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New Delay-Dependent Stability of Uncertain Discrete-Time Switched Systems with Time-Varying Delays

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Abstract This paper deals with the issues of robust stability for uncertain discrete-time switched systems with mode-dependent time delays. Based on a novel difference inequality and a switched Lyapunov function, new delay-dependent stability criteria are formulated in terms of linear matrix inequalities (LMIs) which are not contained in known literature. A numerical example is given to demonstrate that the proposed criteria improves some existing results significantly with much less computational effort.

Keywords discrete-time switched system; difference inequality; switched Lyapunov function; delay-dependent stability; linear matrix inequality (LMI).

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1. Introduction

Switched system is a class of hybrid dynamical systems consisting of a family of continuous- (or discrete-) time subsystems, and a rule that orchestrates the switching between them. It has gained a great deal of attention mainly because various real-world systems, such as chemical processing, communication networks, traffic control, the control of manufacturing systems, and automotive engine control and aircraft control can be modeled as switched systems. In the last two decades, there has been increasing interest in stability analysis and controller design for switched systems, see the survey paper [1] and the references therein. Besides the continuous switched systems, the discrete switched systems have also been considered by many authors, see

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[2]–[7] and the references therein. It has been recognized that time delays, which are the inherent features of many physical process, are the big sources of instability and poor performances. For time delay systems, stability criteria are usually classified into two types: delay-independent criteria and delay-dependent ones. In general, delay-independent criteria are conservative since they cannot handle the systems whose stability depends on the size of time delay. Recently, [8] and [9] used a descriptor systems method and introduced a switched Lyapunov functional respectively, and obtained the delay dependent stability condition of uncertain discrete-time switched systems. However, to some extent, there are few results of discrete delay. It may be improved significantly with some useful approaches, and this has motivated our research.

In this paper, we are interested in establishing delay-dependent stability criteria in terms of linear matrix inequalities (LMIs) for the uncertain discrete-time switched delay systems with mode-dependent time delays under arbitrary switching sequences. The object of this paper is to seek for and introduce an improved LMI test to ensure a large bound for the time-delay. The main idea of our method is inspired by Zhang's recent work [10], where some novel integrate inequalities were introduced for stability analysis and controller synthesis of continuous deterministic delay systems. We extend this approach to uncertain discrete-time switched delay systems by constructing a switched Lyapunov function [11]. The proposed results for the stability analysis have three advantages. Firstly, to introduce difference inequality in which free-weighting matrices are contained would get less conservative asymptotic stability conditions; Secondly, the difference inequality reduces the conservatism considerably entailed in the previously developed transformation methods since it does not transform the systems which could introduce additional dynamics in the sense defined in [12]; Thirdly, some free weighting matrices are introduced properly to counteract the influence brought to the delays by the difference inequality. Note that these advantages are not obtained at the cost of high computational complexity. Finally, numerical example is given to illustrate the superiority of present result to those in the literature.

This paper is organized as follows. In Section 2, we give the problem formulation and introduce an important lemma for our later results. Section 3 is dedicated to stability analysis of switched systems by means of a switched quadratic Lyapunov function and our lemma. One numerical evaluation is given in Section 4.

2. Preliminaries

We will use the following notations.

R^n n-dimensional real space

$R^{n\times n}$ set of all real n by n matrices

x^{T} or A^{T} transpose of vector x (or matrix A)

$P>0$ (respectively, $P<0$) matrix P is symmetric positive (respectively, negative) definite

$P\geq 0$ (respectively, $P\leq 0$) matrix P is symmetric positive (respectively, negative) semi-definite

$*$ the elements below the main diagonal of a symmetric block matrix.

Consider linear switched system in the domain of discrete time:

$$\begin{aligned} &x(k+1) = A(k,r(k))x(k) + B(k,r(k))x(k-d(k,r(k))), \\ &x(s) = \phi(s), \quad s = -d, \ldots, -1, 0, \end{aligned} \tag{1}$$

where $x(k) \in R^n$ is the system state, $r(k) : Z^+ = \{0,1,2,\ldots\} \to \overline{N} = \{1,2,\ldots,N\}$ is the control signal. Here we assume that $d(k,r(k))$ is bounded for $k > 0$. Let $d = \max_{k\geqslant 0}$ $\{d(k,1), d(k,2), \ldots, d(k,N)\}$. $\phi : \{-d, -d+1, \ldots, 0\} \to R^n$ represents the initial condition. For each $i \in \overline{N}$, the system matrices are assumed to be uncertain and satisfy:

