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Notes on "Generalised Fuzzy Soft Sets"

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Abstract In this paper, we point out that several assertions (Propositions 3.15 and 5.2 (v)) in a previous paper by Majumdar and Samanta are not true in general, by counterexamples.

Keywords generalised fuzzy soft set; union; intersection; complement.

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1. Introduction

Molodtsov [1] initiated the theory of soft sets, which can be seen as a new mathematical tool for dealing with uncertainties which traditional mathematical tools cannot handle. Maji et al. [2] introduced the concept of fuzzy soft set, a more generalised concept, which is a combination of fuzzy set and soft set and studied its properties. Recently, fuzzy soft set theory has been applied to decision making problems [3] and forecasting [4]. Majumdar and Samanta [5] further generalised the concept of fuzzy soft sets as introduced by Maji et al. [2]. In this paper, we give the related concepts and the assertions in [5], then we verify that the assertions (Proposition 3.15 and Proposition 5.2 (v)) in [5] are incorrect by counterexamples.

2. Preliminaries

In this section, we recall some basic notions in generalised fuzzy soft set theory.

Definition 2.1 ([5]) *Let* $U=\{x_1,x_2,\ldots,x_n\}$ *be the universal set of elements and* $E=\{e_1,e_2,\ldots,e_m\}$ *be the universal set of parameters. The pair* (U,E) *will be called a soft universe. Let* $F:E\longrightarrow I^U$ *and* μ *be a fuzzy subset of* E*, i.e.,* $\mu:E\longrightarrow I=[0,1]$*, where* I^U *is the collection of all fuzzy subsets of* U*. Let* F_μ *be the mapping* $F_\mu:E\longrightarrow I^U\times I$ *defined as follows:* $F_\mu(e)=(F(e),\mu(e))$*, where* $F(e)\in I^U$*. Then* F_μ *is called a generalised fuzzy soft set (GFSS in short) over the soft universe* (U,E)*.*

Remark 2.2 ([5, 6]) Let c be an involutive fuzzy complement and g be an increasing generator of

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c. Let $*$ and $\circ$ be two binary operations on $[0,1]$ defined as follows: $a*b = g^{-1}(g(a)+g(b)-g(1))$ and $a\circ b = g^{-1}(g(a)+g(b))$. Then $*$ is a t-norm and $\circ$ is a t-conorm. Moreover $(*,\circ,c)$ becomes a dual triple.

Henceforth in the rest of the paper we will take such an involutive dual triple to consider the general case.

Definition 2.3 ([5]) *Let F_μ be a GFSS over (U,E). Then the complement of F_μ, denoted by F_μ^c, is defined by $F_\mu^c = G_\delta$, where $\delta(e) = \mu^c(e)$ and $G(e) = F^c(e)$, $\forall e \in E$.*

Definition 2.4 ([5]) *Union of two GFSS F_μ and G_δ, denoted by $F_\mu \widetilde{\cup} G_\delta$, is a GFSS H_ν, defined as $H_\nu : E \longrightarrow I^U \times I$ such that $H_\nu(e) = (H(e),\nu(e))$, where $H(e) = F(e)\circ G(e)$ and $\nu(e) = \mu(e)\circ\delta(e)$.*

Definition 2.5 ([5]) *Intersection of two GFSS F_μ and G_δ, denoted by $F_\mu \widetilde{\cap} G_\delta$, is a GFSS H_ν, defined as $H_\nu : E \longrightarrow I^U \times I$ such that $H_\nu(e) = (H(e),\nu(e))$, where $H(e) = F(e) * G(e)$ and $\nu(e) = \mu(e) * \delta(e)$.*

Definition 2.6 ([5]) *A GFSS is said to be a generalised null fuzzy soft set, denoted by Φ_θ, if $\Phi_\theta : E \longrightarrow I^U \times I$ such that $\Phi_\theta(e) = (F(e),\theta(e))$, where $F(e) = \overline{0}$ $\forall e \in E$ ($\overline{0}(x) = 0$, $\forall x \in U$) and $\theta(e) = 0$, $\forall e \in E$.*

Definition 2.7 ([5]) *A GFSS is said to be a generalised absolute fuzzy soft set, denoted by $\widetilde{A}_\alpha$, if $\widetilde{A}_\alpha : E \longrightarrow I^U \times I$ such that $\widetilde{A}_\alpha(e) = (A(e),\alpha(e))$, where $A(e) = \overline{1}$, $\forall e \in E$ ($\overline{1}(x) = 1$, $\forall x \in U$) and $\alpha(e) = 1$, $\forall e \in E$.*

3. Counterexamples

We begin this section with a result given by Majumdar and Samanta in [5].

