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Existence and Global Exponential Stability of Periodic Solutions for General Shunting Inhibitory Cellular Neural Networks with Delays

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Abstract By using the Leray-Schauder fixed point theorem, differential inequality techniques and constructing suitable Lyapunov functional, several sufficient conditions are obtained for the existence and global exponential stability of periodic solutions for general shunting inhibitory cellular neural networks with delays. Some new results are obtained and some previously known results are improved. An example is employed to illustrate our feasible results.

Keywords shunting inhibitory cellular neural networks; global exponential stability; periodic solution.

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1. Introduction

In recent years, the shunting inhibitory cellular neural networks (SICNNs) have been extensively studied and found many important applications in different areas such as psychophysics, speech, perception, robotics, adaptive pattern recognition, vision, and image processing. Hence, they have been the object of intensive analysis by numerous authors and some interesting results have been obtained. In particular, there have been some results on the existence of (almost) periodic solutions for SICNNs with delays [1–9]. In 2008, the authors of [6] investigated the existence and global exponential stability of almost periodic solutions for the SICNNs with variable coefficients and obtained some new results, which complemented some of previously known results. The model of SICNNs in [6] is

$$\begin{cases} x'_{ij}(t) = -a_{ij}(t)x_{ij}(t) + \displaystyle\sum_{C^{kl}\in N_r(i,j)} C^{kl}_{ij}(t) f_{ij}(t, x_{kl}(t-\tau_{kl}(t)))x_{ij}(t) + L_{ij}(t), \\ x_{ij}(t) = \varphi_{ij}(t),\ t\in[-\tau, 0], \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$. $\tau_{ij}(t)$ represents axonal signal transmission delays and is continuous with $0 \le \tau_{kl}(t) \le \tau$; $C_{ij}(t)$ denotes the cell at the (i, j) position of the lattice at the

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t, the r-neighborhood $N_r(i,j)$ of $C_{ij}(t)$ is

$$N_r(i,j) = \{C_{kl}(t) : \max(|k-i|, |l-j|) \le r, 1 \le k \le m, 1 \le l \le n\}.$$

$x_{ij}(t)$ is the activity of the cell $C_{ij}(t)$, $L_{ij}(t)$ is the external input to $C_{ij}(t)$, $a_{ij}(t) > 0$ represents the passive decay rate of the cell activity; $C_{ij}^{kl}(t) \ge 0$ is the connection or coupling strength of postsynaptic activity of the cell transmitted to the cell $C_{ij}(t)$, the activity function $f_{ij}(t,\cdot)$ is a continuous function representing the output or firing rate of the cell $C_{kl}(t)$; $\varphi_{ij}(t)$ is the initial function, and is assumed to be bounded and continuous on $[-\tau, 0]$. $a_{ij}(t)$, $C_{ij}^{kl}(t)$, $f_{ij}(t,\cdot)$, $L_{ij}(t)$, $\varphi_{ij}(t)$ are all continuous almost periodic functions.

However, the conditions obtained in [6] for the existence and exponential stability for (1) are not sufficient (see the Remarks 2, 3 and 4 of this paper).

In this paper, we generalize (1) as follows,

$$x'_{ij}(t) = -a_{ij}(t, x_{ij}(t)) - \sum_{C^{kl} \in N_r(i,j)} C_{ij}^{kl}(t) f_{ij}(t, x_{kl}(t - \tau_{kl}(t))) x_{ij}(t) + I_{ij}(t). \tag{2}$$

For convenience, we denote $\tau = \max_{(i,j)}\{\tau_{ij}(t) | t \in [0,\omega]\}$, $\overline{I}_{ij} = \max_{t\in[0,\omega]} |I_{ij}(t)|$, $\underline{\mu}_{ij} = \min_{t\in[0,\omega]} \mu_{ij}(t)$, and let $x = (x_{11}, x_{12}, \ldots, x_{1m}, \ldots, x_{n1}, x_{n2}, \ldots, x_{nm})^{\mathrm{T}}$ be a column vector, in which the symbol $(^{\mathrm{T}})$ denotes the transpose of a vector.

The initial condition $\phi = (\phi_{11}, \ldots, \phi_{1m}, \ldots, \phi_{n1}, \ldots, \phi_{nm})^{\mathrm{T}}$ of (2) is of the form

$$x_{ij}(s) = \phi_{ij}(s), \quad s \in (-\tau, 0],$$

where $\phi_{ij}(s)$, $i = 1, 2, \ldots, n$, $j = 1, \ldots, m$, are continuous ω-periodic solutions.

The main purpose of this paper is to obtain sufficient conditions for the existence and global exponential stability of periodic solutions for (2). The results of this paper are new and they complement results of [1–8] and references cited therein.

The main methods used in this paper are Leray-Schauder's fixed point theorem, differential inequality techniques and Lyapunov functional. An example is employed to illustrate our feasible results.

The remaining parts of this paper are organized as follows. In Section 2, preliminaries and assumptions are given. In Section 3, we study the existence of periodic solutions of system (2) by using the Leray-Schauder's fixed point theorem. In Section 4, by constructing Lyapunov functional we shall derive sufficient conditions for the global exponential stability of the periodic solution of system (2). At last, an example is provided to illustrate our results.

2. Preliminaries and assumptions

The following definition and lemma will be used to prove our main results in Sections 3 and 4.

