omega)}+\frac{\mathrm{d}}{\mathrm{d}t}\|p(\cdot,t)\|^{2m}_{L^{2m}(\Omega)}+\\
&\quad 2m(2m-1)\int_\Omega a(\varepsilon,n,\gamma_n)n^{2m-2}|\nabla n|^2\mathrm{d}x+2m(2m-1)\int_\Omega a(\varepsilon,p,\gamma_p)p^{2m-2}|\nabla p|^2\mathrm{d}x\\
&\le 2m\int_\Omega(p_k-n_k)(n^{2m}-p^{2m})\mathrm{d}x+2m\int_\Omega C(x)(n^{2m}+p^{2m})\mathrm{d}x+\\
&\quad 2m\int_\Omega(\overline{r}+g)(n^{2m-1}+p^{2m-1})\mathrm{d}x.
\end{aligned}\tag{2.11}$$

Since $(p_k-n_k)(n^{2m}-p^{2m})\le 0$, using Young inequality, we get

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\|n(\cdot,t)\|^{2m}_{L^{2m}(\Omega)}+\frac{\mathrm{d}}{\mathrm{d}t}\|p(\cdot,t)\|^{2m}_{L^{2m}(\Omega)}\\
&\le 2m\|\overline{r}+g\|^{2m}_{L^{2m}(\Omega)}+2m(\|C(x)\|_{L^\infty(\Omega)}+1)(\|n(\cdot,t)\|^{2m}_{L^{2m}(\Omega)}+\|p(\cdot,t)\|^{2m}_{L^{2m}(\Omega)}).
\end{aligned}\tag{2.12}$$

Then the Gronwall inequality leads to

$$\begin{aligned}
&\|n(\cdot,t)\|^{2m}_{L^{2m}(\Omega)}+\|p(\cdot,t)\|^{2m}_{L^{2m}(\Omega)}\\
&\le(2m\|\overline{r}+g\|^{2m}_{L^{2m}(\Omega)}T+\|n_0\|^{2m}_{L^{2m}(\Omega)}+\|p_0\|^{2m}_{L^{2m}(\Omega)})e^{2m(\|C(x)\|_{L^\infty(\Omega)}+1)t}.
\end{aligned}\tag{2.13}$$

Simplifying it and then taking the limit as $m\to\infty$, we obtain

$$\begin{aligned}
&\sup_{0\le t\le T}(\|n(\cdot,t)\|_{L^\infty(\Omega)}+\|p(\cdot,t)\|_{L^\infty(\Omega)})\\
&\le C(\|\overline{r}+g\|_{L^\infty(\Omega)}+\|n_0\|_{L^\infty(\Omega)}+\|p_0\|_{L^\infty(\Omega)})e^{(\|C(x)\|_{L^\infty(\Omega)}+1)T}.
\end{aligned}\tag{2.14}$$

Moreover, the L^∞ estimate for ψ can be obtained from Stampacchia method [12]. □

Corollary 2.1 *Choosing $k\ge K(T)$, we have*

$$n_k=n,\quad a(\varepsilon,n,\gamma_n)=\gamma_n(\varepsilon^2+n^2)^{\frac{\gamma_n-1}{2}},\tag{2.15}$$

$$p_k=p,\quad a(\varepsilon,p,\gamma_p)=\gamma_p(\varepsilon^2+p^2)^{\frac{\gamma_p-1}{2}}.\tag{2.16}$$

Lemma 2.2 *The solutions of problem (1.5), (2.2)–(2.7) satisfy the estimates*

$$\|a^{\frac{1}{2}}(\varepsilon,n,\gamma_n)\nabla n\|_{L^2(Q_T)}\le C,\tag{2.17}$$

$$\|a^{\frac{1}{2}}(\varepsilon,p,\gamma_p)\nabla p\|_{L^2(Q_T)}\le C. \tag{2.18}$$