$$\left[\begin{array}{cc} A(k,i) & B(k,i) \end{array}\right] = \left[\begin{array}{cc} A_i & B_i \end{array}\right] + H_i F(k,i) \left[\begin{array}{cc} E_{i1} & E_{i2} \end{array}\right], \tag{2}$$

where A_i, B_i are constant matrices that describe the i-th nominal mode, H_i, E_{i1} and E_{i2} are given constant matrices which characterize the structure of the uncertainty, and $F(k,i)$ are uncertainties satisfying $F^{\mathrm{T}}(k,i)F(k,i) \leqslant I$ for $k \in Z^+$. One reason for assuming that the system uncertainty has the structure given in (2) is that a linear interconnection of a nominal plant with the uncertainty $F(k,i)$ leads to the structure of the form (2). The other comes from the fact that uncertainties in many physical systems can be modeled in this manner, e.g. satisfying matching conditions.

We are here interested to establish delay-dependent robust stability criteria for systems (1) and (2) by introducing a novel difference inequality and using linear matrix inequality technique. Before giving the main theorem of this paper, we firstly provide the following lemmas which play an important role in our later development.

Lemma 1 (Finsler's lemma) *For vector $x \in R^n$, matrix $P \in R^{n\times n}$ and $H \in R^{m\times n}$, satisfying* $\mathrm{rank}(H) = r < n$, *the following statements are equivalent,*

(i) $\forall x \neq 0$ *and* $Hx = 0$, *satisfying* $x^{\mathrm{T}}Px < 0$;

(ii) $\exists X \in R^{n\times m}$, *satisfying* $P + XH + H^{\mathrm{T}}X^{\mathrm{T}}$.

Lemma 2 *For any constant symmetric matrix $Q \in R^{n\times n}$, $Q = Q^{\mathrm{T}} > 0$, and any appropriate dimensional matrices, $M_1 \in R^{n\times n}$, $M_2 \in R^{n\times n}$, $Z = \begin{pmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{pmatrix} \in R^{2n\times 2n}$,* $Y = \left[\begin{array}{cc} M_1 & M_2 \end{array}\right] \in R^{n\times 2n}$, *if* $\begin{pmatrix} Q & Y \\ * & Z \end{pmatrix} > 0$, *we have*

$$-2\sum_{l=k-d(k,r(k))}^{k-1} x^{\mathrm{T}}(l)Qx(l) \leq \xi^{\mathrm{T}}(k) \begin{pmatrix} \Lambda_{11} & \Lambda_{11} \\ * & \Lambda_{22} \end{pmatrix} \xi(k)$$

with $\xi^{\mathrm{T}}(k) = \left[\begin{array}{cc} x^{\mathrm{T}}(k) & x^{\mathrm{T}}(k-d(k,r(k))) \end{array}\right]$, *where,*

$$\Lambda_{11} = M_1 + M_1^{\mathrm{T}} + \mathrm{d}Z_{11} + Q + \mathrm{d}M_1^{\mathrm{T}}Q^{-1}M_1,$$

$$\Lambda_{12} = -M_1^{\mathrm{T}} + M_2 + \mathrm{d}Z_{12} + \mathrm{d}M_1^{\mathrm{T}}Q^{-1}M_2,$$

$$\Lambda_{22} = -M_2 - M_2^{\mathrm{T}} + \mathrm{d}Z_{22} - Q + \mathrm{d}M_2^{\mathrm{T}}Q^{-1}M_2.$$

Proof With the fact,

$$x(k) - x(k - d(k, r(k)) - \sum_{l=k-d(k,r(k)}^{k-1} (x(l+1) - x(l)) = 0,$$

$\forall N_1, N_2 \in R^{n\times n}$, we have

$$\begin{aligned}
0 =& 2[x^{\mathrm{T}}(k)N_1^{\mathrm{T}} + x^{\mathrm{T}}(k - d(k, r(k))N_2^{\mathrm{T}}]\times \\
& [x(k) - x(k - d(k, r(k)) - \sum_{l=k-d(k,r(k)}^{k-1} (x(l+1) - x(l))] \\
=& 2\xi^{\mathrm{T}}(k)N^{\mathrm{T}} \begin{bmatrix} I & -I \end{bmatrix} \xi(k) - 2\xi^{\mathrm{T}}(k)N^{\mathrm{T}} \sum_{l=k-d(k,r(k)}^{k-1} x(l+1) + \\
& 2\xi^{\mathrm{T}}(k)N^{\mathrm{T}} \sum_{l=k-d(k,r(k)}^{k-1} x(l),
\end{aligned} \tag{3}$$