Definition 2.1 *Let $x^*(t)$ be an ω-periodic solution of (2) with initial value ϕ^*. If there exist constants $\alpha > 0$ and $P > 1$ such that for every solution $x(t)$ of (2) with initial value ϕ,*

$$|x_{ij}(t) - x_{ij}^*(t)| \le P\|\phi - \phi^*\| e^{-\alpha t}, \quad \forall t > 0, \ i = 1, 2, \ldots, n, j = 1, 2, \ldots, m,$$

where $\|\phi - \phi^*\| = \max_{(i,j)} \sup_{-\tau \le s \le 0} \{|\phi_{ij}(s) - \phi^*_{ij}(s)|\}$, *then* $x^*(t)$ *is said to be globally exponentially stable.*

Lemma 2.1 (Leray-Schauder) *Let* E *be a Banach space, and let the operator* $A : E \to E$ *be completely continuous. If the set* $\{\|x\| | x \in E, x = \lambda A x, 0 < \lambda < 1\}$ *is bounded, then* A *has a fixed point in* T, *where*

$$T = \{x | x \in E, \|x\| \le R\}, \quad R = \sup\{\|x\| | x = \lambda A x, 0 < \lambda < 1\}.$$

Throughout this paper, we assume that

(H₁) $C^{kl}_{ij}(t) \ge 0$, $\tau_{ij}(t) \ge 0$, $I_{ij}(t)$ *are continuous* ω*-periodic functions.* $\omega > 0$ *is a constant,* $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$;

(H₂) $a_{ij}(t,u) \in C(R^2, R)$ *are* ω*-periodic about the first argument,* $a_{ij}(t, 0) = 0$ *and there are positive continuous* ω*-periodic functions* $\mu_{ij}(t)$ *such that* $\frac{\partial a_{ij}(t,u)}{\partial u} \ge \mu_{ij}(t)$, $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$;

(H₃) $f_{ij}(t,u) \in C(R^2, R)$ *are* ω*-periodic about the first argument. There are continuous* ω*-periodic solutions* $\gamma_{ij}(t)$ *such that* $\gamma_{ij}(t) = \sup_{u \in R} |f_{ij}(t,u)|$, $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$;

(H₄) $\max_{(i,j)} \sup_{0 \le t \le \omega} \{\frac{\sum_{C^{kl} \in N_r(i,j)} C^{kl}_{ij}(t) \gamma_{ij}(t)}{\mu_{ij}(t)}\} = \theta < 1$;

(H₅) *There are non-negative continuous* ω*-periodic solutions* $\beta_{ij}(t)$ *such that*

$\beta_{ij}(t) = \sup_{u \ne v} |\frac{f_{ij}(t,u) - f_{ij}(t,v)}{u - v}|$ *for all* $u, v \in R$, $u \ne v$, $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$.

3. Existence of periodic solutions

Let ξ_{ij}, $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$ be constants. Make the change of variables

$$x_{ij} = \xi_{ij} y_{ij}(t), \quad i = 1, 2, \dots, n, \ j = 1, 2, \dots, m, \tag{3}$$

then (2) can be reformulated as

$$y'_{ij}(t) = -\xi^{-1}_{ij} a_{ij}(t, \xi_{ij} y_{ij}(t)) - \sum_{C^{kl} \in N_r(i,j)} C^{kl}_{ij}(t) f_{ij}(t, \xi_{kl} y_{kl}(t - \tau_{kl}(t))) y_{ij}(t) + \xi^{-1}_{ij} I_{ij}(t). \tag{4}$$

System (4) can be rewritten as

$$y'_{ij}(t) = -d_{ij}(t, y_{ij}(t)) y_{ij}(t) - \sum_{C^{kl} \in N_r(i,j)} C^{kl}_{ij}(t) f_{ij}(t, \xi_{kl} y_{kl}(t - \tau_{kl}(t))) y_{ij}(t) + \xi^{-1}_{ij} I_{ij}(t), \tag{5}$$

where $d_{ij}(t, y_{ij}(t)) \doteq \frac{\partial a_{ij}(t,z)}{\partial z}|_{z = d_{ij}}$, d_{ij} is between 0 and $\xi_{ij} y_{ij}(t)$, $d_{ij} \in R$. By (H₂), we obtain $a_{ij}(t, \xi_{ij} y_{ij})$ is strictly monotone increasing about y_{ij}. Hence, $d_{ij}(t, y_{ij}(t))$ is unique for any $y_{ij}(t)$. Obviously, $d_{ij}(t, y_{ij}(t))$ is continuous ω-periodic about the first argument and $d_{ij}(t, y_{ij}(t)) \ge \mu_{ij}(t)$.

Lemma 3.1 *Suppose that (H₁)–(H₃) hold and let* $x(t)$ *be an* ω*-periodic solution of system (5). Then,*

$$y_{ij}(t) = \int_0^\omega H^y_{ij}(t,s) \Big[- \sum_{C^{kl} \in N_r(i,j)} C^{kl}_{ij}(s) f_{ij}(s, \xi_{kl} y_{kl}(s - \tau_{kl}(s))) y_{ij}(s) + \xi^{-1}_{ij} I_{ij}(s) \Big] \mathrm{d}s,$$
$$t \in [0, \omega], \quad i = 1, 2, \dots, n, \ j = 1, 2, \dots, m, \tag{6}$$

where

$$H_{ij}^{y}(t,s) = \frac{1}{1 - e^{-\int_0^\omega d_{ij}(v,y_{ij}(v))\mathrm{d}v}} \begin{cases} e^{-\int_s^t d_{ij}(v,y_{ij}(v))\mathrm{d}v}, & 0 \le s \le t \le \omega, \\ e^{-(\int_0^\omega d_{ij}(v,y_{ij}(v))\mathrm{d}v - \int_t^s d_{ij}(v,y_{ij}(v))\mathrm{d}v)}, & 0 \le t \le s \le \omega. \end{cases}$$

Proof From system (5) we have

$$\begin{aligned}(y_{ij}(t)e^{\int_0^t d_{ij}(s,y_{ij}(s))\mathrm{d}s})' = \Big[- \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t) f_{ij}(t, \xi_{kl} y_{kl}(t-\tau_{kl}(t))) y_{ij}(t) + \\ \xi_{ij}^{-1} I_{ij}(t)\Big] e^{\int_0^t d_{ij}(s,y_{ij}(s))\mathrm{d}s}. \qquad (7)\end{aligned}$$

Integrating (7) from 0 to t, we have

$$\begin{aligned}y_{ij}(t) =& e^{-\int_0^t d_{ij}(s,y_{ij}(s))\mathrm{d}s} y_{ij}(0) + \int_0^t \Big[- \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(s) f_{ij}(s, \xi_{kl} y_{kl}(s-\tau_{kl}(s))) y_{ij}(s) + \\ & \xi_{ij}^{-1} I_{ij}(s)\Big] e^{-\int_s^t d_{ij}(v,y_{ij}(v))\mathrm{d}v} \mathrm{d}s. \qquad (8)\end{aligned}$$