Proof It follows from (2.11) with $m=1$ that

$$\int_0^T\int_\Omega a(\varepsilon,n,\gamma_n)|\nabla n|^2\mathrm{d}x\mathrm{d}t+\int_0^T\int_\Omega a(\varepsilon,p,\gamma_p)|\nabla p|^2\mathrm{d}x\mathrm{d}t$$
$$\le \|n_0\|^2_{L^2(\Omega)}+\|p_0\|^2_{L^2(\Omega)}+2\|C(x)\|_{L^\infty(\Omega)}\int_0^T\int_\Omega(n^2+p^2)\mathrm{d}x\mathrm{d}t+$$
$$2\int_0^T\int_\Omega(\overline{r}+g)(n+p)\mathrm{d}x$$

which implies (2.17) and (2.18). This completes the proof. □

Lemma 2.3 *The solutions of problem (1.5), (2.2)–(2.7) satisfy the estimates*

$$\|a(\varepsilon,n,\gamma_n)\nabla n\|_{L^2(Q_T)}\le C, \tag{2.19}$$

$$\|a(\varepsilon,p,\gamma_p)\nabla p\|_{L^2(Q_T)}\le C. \tag{2.20}$$

Proof We only prove (2.19). The proof of (2.20) is completely the same as that of (2.19). Let

$$A(n,\gamma_n)=\gamma_n\int_0^n(\varepsilon^2+s^2)^{\frac{\gamma_n-1}{2}}\mathrm{d}x.$$

Then by Lemma 2.2 we conclude from

$$\int_0^T\int_\Omega|\nabla A(n,\gamma_n)|^2\mathrm{d}x\mathrm{d}t\le\gamma_n\varepsilon^{\gamma_n-1}\|a^{\frac{1}{2}}(\varepsilon,n,\gamma_n)\nabla n\|^2_{L^2(Q_T)}\le C(\varepsilon)$$

that the function $A(n,\gamma_n)$ (or $A(p,\gamma_p)$) can be taken as the test function in the (2.2) (or (2.3)).

Using $A(n,\gamma_n)$ and $A(p,\gamma_p)$ as test functions in (2.2) and (2.3) respectively, and employing Hölder inequality, we have

$$\int_0^T\int_\Omega|a(\varepsilon,n,\gamma_n)\nabla n|^2\mathrm{d}x\mathrm{d}t$$
$$=-\int_0^T\int_\Omega n_tA(n,\gamma_n)\mathrm{d}x\mathrm{d}t+\int_0^T\int_\Omega n\nabla\psi\cdot a(\varepsilon,n,\gamma_n)\nabla n\mathrm{d}x\mathrm{d}t+$$
$$\int_0^T\int_\Omega(r(n,p)(1-np)+g)A(n,\gamma_n)\mathrm{d}x\mathrm{d}t$$
$$\le-\int_0^T\int_\Omega n_tA(n,\gamma_n)\mathrm{d}x\mathrm{d}t+\delta\|a(\varepsilon,n,\gamma_n)\nabla n\|^2_{L^2(Q_T)}+$$
$$C(\delta)K^2\|\nabla\psi\|^2_{L^2(Q_T)}+(\overline{r}+\|g\|_{L^\infty(\Omega)})\int_0^T\int_\Omega A(n,\gamma_n)\mathrm{d}x\mathrm{d}t.$$

For the term $I=\int_0^T\int_\Omega n_tA(n,\gamma_n)\mathrm{d}x\mathrm{d}t$, we see that

$$|I|\le\Big|\int_0^T\int_\Omega nA_t(n,\lambda_n)\mathrm{d}x\mathrm{d}t\Big|+\Big|\int_\Omega n(T)A(n,\lambda_n)(T)\mathrm{d}x\Big|+\Big|\int_\Omega n(0)A(n,\lambda_n)(0)\mathrm{d}x\Big|$$
$$\le\gamma_n\Big|\int_0^T\int_\Omega nn_t(\varepsilon^2+n^2)^{\frac{\gamma_n-1}{2}}\mathrm{d}x\mathrm{d}t\Big|+2KM|\Omega|$$
$$=\gamma_n\Big|\int_0^T\int_\Omega\frac{\partial}{\partial t}\Big(\int_0^n s(\varepsilon^2+s^2)^{\frac{\gamma_n-1}{2}}\mathrm{d}s\Big)\mathrm{d}x\mathrm{d}t\Big|+2KM|\Omega|$$

$$= \gamma_n \Big| \int_\Omega \Big(\int_{n_0(x)}^{n(x,T)} s(\varepsilon^2+s^2)^{\frac{\gamma_n-1}{2}} \mathrm{d}s \Big) \mathrm{d}x \Big| + 2KM|\Omega|, \tag{2.22}$$