Theorem 3.1 (Proposition 3.15 ([5])) *The following laws also hold true:*

(a) $F_\mu \widetilde{\cup} F_\mu^c = \widetilde{A}_\alpha$ and (b) $F_\mu \widetilde{\cap} F_\mu^c = \Phi_\theta$.

Remark 3.2 The assertions above were not proved in detail in [5]. In fact, the assertions are incorrect in general. Let F_μ be a GFSS over (U,E). Take $a*b = a\cdot b$, $a\circ b = a+b-a\cdot b$, and $a^c = 1-a$. Then $(F_\mu \widetilde{\cup} F_\mu^c)(e) = (F(e)\circ F^c(e), \mu(e)\circ\mu^c(e)) = (\overline{1}-F(e)\cdot(\overline{1}-F(e)), 1-\mu(e)\cdot(1-\mu(e)))$; $(F_\mu \widetilde{\cap} F_\mu^c)(e) = (F(e) * F^c(e), \mu(e) * \mu^c(e)) = (F(e)\cdot(\overline{1}-F(e)), \mu(e)\cdot(1-\mu(e)))$. If $F(e) \neq \overline{0}$ and $F(e) \neq \overline{1}$, or $\mu(e) \neq 0$ and $\mu(e) \neq 1$, then $(F_\mu \widetilde{\cup} F_\mu^c)(e) \neq (\overline{1},1)$ and $(F_\mu \widetilde{\cap} F_\mu^c)(e) \neq (\overline{0},0)$. So $F_\mu \widetilde{\cup} F_\mu^c \neq \widetilde{A}_\alpha$, $F_\mu \widetilde{\cap} F_\mu^c \neq \Phi_\theta$.

Next we give a concrete example.

Example 3.3 Let $U = \{x_1,x_2,x_3\}$ be a set of three shirts under consideration. Let $E = \{e_1,e_2,e_3\}$ be a set of qualities where e_1 = bright, e_2 = cheap, e_3 = colorful. Let $\mu : E \longrightarrow I = [0,1]$ be defined as follows: $\mu(e_1) = 0.1$, $\mu(e_2) = 0.4$, $\mu(e_3) = 0.7$.

Let us define the t-norm $*$ on $[0,1]$ as follows: $a * b = a\cdot b$ and the t-conorm $\circ$ on $[0,1]$ as follows: $a \circ b = a + b - a \cdot b$. Let us also take c as the fuzzy complement i.e., $a^c = 1 - a$. Then $(*, \circ, c)$ forms an involutive dual triple.

The GFSS $F_\mu : E \longrightarrow I^U \times I$ is defined as follows:

$$F_\mu(e_1) = \big(\{\frac{x_1}{0.7}, \frac{x_2}{0.4}, \frac{x_3}{0.2}\}, 0.1\big),\ F_\mu(e_2) = \big(\{\frac{x_1}{0.1}, \frac{x_2}{0.3}, \frac{x_3}{0.9}\}, 0.4\big),$$
$$F_\mu(e_3) = \big(\{\frac{x_1}{0.8}, \frac{x_2}{0.5}, \frac{x_3}{0.4}\}, 0.7\big).$$

By Definition 2.3, we conclude F_μ^c as follows:

$$F_\mu^c(e_1) = \big(\{\frac{x_1}{0.3}, \frac{x_2}{0.6}, \frac{x_3}{0.8}\}, 0.9\big),\ F_\mu^c(e_2) = \big(\{\frac{x_1}{0.9}, \frac{x_2}{0.7}, \frac{x_3}{0.1}\}, 0.6\big),$$
$$F_\mu^c(e_3) = \big(\{\frac{x_1}{0.2}, \frac{x_2}{0.5}, \frac{x_3}{0.6}\}, 0.3\big).$$

By Definition 2.4, we conclude $F_\mu \widetilde{\cup} F_\mu^c$ as follows:

$$\big(F_\mu \widetilde{\cup} F_\mu^c\big)(e_1) = \big(\{\frac{x_1}{0.79}, \frac{x_2}{0.76}, \frac{x_3}{0.84}\}, 0.91\big),\ \big(F_\mu \widetilde{\cup} F_\mu^c\big)(e_2) = \big(\{\frac{x_1}{0.91}, \frac{x_2}{0.79}, \frac{x_3}{0.91}\}, 0.76\big),$$
$$\big(F_\mu \widetilde{\cup} F_\mu^c\big)(e_3) = \big(\{\frac{x_1}{0.84}, \frac{x_2}{0.75}, \frac{x_3}{0.76}\}, 0.79\big).$$

Thus $F_\mu \widetilde{\cup} F_\mu^c$ is not a generalised absolute fuzzy soft set. This implies that the assertion $F_\mu \widetilde{\cup} F_\mu^c = \widetilde{A}_\alpha$ ((a) in Theorem 3.1) does not hold.