where $N = \begin{bmatrix} N_1 & N_2 \end{bmatrix}$, $\xi^{\mathrm{T}}(k) = \begin{bmatrix} x^{\mathrm{T}}(k) & x^{\mathrm{T}}(k - (k, r(k)) \end{bmatrix}$, by using the Moon's inequality [13], we have

$$\begin{aligned}
-2\xi^{\mathrm{T}}(k)N^{\mathrm{T}} \sum_{l=k-d(k,r(k)}^{k-1} x(l+1) \leqslant & \sum_{l=k-d(k,r(k)}^{k-1} \begin{pmatrix} x(l+1) \\ \xi(k) \end{pmatrix}^{\mathrm{T}} \times \\
& \begin{pmatrix} Q & Y - N \\ Y^{\mathrm{T}} - N^{\mathrm{T}} & Z \end{pmatrix} \begin{pmatrix} x(l+1) \\ \xi(k) \end{pmatrix} \\
\leqslant & \sum_{l=k-d}^{k-1} x^{\mathrm{T}}(l+1)Qx(l+1) + \mathrm{d}\xi^{\mathrm{T}}(k)Z\xi^{\mathrm{T}}(k) + \\
& 2\xi^{\mathrm{T}}(k)(Y^{\mathrm{T}} - N^{\mathrm{T}}) \begin{bmatrix} I & -I \end{bmatrix} \xi(k) + \\
& 2\xi^{\mathrm{T}}(k)(Y^{\mathrm{T}} - N^{\mathrm{T}}) \sum_{l=k-d}^{k-1} x(l).
\end{aligned} \tag{4}$$

Substituting (4) into (3), and with the fundamental inequality, we get

$$\begin{aligned}
0 \leqslant & 2\xi^{\mathrm{T}}(k)Y^{\mathrm{T}} \begin{bmatrix} I & -I \end{bmatrix} \xi(k) + \sum_{l=k-d(k,r(k))}^{k-1} x^{\mathrm{T}}(l+1)Qx(l+1) + \\
& \mathrm{d}\xi^{\mathrm{T}}(k)Z\xi(k) + 2\xi^{\mathrm{T}}(k)Y^{\mathrm{T}} \sum_{l=k-d(k,r(k))}^{k-1} x(l) \\
\leqslant & 2\xi^{\mathrm{T}}(k)Y^{\mathrm{T}} \begin{bmatrix} I & -I \end{bmatrix} \xi(k) + \mathrm{d}\xi^{\mathrm{T}}(k)Z\xi(k) + \mathrm{d}\xi^{\mathrm{T}}(k)Y^{\mathrm{T}}Q^{-1}Y\xi(k) + \\
& \sum_{l=k-d(k,r(k))}^{k-1} x^{\mathrm{T}}(l)Qx(l) + \sum_{l=k-d(k,r(k))}^{k-1} x^{\mathrm{T}}(l+1)Qx(l+1) \\
= & 2\xi^{\mathrm{T}}(k)Y^{\mathrm{T}} \begin{bmatrix} I & -I \end{bmatrix} \xi(k) + \mathrm{d}\xi^{\mathrm{T}}(k)Z\xi(k) + \mathrm{d}\xi^{\mathrm{T}}(k)Y^{\mathrm{T}}Q^{-1}Y\xi(k) +
\end{aligned}$$

$$2\sum_{l=k-d(k,r(k))}^{k-1} x^{\mathrm{T}}(l)Qx(l)+\xi^{\mathrm{T}}(k)\begin{pmatrix} Q & 0 \\ 0 & -Q \end{pmatrix}\xi(k).$$

So the conclusion is true.

Lemma 3 ([14]) *Let D, E and F be matrices with appropriate dimensions. Suppose $F^{\mathrm{T}}F \leqslant I$. Then for any scalar $\lambda > 0$, we have*

$$DFE + E^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}} \leqslant \lambda DD^{\mathrm{T}} + \lambda^{-1}E^{\mathrm{T}}E.$$

3. Main result

In this section, we present asymptotical stability criteria dependent on delays for the uncertain discrete-time switched systems described by (1) and (2) with strict LMI approaches.