where $M = \max A(n,\gamma_n) \le \gamma_n \int_0^K (\varepsilon^2+s^2)^{\frac{\gamma_n-1}{2}} \mathrm{d}s$. By virtue of Lemma 2.1, $|I| \le C$. Together with

$$\|\nabla\psi\|_{L^2(\Omega)} \le C(1+\|n\|_{L^2(\Omega)} + \|p\|_{L^2(\Omega)}) \le C, \tag{2.23}$$

and choosing δ sufficiently small, we arrive at the inequality

$$\int_0^T \int_\Omega a^2(\varepsilon, n, \gamma_n)|\nabla n|^2 \mathrm{d}x\mathrm{d}t \le C.$$

The proof is completed. □

Lemma 2.4 *The solutions of problem (1.5), (2.2)–(2.7) satisfy the following estimate*

$$\|\nabla n\|_{L^2(Q_T)} + \|\nabla p\|_{L^2(Q_T)} + \|\nabla\psi\|_{L^2(Q_T)} \le C. \tag{2.24}$$

Proof Since $0 \le n \le K$ a.e. $x \in Q_T$ and $0 < \varepsilon < 1$, we have

$$\begin{aligned} C &\ge \int_0^T \int_\Omega |a(\varepsilon,n,\gamma_n)|^2 |\nabla n|^2 \mathrm{d}x\mathrm{d}t = \gamma_n^2 \int_0^T \int_\Omega (\frac{1}{n^2+\varepsilon^2})^{1-\gamma_n} |\nabla n|^2 \mathrm{d}x\mathrm{d}t \\ &\ge \gamma_n^2 (\frac{1}{K^2+1})^{1-\gamma_n} \int_0^T \int_\Omega |\nabla n|^2 \mathrm{d}x\mathrm{d}t, \end{aligned} \tag{2.25}$$

which gives

$$\|\nabla n\|_{L^2(Q_T)} \le C.$$

A similar estimate holds for p and $\|\nabla\psi\|_{L^2(Q_T)} \le C$ can be obtained by (2.23). □

Using Lemmas 2.1 and 2.3, the following lemma can be easily proved and we omit the proof here.

Lemma 2.5 *The solutions of problem (1.5), (2.2)–(2.7) have the weak derivative*

$$n_t, p_t \in L^2(0,T;H^{-1}(\Omega)). \tag{2.26}$$

Proof of Theorem 2.1 Let $\{\psi_\varepsilon, n_\varepsilon, p_\varepsilon\}$ be the sequence of solutions of problem (1.5), (2.2)–(2.7). By passing to a subsequence if necessary, from (2.8), (2.19), (2.20), (2.24) and (2.26), we infer that

$$\begin{cases} ((n_\varepsilon)_t, (p_\varepsilon)_t) \to (n_t, p_t) \text{ weakly in } L^2(0,T;H^{-1}(\Omega)), \\ (\psi_\varepsilon, n_\varepsilon, p_\varepsilon) \to (\psi, n, p) \text{ strongly in } L^2(Q_T) \text{ and a.e. in } Q_T, \\ (\psi_\varepsilon, n_\varepsilon, p_\varepsilon) \to (\psi, n, p) \text{ weakly in } L^2(0,T;H^1(\Omega)). \end{cases} \tag{2.27}$$

(2.27) together with the weak $*$ convergence of $(\psi_\varepsilon, n_\varepsilon, p_\varepsilon)$ to (ψ, n, p) in $L^\infty(Q_T)$ implies that

$$(\psi_\varepsilon, n_\varepsilon, p_\varepsilon) \to (\psi, n, p) \text{ strongly in } L^q(Q_T) \text{ with any } 1 < q < \infty. \tag{2.28}$$

Let

$$h(n) = n(\varepsilon^2+n^2)^{\frac{\gamma_n-1}{2}}.$$

It follows that $\nabla h(n) \in L^2(Q_T)$ from

$$|\nabla h(n)|^2 = \Big| (\varepsilon^2+n^2)^{\frac{\gamma_n-1}{2}} \nabla n \cdot \frac{\varepsilon^2+\gamma_n n^2}{\varepsilon^2+n^2} \Big|^2 \le C|a(\varepsilon,n,\gamma_n)\nabla n|^2. \tag{2.29}$$