By Definition 2.5, we conclude $F_\mu \widetilde{\cap} F_\mu^c$ as follows:

$$(F_\mu \widetilde{\cap} F_\mu^c)(e_1) = \big(\{\frac{x_1}{0.21}, \frac{x_2}{0.24}, \frac{x_3}{0.16}\}, 0.09\big),\ (F_\mu \widetilde{\cap} F_\mu^c)(e_2) = \big(\{\frac{x_1}{0.09}, \frac{x_2}{0.21}, \frac{x_3}{0.09}\}, 0.24\big),$$
$$(F_\mu \widetilde{\cap} F_\mu^c)(e_3) = \big(\{\frac{x_1}{0.16}, \frac{x_2}{0.25}, \frac{x_3}{0.24}\}, 0.21\big).$$

Thus $F_\mu \widetilde{\cap} F_\mu^c$ is not a generalised null fuzzy soft set. This implies that the assertion $F_\mu \widetilde{\cap} F_\mu^c = \Phi_\theta$ ((b) in Theorem 3.1) does not hold. This shows that Theorem 3.1 is actually not true.

Theorem 3.4 (Proposition 5.2 (v) ([5])) *Let F_μ and G_δ be two GFSS over (U, E). Then the following holds:*

(v) $F_\mu \widetilde{\cap} G_\delta = \Phi_\theta \Longleftrightarrow S(F_\mu, G_\delta) = 0$, where minimum operation has been taken as generalised fuzzy intersection and $S(F_\mu, G_\delta)$ is the similarity between F_μ and G_δ (see [5, p. 1430).

Remark 3.5 In [5], Majumdar and Samanta point out that the proof is straightforward and follows from definition. In fact, the sufficiency of (v) in Theorem 3.4 does not hold in general. If $S(F_\mu, G_\delta) = 0$, then $M(\widehat{F}, \widehat{G}) = 0$ or $m(\mu, \delta) = 0$ which cannot imply $F_\mu \widetilde{\cap} G_\delta = \Phi_\theta$. If two GFSS F_μ and G_δ satisfy $m(\mu, \delta) = 0$ and $F(e) * G(e) \neq \overline{0}$, then $S(F_\mu, G_\delta) = 0$ by definition, but $F_\mu \widetilde{\cap} G_\delta \neq \Phi_\theta$.

Next we give a concrete example.

Example 3.6 Consider the following two GFSS where $U = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3\}$:

$$F_\mu(e_1) = \big(\{\frac{x_1}{0.7}, \frac{x_2}{0.4}, \frac{x_3}{0.2}\}, 0\big),\ F_\mu(e_2) = \big(\{\frac{x_1}{0.2}, \frac{x_2}{0.3}, \frac{x_3}{0.9}\}, 0.4\big),\ F_\mu(e_3) = \big(\{\frac{x_1}{0.8}, \frac{x_2}{0.5}, \frac{x_3}{0.4}\}, 0.6\big)$$

and

$$G_\delta(e_1) = \big(\{\frac{x_1}{0.5}, \frac{x_2}{0.4}, \frac{x_3}{0.6}\}, 0.1\big),\ G_\delta(e_2) = \big(\{\frac{x_1}{0.3}, \frac{x_2}{0.2}, \frac{x_3}{0.7}\}, 0\big),\ G_\delta(e_3) = \big(\{\frac{x_1}{0.4}, \frac{x_2}{0.8}, \frac{x_3}{0.9}\}, 0\big).$$

Define the t-norm $*$ on $[0,1]$ as follows: $a*b=a\wedge b$.

By the similarity formula between two fuzzy sets [5, p. 1430], we can easily conclude the similarity between the two fuzzy sets μ and δ: $m(\mu,\delta)=0$. Therefore, by the similarity formula between two GFSS [5, p. 1430], we can easily conclude the similarity between the two GFSS F_μ and G_δ: $S(F_\mu,G_\delta)=M(\widehat{F},\widehat{G})\cdot m(\mu,\delta)=0$. However

$$(F_\mu\widetilde{\cap}G_\delta)(e_1)=\big(\{\frac{x_1}{0.5},\frac{x_2}{0.4},\frac{x_3}{0.2}\},0\big),\ (F_\mu\widetilde{\cap}G_\delta)(e_2)=\big(\{\frac{x_1}{0.2},\frac{x_2}{0.2},\frac{x_3}{0.7}\},0\big),$$
$$(F_\mu\widetilde{\cap}G_\delta)(e_3)=\big(\{\frac{x_1}{0.4},\frac{x_2}{0.5},\frac{x_3}{0.4}\},0\big).$$

This implies that $F_\mu\widetilde{\cap}G_\delta\neq\Phi_\theta$.

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