For system (1), we define the following switched Lyapunov function:

$$V(k,x(k)) = x^{\mathrm{T}}(k)P_{r(k)}x(k) + 2\sum_{\theta=-d(k,r(k))+1}^{0}\sum_{l=k-1+\theta}^{k-1} x^{\mathrm{T}}(l)Qx(l) \tag{5}$$

with $P_1, P_2, \ldots, P_N$, Q being symmetric positive definite matrices.

If such a Lyapunov function exists and its difference $\triangle V(k,x(k)) = V(k+1,x(k+1)) - V(k,x(k))$ is negative definite along the solution of (1), the origin of the system (1) is globally asymptotically stable as shown by the following general lemma.

Lemma 4 ([15]) *The equilibrium 0 of*

$$x(k+1) = f(x(k)) \tag{6}$$

is globally uniformly asymptotically stable if there is a function $V : Z^+ \times R^n \to R$ such that,

(i) V is a positive definite function, decrescent, and radially unbounded;

(ii) $\triangle V(k,x(k)) = V(k+1,x(k+1)) - V(k,x(k))$ is negative definite along the solution of equation (6).

For the asymptotical stability of systems described by (1), we have the following result.

Theorem 1 *The system (1) with uncertainty described by (2) is of asymptotical stability, if there exist constants $\lambda_i > 0$, symmetric matrices $P_1, P_2, \ldots, P_N, Q, Z_{11}$, $Z_{22} \in R^{n\times n}$ and any appropriate dimensional matrices $G_i, T_i, U_i, M_1, M_2 \in R^{n\times n}$, such that the following LMIs hold,*

$$\begin{pmatrix} Q & M_1 & M_2 \\ * & Z_{11} & Z_{12} \\ * & * & Z_{22} \end{pmatrix} > 0, \tag{7}$$

$$\Psi = \begin{pmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & 0 & G_iH_i \\ * & \Psi_{22} & \Psi_{23} & \mathrm{d}M_1 & U_iH_i \\ * & * & \Psi_{33} & \mathrm{d}M_2 & W_iH_i \\ * & * & * & -\mathrm{d}Q & 0 \\ * & * & * & * & -\lambda_i I \end{pmatrix} < 0, \tag{8}$$

where

$$\Psi_{11} = P_j - G_i^{\mathrm{T}} - G_i, \quad \Psi_{12} = G_iA_i - U_i^{\mathrm{T}}, \quad \Psi_{13} = G_iB_i - W_i^{\mathrm{T}},$$

$$\Psi_{22} = -P_i + M_1 + M_1^{\mathrm{T}} + U_iA_i + A_i^{\mathrm{T}}U_i^{\mathrm{T}} + (2d+1)Q + \mathrm{d}Z_{11} + \lambda_i E_{i1}^{\mathrm{T}}E_{i1},$$

$$\Psi_{23} = -M_1^{\mathrm{T}} + M_2 + U_iB_i + A_i^{\mathrm{T}}W_i^{\mathrm{T}} + \lambda_i E_{i1}^{\mathrm{T}}E_{i2} + \mathrm{d}Z_{12},$$

$$\Psi_{33} = -M_2 - M_2^{\mathrm{T}} - Q + W_iB_i + B_i^{\mathrm{T}}W_i^{\mathrm{T}} + \mathrm{d}Z_{22} + \lambda_i E_{i2}^{\mathrm{T}}E_{i2}.$$

Proof Choose a switching Lyapunov function candidate for system (1) as follows:

$$V(k, x(k)) = x^{\mathrm{T}}(k)P_{r(k)}x(k) + 2\sum_{\theta=-d(k,r(k))+1}^{0}\ \sum_{l=k-1+\theta}^{k-1} x^{\mathrm{T}}(l)Qx(l).$$

Let the mode at time k and $k+1$ be i and j, respectively. That is, $r(k) = i$ and $r(k+1) = j$ for any $i, j \in N$. Along the solution of (1), and using Lemma 2, we have

$$\begin{aligned} \triangle V(k, x(k)) =& V(k+1, x(k+1)) - V(k, x(k)) = x^{\mathrm{T}}(k+1)P_jx(k+1) - x^{\mathrm{T}}(k)P_ix(k) + \\ & 2d(k, r(k))x^{\mathrm{T}}(k)Qx(k) - 2\sum_{l=k-d(k,r(k))}^{k-1} x(l)^{\mathrm{T}}Qx(l) \\ \leqslant& \zeta^{\mathrm{T}}\Phi(i,j)\zeta, \end{aligned}$$