From $x_{ij}(\omega) = x_{ij}(0)$ and (3) we have $y_{ij}(\omega) = y_{ij}(0)$. By (8) we obtain

$$\begin{aligned}y_{ij}(0) =& \frac{1}{1 - e^{-\int_0^\omega d_{ij}(s,y_{ij}(s))\mathrm{d}s}} \int_0^\omega \Big[- \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(s) f_{ij}(s, \xi_{kl} y_{kl}(s-\tau_{kl}(s))) y_{ij}(s) + \\ & \xi_{ij}^{-1} I_{ij}(s)\Big] e^{-\int_s^\omega d_{ij}(v,y_{ij}(v))\mathrm{d}v} \mathrm{d}s. \qquad (9)\end{aligned}$$

Substituting (9) into (8), we obtain

$$\begin{aligned}y_{ij}(t) =& \frac{e^{-\int_0^t d_{ij}(s,y_{ij}(s))\mathrm{d}s}}{1 - e^{-\int_0^\omega d_{ij}(v,y_{ij}(v))\mathrm{d}v}} \int_0^\omega \Big[- \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(s) f_{ij}(s, \xi_{kl} y_{kl}(s-\tau_{kl}(s))) y_{ij}(s) + \\ & \xi_{ij}^{-1} I_{ij}(s)\Big] e^{-\int_s^\omega d_{ij}(v,y_{ij}(v))\mathrm{d}v} \mathrm{d}s + \\ & \int_0^t \Big[- \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(s) f_{ij}(s, \xi_{kl} y_{kl}(s-\tau_{kl}(s))) y_{ij}(s) + \xi_{ij}^{-1} I_{ij}(s)\Big] e^{-\int_s^t d_{ij}(v,y_{ij}(v))\mathrm{d}v} \mathrm{d}s \\ =& \int_0^\omega H_{ij}^{y}(t,s) \Big[- \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(s) f_{ij}(s, \xi_{kl} y_{kl}(s-\tau_{kl}(s))) y_{ij}(s) + \xi_{ij}^{-1} I_{ij}(s)\Big] \mathrm{d}s.\end{aligned}$$

The proof is completed. □

In order to use Lemma 2.1, we take $X = \{y | y \in C([0,\omega], R^{nm})\}$. Then X is a Banach space with the norm

$$\|y\| = \max_{(i,j)} \{|y_{ij}|_0\}, \ |y_{ij}|_0 = \sup_{0\le t\le\omega} |y_{ij}(t)|, \quad i = 1, \ldots, n, \ j = 1, \ldots, m.$$

Set a mapping $\Phi : X \to X$ by setting

$$(\Phi y)(t) = y(t),$$

where,

$$(\Phi y)_{ij}(t) = \int_t^{t+\omega} H_{ij}^{y}(t,s) \Big[- \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(s) f_{ij}(s, \xi_{kl} y_{kl}(s-\tau_{kl}(s))) y_{ij}(s) + \xi_{ij}^{-1} I_{ij}(s)\Big] \mathrm{d}s,$$

$$i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m.$$

It is easy to know the fact that the existence of ω-periodic solution of (2) is equivalent to the existence of fixed point of the mapping Φ in X.

Lemma 3.2 *Suppose that* (H_1)–(H_4) *hold. Then* $\Phi : X \to X$ *is completely continuous.*

Proof Under our assumptions, it is clear that the operator Φ is continuous. Next, we show that Φ is compact.

For any constant $D > 0$, let $\Omega = \{y | y \in X, \|y\| < D\}$. For any $y \in \Omega$, we have

$$\begin{aligned}\|\Phi y\| =& \max_{(i,j)} \sup_{0\le t\le\omega} \Big\{\Big| \int_0^\omega H_{ij}^y(t,s)\Big[-\sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t) f_{ij}(s,\xi_{kl}y_{kl}(s-\tau_{kl}(s)))y_{ij}(s)+\\ & \xi_{ij}^{-1}I_{ij}(s)\Big]\mathrm{d}s\Big|\Big\}\\ \le& \max_{(i,j)} \sup_{0\le t\le\omega} \Big\{ \int_0^\omega H_{ij}^y(t,s)\Big[\sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\gamma_{ij}(s)|y_{ij}(s)| + \xi_{ij}^{-1}|I_{ij}(s)|\Big]\mathrm{d}s\Big\}\\ \le& \max_{(i,j)} \sup_{0\le t\le\omega} \Big\{ \int_0^\omega H_{ij}^y(t,s)\mu_{ij}(s)\theta\mathrm{d}s\Big\}\|y\| + \max_{(i,j)}\{\frac{\overline{I}_{ij}}{\xi_{ij}\underline{\mu}_{ij}}\}\\ <& \theta D + \max_{(i,j)}\{\frac{\overline{I}_{ij}}{\xi_{ij}\underline{\mu}_{ij}}\},\end{aligned}$$

which implies that $\Phi(\Omega)$ is uniformly bounded. Where,

$$\begin{aligned}&\int_0^\omega H_{ij}^y(t,s)\mu_{ij}(s)\mathrm{d}s\\ &= \frac{1}{1-e^{-\int_0^\omega d_{ij}(v,x_{ij}(v))\mathrm{d}v}}\Big\{\int_0^t e^{-\int_s^t d_{ij}(v,x_{ij}(v))\mathrm{d}v}\mu_{ij}(s)\mathrm{d}s+\\ &\quad e^{-\int_0^\omega d_{ij}(v,x_{ij}(v))\mathrm{d}v}\int_t^\omega e^{\int_t^s d_{ij}(v,x_{ij}(v)\mathrm{d}v)}\mu_{ij}(s)\mathrm{d}s\Big\}\\ &\le \frac{1}{1-e^{-\int_0^\omega d_{ij}(v,x_{ij}(v))\mathrm{d}v}}\Big\{\int_0^t e^{-\int_s^t \mu_{ij}(v)\mathrm{d}v}\mu_{ij}(s)\mathrm{d}s+\\ &\quad e\int_t^\omega e^{\int_t^s d_{ij}(v,x_{ij}(v))\mathrm{d}v} d_{ij}(s,x_{ij}(s))\mathrm{d}s\Big\}\\ &= \frac{1}{1-e^{-\int_0^\omega d_{ij}(v,x_{ij}(v))\mathrm{d}v}}\Big\{1-e^{-\int_0^t \mu_{ij}(s)\mathrm{d}v} + e^{-\int_0^\omega d_{ij}(v,x_{ij}(v))\mathrm{d}v}(e^{\int_t^\omega d_{ij}(v,x_{ij}(v)\mathrm{d}v)}-1)\Big\}\\ &= \frac{1}{1-e^{-\int_0^\omega d_{ij}(v,x_{ij}(v))\mathrm{d}v}}\Big\{1-e^{-\int_0^t \mu_{ij}(s)\mathrm{d}v} + e^{-\int_0^t d_{ij}(v,x_{ij}(v))\mathrm{d}v} - e^{-\int_0^\omega d_{ij}(v,x_{ij}(v))\mathrm{d}v}\Big\} \le 1.\end{aligned}$$