Then we can extract subsequences (labeled again by ε) such that

$$\begin{cases} h(n_\varepsilon) = n_\varepsilon(\varepsilon^2 + n_\varepsilon^2)^{\frac{\gamma_n - 1}{2}} \to n^{\gamma_n} \\ \text{a.e. in } Q_T \text{ and strongly in } L^q(Q_T) \text{ with } 1 < q < \infty, \\ \nabla h(n_\varepsilon) \to \nabla \xi \text{ weakly in } L^2(Q_T). \end{cases} \tag{2.30}$$

In the following we will identify the limit $\nabla\xi$. For every function $\varphi \in C_0^\infty(Q_T)$, we have

$$\begin{aligned} \int_0^T \int_\Omega \nabla\xi \cdot \nabla\varphi \mathrm{d}x\mathrm{d}t &= \lim_{\varepsilon\to 0} \int_0^T \int_\Omega \nabla h(n_\varepsilon) \cdot \nabla\varphi \mathrm{d}x\mathrm{d}t = -\lim_{\varepsilon\to 0} \int_0^T \int_\Omega h(n_\varepsilon)\nabla^2\varphi \mathrm{d}x\mathrm{d}t \\ &= -\int_0^T \int_\Omega n^{\gamma_n} \nabla^2\varphi \mathrm{d}x\mathrm{d}t = \int_0^T \int_\Omega \nabla(n^{\gamma_n}) \cdot \nabla\varphi \mathrm{d}x\mathrm{d}t, \end{aligned} \tag{2.31}$$

whence $\nabla\xi = \nabla(n^{\gamma_n})$.

Now we show that the limit (ψ, n, p) is a solution of the problem (1.1)–(1.5). Let $v \in L^2(0, T; H^1(\Omega))$ be a test function. By (2.27) and the continuity of r it is clear that

$$\langle (n_\varepsilon)_t, v\rangle \to \langle n, v\rangle \quad \text{and} \quad \int_0^T \int_\Omega r(n_\varepsilon, p_\varepsilon)(1 - n_\varepsilon p_\varepsilon) v \mathrm{d}x\mathrm{d}t \to \int_0^T \int_\Omega r(n, p)(1 - np) v \mathrm{d}x\mathrm{d}t.$$

A standard elliptic estimate gives

$$\|\nabla(\psi_\varepsilon - \psi)\|_{L^2(Q_T)} \le C\|(p_\varepsilon - p) - (n_\varepsilon - n)\|_{L^2(Q_T)}$$

from which we conclude that $\nabla\psi_\varepsilon \to \nabla\psi$ strongly in $L^2(Q_T)$. Hence

$$\int_0^T \int_\Omega n_\varepsilon \nabla\psi_\varepsilon \cdot \nabla v \mathrm{d}x\mathrm{d}t \to \int_0^T \int_\Omega n\nabla\psi \cdot \nabla v \mathrm{d}x\mathrm{d}t.$$

Finally we deduce from (2.27) and (2.30)

$$\begin{aligned} \int_0^T \int_\Omega a(\varepsilon, n_\varepsilon, \lambda_n)\nabla n_\varepsilon \cdot \nabla v \mathrm{d}x\mathrm{d}t &= \gamma_n \int_0^T \int_\Omega (\varepsilon^2 + n_\varepsilon^2)^{\frac{\gamma_n - 1}{2}} \nabla n_\varepsilon \cdot \nabla v \mathrm{d}x\mathrm{d}t \\ &= \gamma_n \int_0^T \int_\Omega \frac{\varepsilon^2 + n^2}{\varepsilon^2 + \gamma_n n^2} \nabla h(n_\varepsilon) \cdot \nabla v \mathrm{d}x\mathrm{d}t \\ &\to \int_0^T \int_\Omega \nabla(n^{\gamma_n}) \cdot \nabla v \mathrm{d}x\mathrm{d}t. \end{aligned}$$

So the limit n has been identified as the second component of a solution of the problem (1.1)–(1.5). The other components are handled in a very similar way and therefore we skip the proof. This completes the proof. □

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