where $\zeta^{\mathrm{T}} = \begin{bmatrix} x^{\mathrm{T}}(k+1) & x^{\mathrm{T}}(k) & x^{\mathrm{T}}(k - d(k, r(k))) \end{bmatrix} \neq 0$, and

$$\Phi(i,j) = \begin{pmatrix} P_j & 0 & 0 \\ * & \Phi_1 & \Phi_2 \\ * & * & \Phi_3 \end{pmatrix} \tag{9}$$

with

$$\Phi_1 = -P_i + M_1 + M_1^{\mathrm{T}} + (2d+1)Q + \mathrm{d}Z_{11} + \mathrm{d}M_1^{\mathrm{T}}Q^{-1}M_1,$$

$$\Phi_2 = -M_1^{\mathrm{T}} + M_2 + \mathrm{d}Z_{12} + \mathrm{d}M_1^{\mathrm{T}}Q^{-1}M_2,$$

$$\Phi_3 = -Q - M_2 - M_2^{\mathrm{T}} + \mathrm{d}Z_{22} + \mathrm{d}M_2^{\mathrm{T}}Q^{-1}M_2,$$

and there exist appropriate dimensional matrices G_i, T_i, U_i such that,

$$\Phi(i,j) + \begin{pmatrix} G_i \\ U_i \\ W_i \end{pmatrix} \begin{pmatrix} -I & A_i(k,i) & B_i(k,i) \end{pmatrix} + \begin{pmatrix} -I \\ A_i^{\mathrm{T}}(k,i) \\ B_i^{\mathrm{T}}(k,i) \end{pmatrix} \begin{pmatrix} G_i^{\mathrm{T}} & U_i^{\mathrm{T}} & W_i^{\mathrm{T}} \end{pmatrix} = \Gamma,$$

where

$$\Gamma = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & 0 \\ * & \Gamma_{22} & \Gamma_{23} & \mathrm{d}M_1 \\ * & * & \Gamma_{33} & \mathrm{d}M_2 \\ * & * & * & -\mathrm{d}Q \end{pmatrix} < 0, \quad i, j \in \overline{N} \tag{10}$$

with

$$\Gamma_{11} = P_j - G_i^{\mathrm{T}} - G_i, \quad \Gamma_{12} = G_i A(k,i) - U_i^{\mathrm{T}}, \quad \Gamma_{13} = G_i B(k,i) - W_i^{\mathrm{T}},$$

$$\Gamma_{22} = -P_i + M_1 + M_1^{\mathrm{T}} + U_i A(k,i) + A^{\mathrm{T}}(k,i) U_i^{\mathrm{T}} + (2d+1)Q + \mathrm{d}Z_{11},$$

$$\Gamma_{23} = -M_1^{\mathrm{T}} + M_2 + U_i B(k,i) + A^{\mathrm{T}}(k,i) W_i^{\mathrm{T}} + \mathrm{d}Z_{12},$$

$$\Gamma_{33} = -M_2 - M_2^{\mathrm{T}} - Q + W_i B(k,i) + B^{\mathrm{T}}(k,i) W_i^{\mathrm{T}} + \mathrm{d}Z_{22}.$$

Using the uncertain condition (2), we have

$$\begin{aligned} \Gamma = {} & \begin{pmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & 0 \\ * & \widetilde{\Gamma}_{22} & \widetilde{\Gamma}_{23} & \mathrm{d}M_1 \\ * & * & \widetilde{\Gamma}_{33} & \mathrm{d}M_2 \\ * & * & * & -\mathrm{d}Q \end{pmatrix} + \begin{pmatrix} G_i H_i \\ U_i H_i \\ W_i H_i \\ 0 \end{pmatrix} F(k,i) \begin{pmatrix} 0 & E_{i1} & E_{i2} & 0 \end{pmatrix} + \\ & \begin{pmatrix} 0 \\ E_{i1}^{\mathrm{T}} \\ E_{i2}^{\mathrm{T}} \\ 0 \end{pmatrix} F^{\mathrm{T}}(k,i) \begin{pmatrix} H_i^{\mathrm{T}} G_i^{\mathrm{T}} & H_i^{\mathrm{T}} U_i^{\mathrm{T}} & H_i^{\mathrm{T}} W_i^{\mathrm{T}} & 0 \end{pmatrix} < 0, \end{aligned} \tag{11}$$