By (H_2), there exists a constant $M > 0$ such that

$$|d_{ij}(t,y_{ij}(t))| \le M, \quad \text{for} \quad t\in[0,\omega]\times\Omega, \quad i=1,2,\dots,\ n, j=1,2,\dots,m.$$

In view of the definition of Φ, we have

$$\begin{aligned}(\Phi y)_{ij}'(t) &= \frac{\mathrm{d}}{\mathrm{d}t}\Big(\int_0^\omega H_{ij}^y(t,s)\Big[-\sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)f_{ij}(s,\xi_{kl}y_{kl}(s-\tau_{kl}(s)))y_{ij}(s)+\xi_{ij}^{-1}I_{ij}(s)\Big]\mathrm{d}s\Big)\\ &= -d_{ij}(t,y_{ij}(t))(\Phi y)_{ij}(t) - \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)f_{ij}(t,\xi_{kl}y_{kl}(t-\tau_{kl}(t)))y_{ij}(t)+\xi_{ij}^{-1}I_{ij}(t).\end{aligned}$$

Hence,

$$\left|(\Phi y)'_{ij}(t)\right| \le M\Big(\theta D + \max_{(i,j)}\{\frac{\overline{I}_{ij}}{\xi_{ij}\underline{\mu}_{ij}}\}\Big) + \max_{(i,j)}\Big\{\sum_{C^{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl}\overline{\gamma}_{ij}D + \frac{\overline{I}_{ij}}{\xi_{ij}}\Big\},$$

where $\overline{C}_{ij}^{kl} = \max_{t\in[0,\omega]} C_{ij}^{kl}(t)$, $\overline{\gamma}_{ij} = \max_{t\in[0,\omega]}\gamma_{ij}(t)$. So, $\Phi(\Omega)\subseteq X$ is a family of uniformly bounded and equi-continuous subsets. By using the Arzela-Ascoli Theorem, $\Phi: X\to X$ is compact. Therefore, $\Phi: X\to X$ is completely continuous. The proof is completed. □

Theorem 3.1 *Suppose that* (H_1)–(H_4) *hold. Let* ξ_{ij}, $i=1,2,\ldots,n$, $j=1,2,\ldots,m$ *be constants. Then system (2) has an* ω*-periodic solution* $x^*(t)$ *with* $\|x^*\|\le \max_{(i,j)}\{\xi_{ij}\}\widetilde{R}\doteq R_0$, *where*

$$\widetilde{R} = \frac{\max_{(i,j)}\{\frac{\overline{I}_{ij}}{\xi_{ij}\underline{\mu}_{ij}}\}}{1-\theta}.$$

Proof Let $y\in X$, $t\in[0,\omega]$. we consider the operator equation

$$y=\lambda\Phi y, \quad \lambda\in(0,1). \tag{10}$$

If y is a solution of (10), for $t\in[0,\omega]$, we obtain

$$\|y\|\le\|\Phi y\|\le\theta\|y\| + \max_{(i,j)}\{\frac{\overline{I}_{ij}}{\xi_{ij}\underline{\mu}_{ij}}\}.$$

This and (H_4) imply that

$$\|y\|\le\widetilde{R}.$$

In view of Lemma 2.1, we obtain that Φ has a fixed point $y^*(t)$ with $\|y^*\|\le\widetilde{R}$. Hence, system (5) has one ω-periodic solution $y^*(t) = (y^*_{11}, y^*_{12},\ldots,y^*_{1m},\ldots,y^*_{n1},y^*_{n2},\ \ldots,y^*_{nm})^{\mathrm{T}}$ with $\|y^*\|\le\widetilde{R}$. It follows from (3) that $x^*(t) = (x^*_{11}(t),\ x^*_{12}(t),\ldots,x^*_{1m}(t),\ldots,x^*_{n1}(t),x^*_{n2}(t),\ldots,x^*_{nm}(t))^{\mathrm{T}} = (\xi_{11}y^*_{11},\xi_{12}y^*_{12},\ldots,\xi_{1m}y^*_{1m},\ldots,\xi_{n1}y^*_{n1},\ \xi_{n2}y^*_{n2},\ \ldots,\xi_{nm}y^*_{nm})^{\mathrm{T}}$ is one ω-periodic solution of (2) with

$$\|x^*\|\le\max_{(i,j)}\{\xi_{ij}\}\widetilde{R}\doteq R_0.$$

The proof is completed. □

4. Global exponential stability of periodic solution

In this section, we shall construct some suitable Lyapunov functionals to derive sufficient conditions ensuring that (2) has a unique ω-periodic solution and all solutions of (2) exponentially converge to its unique ω-periodic solution.

Theorem 4.1 *Assume* (H_1)–(H_3) *and* (H_5) *hold and*

(H_6) *There are a set of positive constants* ξ_{ij}, $i=1,2,\ldots,n$, $j=1,2,\ldots,m$, *such that*

$$\max_{(i,j)}\sup_{0\le t\le\omega}\Big\{\frac{\sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)(\gamma_{ij}(t)\xi_{ij}+R_0\xi_{kl}\beta_{ij}(t))}{\mu_{ij}(t)\xi_{ij}}\Big\} = \kappa < 1.$$

Then system (2) has exactly one ω*-periodic solution, which is globally exponentially stable.*

Proof Obviously, (H_6) holds implies (H_4) holds. By Theorem 3.1, there exists an ω-periodic solution $x^*(t)$ of (2) with initial value $\phi^*(t) = (\phi^*_{11}(t), \ldots, \phi^*_{1m}(t), \ldots, \phi^*_{n1}(t), \ldots, \phi^*_{nm}(t))^{\mathrm{T}}$ and $\|x^*\| \le R_0$. Suppose that $x(t)$ is an arbitrary solution of system (2) with initial value $\phi(t) = (\phi_{11}(t), \ldots, \phi_{1m}(t), \ldots, \phi_{n1}(t), \ldots, \phi_{nm}(t))^{\mathrm{T}}$. Set $z(t) = (z_{11}(t), \ldots, z_{1m}(t), \ldots, z_{n1}(t), \ldots, z_{nm}(t))^{\mathrm{T}}$ $= x(t) - x^*(t)$. Then, from system (2) we have

$$\begin{aligned} z'_{ij}(t) = &- [a_{ij}(t, x_{ij}(t)) - a_{ij}(t, x^*_{ij}(t))] - \sum_{C^{kl} \in N_r(i,j)} C^{kl}_{ij}(t)[f_{ij}(t, x_{kl}(t - \tau_{kl}(t)))x_{ij}(t) - \\ & f_{ij}(t, x^*_{kl}(t - \tau_{kl}(t)))x^*_{ij}(t)]. \end{aligned} \tag{11}$$

From (H_6) we have

$$\begin{aligned} -\mu_{ij}(t)\xi_{ij} + \sum_{C^{kl} \in N_r(i,j)} C^{kl}_{ij}(t)(\gamma_{ij}(t)\xi_{ij} + R_0 d_{kl}\beta_{ij}(t)) \le (\kappa - 1)\mu_{ij}(t) \le (\kappa - 1)\underline{\mu}_{ij} < 0, \\ i = 1, 2, \ldots,\ n, j = 1, 2, \ldots, m. \end{aligned} \tag{12}$$

Set

$$h_{ij}(\lambda) = (\lambda - \mu_{ij}(t))\xi_{ij} + \sum_{C^{kl} \in N_r(i,j)} C^{kl}_{ij}(t)(\gamma_{ij}(t)\xi_{ij} + R_0\xi_{kl}\beta_{ij}(t)e^{\lambda\tau}).$$

Clearly, $h_{ij}(\lambda)$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, are continuous functions on R. Since $h_{ij}(0) < 0$,

$$\frac{\mathrm{d}h_{ij}(\lambda)}{\mathrm{d}\lambda} = \xi_{ij} + \lambda \sum_{C^{kl} \in N_r(i,j)} C^{kl}_{ij}(t) R_0 \xi_{kl}\beta_{ij}(t)e^{\lambda\tau} > 0,$$

and $h_{ij}(+\infty) = +\infty$, we see that $h_{ij}(\lambda)$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, are strictly monotone increasing functions. Therefore, for any $i \in \{1, 2, \ldots, n\}$, $j \in \{1, 2, \ldots, m\}$ and $t \ge 0$, there is unique $\lambda(t)$ such that

$$(\lambda(t) - \mu_{ij}(t))\xi_{ij} + \sum_{C^{kl} \in N_r(i,j)} C^{kl}_{ij}(t)(\gamma_{ij}(t)\xi_{ij} + R_0\xi_{kl}\beta_{ij}(t)e^{\lambda(t)\tau}) = 0.$$

Let $\lambda^*_{ij} = \inf_{t \ge 0}\{\lambda(t) | (\lambda(t) - \mu_{ij}(t))\xi_{ij} + \sum_{C^{kl} \in N_r(i,j)} C^{kl}_{ij}(t)(\gamma_{ij}(t)\xi_{ij} + R_0\xi_{kl}\beta_{ij}(t)e^{\lambda(t)\tau}) = 0\}$. Obviously, $\lambda^*_{ij} \ge 0$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. Now, we shall prove that $\lambda^*_{ij} > 0$. Suppose this is not true. From (12), there exists a positive constant η such that

$$\inf_{t \ge 0, (i,j)} \left\{ \frac{\mu_{ij}(t)\xi_{ij} - \sum_{C^{kl} \in N_r(i,j)} C^{kl}_{ij}(t)(\gamma_{ij}(t)\xi_{ij} + R_0\xi_{kl}\beta_{ij}(t))}{\xi_{ij} + \sum_{C^{kl} \in N_r(i,j)} 1.5\tau C^{kl}_{ij}(t)R_0\xi_{kl}\beta_{ij}(t)} \right\} \ge \eta.$$

Take small $\varepsilon > 0$, then there exists $t_0 \ge 0$ such that

$$0 < \lambda^*_{ij}(t_0) < \varepsilon < \eta.$$

Let us recall the inequality $e^x < 1 + 1.5x$ for sufficiently small $x > 0$. Then we obtain

$$\begin{aligned} 0 = &(\lambda^*_{ij}(t_0) - \mu_{ij}(t))\xi_{ij} + \sum_{C^{kl} \in N_r(i,j)} C^{kl}_{ij}(t)(\gamma_{ij}(t)\xi_{ij} + R_0\xi_{kl}\beta_{ij}(t)e^{\lambda^*_{ij}(t_0)\tau}) \\ < &(\varepsilon - \mu_{ij}(t))\xi_{ij} + \sum_{C^{kl} \in N_r(i,j)} C^{kl}_{ij}(t)(\gamma_{ij}(t)\xi_{ij} + R_0\xi_{kl}\beta_{ij}(t)e^{\varepsilon\tau}) \\ < &(\eta - \mu_{ij}(t))\xi_{ij} + \sum_{C^{kl} \in N_r(i,j)} C^{kl}_{ij}(t)(\gamma_{ij}(t)\xi_{ij} + R_0\xi_{kl}\beta_{ij}(t)(1 + 1.5\eta\tau)) \end{aligned}$$