where $\widetilde{\Gamma}_{22}$, $\widetilde{\Gamma}_{23}$ and $\widetilde{\Gamma}_{33}$ are taken from Γ_{22}, Γ_{23} and Γ_{33} by replacing $A(k,i)$ and $B(k,i)$ with A_i and B_i, respectively. By Lemma 3, a sufficient condition guaranteeing $\Gamma < 0$ is that there exist positive constants λ_i such that

$$\begin{aligned} \Gamma = {} & \begin{pmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & 0 \\ * & \widetilde{\Gamma}_{22} & \widetilde{\Gamma}_{23} & \mathrm{d}M_1 \\ * & * & \widetilde{\Gamma}_{33} & \mathrm{d}M_2 \\ * & * & * & -\mathrm{d}Q \end{pmatrix} + \lambda_i \begin{pmatrix} 0 \\ E_{i1}^{\mathrm{T}} \\ E_{i2}^{\mathrm{T}} \\ 0 \end{pmatrix} \begin{pmatrix} 0 & E_{i1} & E_{i2} & 0 \end{pmatrix} + \\ & \lambda_i^{-1} \begin{pmatrix} G_i H_i \\ U_i H_i \\ W_i H_i \\ 0 \end{pmatrix} \begin{pmatrix} H_i^{\mathrm{T}} G_i^{\mathrm{T}} & H_i^{\mathrm{T}} U_i^{\mathrm{T}} & H_i^{\mathrm{T}} W_i^{\mathrm{T}} & 0 \end{pmatrix} < 0. \end{aligned} \tag{12}$$

Applying Schur's complement [16], we have that (8) is equivalent to (12). Therefore, we have $\triangle V(k, x(k)) \leqslant \zeta^{\mathrm{T}} \Phi(i,j) \zeta < 0$ for all $k \geqslant 0$ from the Finsler's lemma. This completes the proof of Theorem 1 according to Lemma 4. □

Remark 1 In this Theorem, the difference inequality which contains free matrices is important to reduce the conservatism of the results on the stability analysis of this discrete-time switched systems.

Remark 2 As is well known, it could bring conservativeness inevitably if one uses inequality analysis technique to analyze the stability of delay systems. In this paper, it may reduce the conservativeness of our results using Finsler's lemma which introduces some free weighting matrices appropriately.

4. Numerical example

In order to show the effectiveness of the approaches presented in Section 3, in this section, a numerical example is provided.

Example 1 Consider the uncertain systems described by (1) and (2) $\overline{N} = 1, 2$ and

$$A_1 = \begin{pmatrix} 0.8 & 0.2 \\ 0 & 0.91 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0.3 & a \\ b & 0.58 \end{pmatrix}, \quad E_{11} = E_{12} = 0.01I, H_1 = cI$$

$$A_2 = \begin{pmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0.12 & 0 \\ 0.11 & 0.11 \end{pmatrix}, \quad E_{21} = E_{22} = 0.01I, H_2 = cI.$$

As $c = 0$, these systems become nominal systems. When $a = 0$ and $b = 0$, by Theorem 1, both the results in [8] and our results are the same, viz.$d \leqslant 1$. However, as $a = 0.2$, $b = 0.1$, the delay d can be obtained as much as 21 by Theorem 1, while d in [8] remains as $d \leqslant 1$. This comparison shows that our result is much less conservative than that in [8].

Applying Corollary 1 to this example shows that the system is robust stable for $d = 1$ as $a = b = 0$, $c = 0.1$, which is much less than that in [9] where $d \leqslant 5$. However, as $a \neq 0$ and $b \neq 0$, our result is much less conservative than that in [9]. Take $a = 0.2$, $b = 0.2$, $c = 0.1$ for example, we can obtain the system is robust stable for $d \leqslant 12$ while $d \leqslant 5$ in [9] which keeps unchanged. It is also very easy to verify that the time delay increases as a,b increase while the results in [9] remain much conservative.

This comparison shows that our result is also less conservative than that in [9] as $B1$ is not a diagonal matrix. This also shows that our results and those in [9] do not contain each other.

5. Conclusion

The robust stability for uncertain discrete-time switched systems with mode-dependent time delays has been investigated. Based on providing with a novel difference inequality, and combining with the switched Lyapunov function and the useful inequalities analysis technique, some novel delay dependent stability criteria have been obtained. The numerical example has shown significant improvements over some existing results.

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