$$
\begin{aligned}
&= -\mu_{ij}(t)\xi_{ij} + \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)(\gamma_{ij}(t)\xi_{ij} + R_0\xi_{kl}\beta_{ij}(t)) + \\
&\quad \eta\{\xi_{ij} + \sum_{C^{kl}\in N_r(i,j)} 1.5\tau C_{ij}^{kl}(t)R_0\xi_{kl}\beta_{ij}(t)\} \\
&\le -\mu_{ij}(t)\xi_{ij} + \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)(\gamma_{ij}(t)\xi_{ij} + R_0\xi_{kl}\beta_{ij}(t)) + \\
&\quad \frac{\mu_{ij}(t)\xi_{ij} - \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)(\gamma_{ij}(t)\xi_{ij} + R_0\xi_{kl}\beta_{ij}(t))}{\xi_{ij} + \sum_{C^{kl}\in N_r(i,j)} 1.5\tau C_{ij}^{kl}(t)R_0\xi_{kl}\beta_{ij}(t)} \times \\
&\quad \{\xi_{ij} + \sum_{C^{kl}\in N_r(i,j)} 1.5\tau C_{ij}^{kl}(t)R_0\xi_{kl}\beta_{ij}(t)\} = 0,
\end{aligned}
$$

which is a contradiction, and hence, $\lambda_{ij}^* > 0$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$.

Let $\alpha = \min_{(i,j)}\{\lambda_{ij}^*\}$. Obviously,

$$
h_{ij}(\alpha) = (\alpha - \mu_{ij}(t))\xi_{ij} + \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)(\gamma_{ij}(t)\xi_{ij} + R_0\xi_{kl}\beta_{ij}(t)e^{\alpha\tau}) \le 0,
$$
$$
i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, m. \tag{13}
$$

We choose a constant $d > 1$ such that

$$
d\xi_{ij}e^{-\alpha t} \ge 1, \ \text{for } t \in (-\tau, 0], \quad i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, m.
$$

It is obvious that

$$
|z_{ij}(t)| \le \|\phi - \phi^*\| \le d\xi_{ij}\|\phi - \phi^*\|e^{-\alpha t}, \ \text{for } t \in (-\tau, 0], \quad i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, m,
$$

where $\|\phi - \phi^*\|$ is defined as that in Definition 1.1.

Define a Lyapunov functional $V(t) = (V_{11}(t), \ldots, V_{1m}(t), \ldots, V_{n1}(t), \ldots, V_{nm}(t),)^{\mathrm{T}}$ by $V_{ij}(t) = e^{\alpha t}|z_{ij}(t)|$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. In view of (11), we obtain

$$
\begin{aligned}
\frac{\mathrm{d}^+V_{ij}(t)}{\mathrm{d}t} =& e^{\alpha t}\operatorname{sgn} z_{ij}\Big\{ -[a_{ij}(t, x_{ij}(t)) - a_{ij}(t, x_{ij}^*(t))] - \\
& \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)[f_{ij}(t, x_{kl}(t - \tau_{kl}(t)))x_{ij}(t) - \\
& f_{ij}(t, x_{kl}^*(t - \tau_{kl}(t)))x_{ij}^*(t)]\Big\} + \alpha e^{\alpha t}|z_{ij}(t)| \\
\le& e^{\alpha t}\Big\{(\alpha - \mu_{ij}(t))|z_{ij}(t)| + \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)[|f_{ij}(t, x_{kl}(t - \tau_{kl}(t)))x_{ij}(t) - \\
& f_{ij}(t, x_{kl}(t - \tau_{kl}(t)))x_{ij}^*(t)| + \\
& |f_{ij}(t, x_{kl}(t - \tau_{kl}(t)))x_{ij}^*(t) - f_{ij}(t, x_{kl}^*(t - \tau_{kl}(t)))x_{ij}^*(t)|]\Big\} \\
\le& e^{\alpha t}\Big\{\Big(\alpha - \mu_{ij}(t) + \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\gamma_{ij}(t)\Big)|z_{ij}(t)| + \\
& \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)R_0\beta_{ij}(t)|z_{kl}(t - \tau_{kl}(t))|\Big\} \\
\le& \Big(\alpha - \mu_{ij}(t) + \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\gamma_{ij}(t)\Big)V_{ij}(t) +
\end{aligned}
$$

$$\sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)R_0\beta_{ij}(t)e^{\alpha\tau}V_{kl}(t-\tau_{kl}(t)). \tag{14}$$

We claim that

$$V_{ij}(t)=|z_{ij}(t)|e^{\alpha t}\le d\xi_{ij}\|\phi-\phi^*\|,\quad i=1,2,\ldots,n,\ j=1,2,\ldots,m,\quad \text{for all}\ \ t>0. \tag{15}$$

Contrarily, there must exist $i_0\in\{1,2,\ldots,n\}$, $j_0\in\{1,2,\ldots,m\}$ and $\widetilde{t}>0$ such that

$$V_{i_0j_0}(\widetilde{t})=d\xi_{i_0j_0}\|\phi-\phi^*\|,\ \frac{\mathrm{d}^+V_{i_0j_0}(\widetilde{t})}{\mathrm{d}t}>0,\quad V_{ij}(t)\le d\xi_{ij}\|\phi-\phi^*\|, \tag{16}$$

$\forall t\in(-\tau,\widetilde{t}]$, $i=1,2,\ldots,n$, $j=1,2,\ldots,m$. Together with (14) and (16), we obtain

$$\begin{aligned}0<&\frac{\mathrm{d}^+V_{i_0j_0}(\widetilde{t})}{\mathrm{d}t}\le\Big(\alpha-\mu_{i_0j_0}(t)+\sum_{C^{kl}\in N_r(i_0,j_0)}C_{i_0j_0}^{kl}(t)\gamma_{i_0j_0}(t)\Big)V_{i_0j_0}(t)+\\&\sum_{C^{kl}\in N_r(i_0,j_0)}C_{i_0j_0}^{kl}(t)R_0\beta_{i_0j_0}(t)e^{\alpha\tau}V_{kl}(t-\tau_{kl}(t))\\&\le d\|\phi-\phi^*\|\Big\{(\alpha-\mu_{i_0j_0}(t))\xi_{i_0j_0}+\sum_{C^{kl}\in N_r(i_0,j_0)}C_{i_0j_0}^{kl}(t)(\gamma_{i_0j_0}(t)\xi_{i_0j_0}+R_0\xi_{kl}\beta_{i_0j_0}(t)e^{\alpha\tau})\Big\}.\end{aligned}$$

Hence,

$$0<(\alpha-\mu_{i_0j_0}(t))\xi_{i_0j_0}+\sum_{C^{kl}\in N_r(i_0,j_0)}C_{i_0j_0}^{kl}(t)(\gamma_{i_0j_0}(t)\xi_{i_0j_0}+R_0\xi_{kl}\beta_{i_0j_0}(t)e^{\alpha\tau}),$$

which contradicts (13). Hence, (15) holds. It follows that

$$|x_{ij}(t)-x_{ij}^*(t)|=|z_{ij}(t)|\le d\xi_{ij}\|\phi-\phi^*\|e^{-\alpha t},\quad \forall t>0,\ i=1,2,\ldots,n,\ j=1,2,\ldots,m.$$

Let $M=\max_{(i,j)}\{d\xi_{ij}+1\}$. Then, we have

$$|x_{ij}(t)-x_{ij}^*(t)|\le M\|\phi-\phi^*\|e^{-\alpha t},\quad \forall t>0,\ i=1,2,\ldots,n,\ j=1,2,\ldots,m.$$

In view of Definition 2.1, the ω-periodic solution $x^*(t)$ of system (2) is globally exponentially stable. The proof is completed.

Now we consider the almost periodic solution of (2). Substituting almost periodic solution for ω-periodic solution of (H_1), (H_2), (H_3) and (H_5) and named by ($\widetilde{\mathrm{H}}_1$), ($\widetilde{\mathrm{H}}_2$), ($\widetilde{\mathrm{H}}_3$) and ($\widetilde{\mathrm{H}}_5$), respectively. For convenience, we still call it (2). Let ξ_{ij}, $i=1,2,\ldots,n$, $j=1,2,\ldots,m$ be constants, $\psi(t)$ be continuous almost periodic solution. Then (2) has a unique almost periodic solution

$$\begin{aligned}y^{\psi(t)}(t)=&\Big\{\int_{-\infty}^{t}e^{-\int_s^t d_{ij}(u,\psi_{ij}(u))\mathrm{d}u}\Big[-\sum_{C^{kl}\in N_r(i,j)}C_{ij}^{kl}(s)f_{ij}(s,\xi_{kl}\psi_{kl}(s-\tau_{kl}(s)))\psi_{ij}(s)+\\&\xi_{ij}^{-1}I_{ij}(s)\Big]\mathrm{d}s\Big\}.\end{aligned}$$

Similarly to the proof of Theorems 3.1 and 4.1, we obtain the following theorem.

Theorem 4.2 *Assume ($\widetilde{H}_1$)–($\widetilde{H}_3$) and ($\widetilde{H}_5$) hold and*

($\widetilde{H}_6$) *There are a set of positive constants* ξ_{ij}, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, *such that*

$$\max_{(i,j)} \sup_{t\in R} \Big\{ \frac{\sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)(\gamma_{ij}(t)\xi_{ij} + R_0\xi_{kl}\beta_{ij}(t))}{\mu_{ij}(t)\xi_{ij}} \Big\} < 1,$$

where $R_0 = \max_{(i,j)}\{\xi_{ij}\}\widetilde{R}$, $\widetilde{R} = \max_{(i,j)}\{\frac{\overline{I}_{ij}}{\xi_{ij}\underline{\mu}_{ij}}\}/(1-\theta)$, $\overline{I}_{ij} = \max_{t\in R}|I_{ij}(t)|$, $\underline{\mu}_{ij} = \min_{t\in R}\mu_{ij}(t)$. Then system (2) has exactly one almost periodic solution, which is globally exponentially stable.

Remark 1 By the assumption ($\widetilde{\mathrm{H}}_2$), all the systems of [1–9] are special cases of system (2). For example, when $\xi_{ij} = 1$ in the solution of system (2), the results in [1] are the same as ours. If $a_{ij}(t, x_{ij}(t)) = a_{ij}x_{ij}(t)$, $C_{ij}^{kl}(t) = C_{ij}^{kl}$, and $f_{ij}(t, x) = x$ in (2), the results in [2, 3, 5, 8] can be obtained from system (2). If $a_{ij}(t, x_{ij}(t)) = a_{ij}(t)x_{ij}(t)$, and $f_{ij}(t, x) = x$ in (2), then the results in [4] can also be obtained from our results. If $a_{ij}(t, x_{ij}(t)) = a_{ij}(t)x_{ij}(t)$ in (2), the results in [6] are the same as our results. If $a_{ij}(t, x_{ij}(t)) = a_{ij}x_{ij}(t)$, $C_{ij}^{kl}(t) = C_{ij}^{kl}$, $f_{ij}(t, x) = x$, and $\tau_{kl}(t) = \tau$ in (2), the results in our paper include those in reference [7].

Remark 2 In [6], the assumption $\sup_{t\in R} a_{ij}^{-1}(t) \le 1$ is needed. While, according to our results, this condition is not necessary.

Remark 3 The following is an important assumption in [6] for global exponential stability of almost periodic solution (Theorem 3 in [6]) for (1):

(H_0) There are a set of positive constants d_{ij}, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, such that

$$\max_{(i,j)} \sup_{t\in R} \Big\{ \frac{\sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)(\gamma_{ij}(t) + Dd_{kl}\beta_{ij}(t))}{a_{ij}(t)} \Big\} < 1, \tag{17}$$

where $D > \max_{(i,j)} \frac{I}{1-\theta} I = \max_{(i,j)}\{d_{ij}^{-1} \sup_{t\in R} \frac{|I_{ij}(t)|}{a_{ij}(t)}\}$.

However, by deduction, the proof of Theorem 3 in [6] is not correct in the case $\eta = 1$ and $q = 1$, where η and q are defined in the proof of Theorem 3 in [6]. Moreover, from $\|\varphi\| < D$ one knows that (9) cannot hold globally. Hence, Theorem 3 in [6] is not correct.

5. Application

In this Section, we give an example to illustrate that our results are feasible. Consider the following general SICNNs with delays

$$x_{ij}'(t) = -a_{ij}(t, x_{ij}(t)) - \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t) f_{ij}(t, x_{kl}(t - \tau_{kl}(t)))x_{ij}(t) + I_{ij}(t), \tag{18}$$

where $r = 1$, $\tau_{ij}(t)$ is any continuous non-negative 2π-periodic function, $i, j = 1, 2, 3$. Take $a_{11}(t, x) = a_{13}(t, x) = a_{21}(t, x) = a_{32}(t, x) = 4x + \sin x + x\sin t$, $a_{12}(t, x) = a_{23}(t, x) = a_{31}(t, x) = a_{33}(t, x) = 5x - \sin x + x\cos t$, $a_{22}(t, x) = 3x + \cos x - x\sin t$, $f_{ij}(t, x) = 0.1\sin x$, $\tau_{ij}(t) = (\cos t)^2$, and

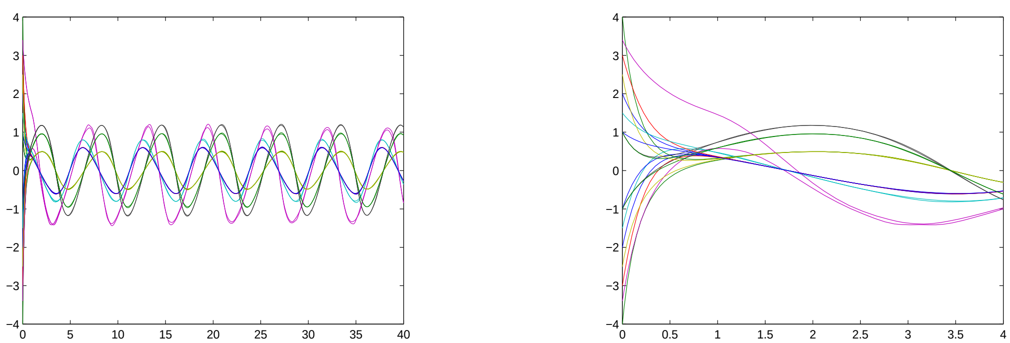
$$\begin{bmatrix} C_{11}(t) & C_{12}(t) & C_{13}(t) \\ C_{21}(t) & C_{22}(t) & C_{23}(t) \\ C_{31}(t) & C_{32}(t) & C_{33}(t) \end{bmatrix} = \begin{bmatrix} 0.2|\cos t| & 0.4|\sin t| & 0.3|\cos t| \\ 0.6|\cos t| & 0 & 0.5|\sin t| \\ 0.5|\sin t| & 0.6|\cos t| & 0.5|\sin t| \end{bmatrix},$$

$$\begin{bmatrix} I_{11}(t) & I_{12}(t) & I_{13}(t) \\ I_{21}(t) & I_{22}(t) & I_{23}(t) \\ I_{31}(t) & I_{32}(t) & I_{33}(t) \end{bmatrix} = \begin{bmatrix} 3\cos t & 2\sin t & 3\cos t \\ 4\cos t & 4\cos t & 2\sin t \\ 5\sin t & 3\cos t & 4\sin t \end{bmatrix}.$$

Then $\omega = 2\pi$, $\mu_{11}(t) = \mu_{13}(t) = \mu_{21}(t) = \mu_{32}(t) = 3 + \sin t$, $\mu_{12}(t) = \mu_{23}(t) = \mu_{31}(t) = \mu_{33}(t) = 4 + \cos t$, $\mu_{22}(t) = 2 - \sin t$, $\gamma_{ij}(t) = \beta_{ij}(t) = 0.1$, $\sum_{C^{kl}\in N_r(1,1)} C_{11}^{kl}(t) = 0.8|\cos t| + 0.4|\sin t|$, $\sum_{C^{kl}\in N_r(1,2)} C_{12}^{kl}(t) = 1.1|\cos t| + 0.9|\sin t|$, $\sum_{C^{kl}\in N_r(1,3)} C_{13}^{kl}(t) = 0.3|\cos t| + 0.9|\sin t|$, $\sum_{C^{kl}\in N_r(2,1)} C_{21}^{kl}(t) = 1.4|\cos t| + 0.9|\sin t|$, $\sum_{C^{kl}\in N_r(2,2)} C_{22}^{kl}(t) = 1.7|\cos t| + 1.9|\sin t|$, $\sum_{C^{kl}\in N_r(2,3)} C_{23}^{kl}(t) = 0.9|\cos t| + 1.4|\sin t|$, $\sum_{C^{kl}\in N_r(3,1)} C_{31}^{kl}(t) = 1.2|\cos t| + 0.5|\sin t|$, $\sum_{C^{kl}\in N_r(3,2)} C_{32}^{kl}(t) = 1.2|\cos t| + 1.5|\sin t|$, $\sum_{C^{kl}\in N_r(3,3)} C_{33}^{kl}(t) = 0.6|\cos t| + |\sin t|$.

Take $\xi_{ij} = 1$, $i,j = 1,2,3$. Computing through MATLAB, we have $\theta \approx 0.22358 < 1$, $R_0 \approx 2.1466$, and $\kappa \approx 0.7035 < 1$. It is easy to check that all the conditions needed in Theorem 4.1 are satisfied. Therefore, system (18) has a unique 2π-periodic solution $x^*(t)$ with $\|x^*\| \le R_0$, which is globally exponentially stable.

Remark 4 System (18) is a simple SICNNs with delays and time-varying coefficients. Obviously, $a_{ij}(t,x)$, $i,j = 1,2,3$, are non-linear about x, hence none of the the results in [1–8] and references cited therein can be applied to (18). Moreover, the periodic solution $x^*(t)$ satisfies $\|x^*\| \le R_0$, which has nothing to do with the initial value of (18). The results of [6] and its example are actually locally exponentially stable. Hence, the results of this paper are completely new and complement previously known results.



(a_1) (a_2)

(a) Solutions of x.

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