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Global Exponential Stability of Periodic Solution for Competitive Neural Networks with Time-Varying and Distributed Delays on Time Scales

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Abstract In this paper, competitive neural networks with time-varying and distributed delays are investigated. By utilizing Lyapunov functional methods, the global exponential stability of periodic solutions of the neural networks is discussed on time scales. In addition, an example is given to illustrate the effectiveness of the theoretical results.

Keywords stability; competitive neural networks; delays; time scales.

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1. Introduction

Since Cohen and Grossberg proposed competitive neural networks (CNNs) as a new cellular neural networks in 1983, CNNs have received a considerable interest and have been applied in the image processing, pattern recognition, signal processing, optimization and control theory and so on [1–13]. The interest of the study of the competitive neural networks lies in the fact that models contain both the neural activity levels, the short-term memory (STM) representing rapid changes in neuronal dynamics and the long-term memory (LTM) describing the slow dynamics of the system.

The competitive neural networks (CNNs) was first studied by Cohen and Grossberg in [2]. Then the stability of CNNs attracted the attention of more researchers [3–13]. Undoubtedly, Meyer-Baese made a huge contribution on studying the stability of CNNs [3–8]. The local stability [3], the global stability [6], the robustness stability [7], the local uniform stability [4], the local exponential stability [5] and the global exponential stability [8] of CNNs without delays were studied by Meyer, respectively. In addition, the stability of CNNs was also studied by other researchers [9–13]. Beyond that, the periodic behavior of competitive neural networks is also a fast growing area of research [13].

Recently, many excellent results have been reported on stability of periodic solution of several types of neural networks on time scales which not only unify the continuous-time and

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discrete-time domains but also "between" them [14–17]. However, there is still little work dedicated to studying the stability of periodic solutions for CNNs on time scales. Motivated by all the above, in this paper, we will discuss global exponential stability of periodic solution for competitive neural networks with time-varying and distributed delays on time scales.

To the best of our knowledge, this is the first time that investigation is carried out on the stability problem for competitive neural networks on time scales. Consequently, the results derived in this paper extend some previously existing results. In addition, by considering the model with time-varying and distributed delays, a novel Lyapunov functional is proposed in our paper, which leads to smaller computational burden. Compared with existing relevant results, the criteria in this paper tend to be less conservative, such as, our results remove the requirement that the activation functions are bounded and zero at the zero which are supposed in [14] and [15], respectively. Therefore, our work is helpful to rich the results on the stability of neural networks in academic circles.

In this paper, the model is described by the following form:

$$\begin{cases} \text{STM}: x_i^{\Delta}(t) = -\alpha_i(t)x_i(t) + \sum\limits_{j=1}^{N} D_{ij}(t)f_j(x_j(t)) + \sum\limits_{j=1}^{N} D_{ij}^{\tau}(t)f_j(x_j(t-\tau_{ij}(t))+ \\ \qquad\qquad \sum\limits_{j=1}^{N} \bar{D}_{ij}(t)\displaystyle\int_0^{+\infty} K_{ij}(u)f_j(x_j(t-u))\Delta u + B_i(t)S_i(t) + I_i(t), \\ \text{LTM}: S_i^{\Delta}(t) = -c_i(t)S_i(t) + E_i(t)f_i(x_i(t)) + J_i(t) \end{cases} \tag{1.1}$$

with the initial values

$$\begin{aligned} x_i(s) &= \phi_i(s), \quad s \in (-\infty, 0]_T, \\ S_i(s) &= \psi_i(s), \quad s \in (-\infty, 0]_T \end{aligned}$$

where $i, j = 1, \ldots, N$, $x_i(t)$ is the neuron current activity level, $\alpha_i(t), c_i(t)$ are the time variable of the neuron, $f_j(x_j(t))$ is the output of neurons, $D_{ij}(t)$ and $D_{ij}^{\tau}(t)$, $\bar{D}_{ij}(t)$ represent the connection weight and the synaptic weight of delayed feedback between the ith neuron and the jth neuron respectively, $B_i(t)$ is the strength of the external stimulus, $E_i(t)$ denotes disposable scale, transmission delays $\tau_{ij}(t)$ satisfies $0 < \tau_{ij}(t) \leqslant \tau_{ij}$, $\tau_{ij}^{\Delta}(t) \le \tau < 1$ (τ_{ij} and τ are constants).

T is an ω-periodic time scale, and $\phi_i(\cdot)$, $\psi_i(\cdot)$ are rd-continuous. For $i = 1, \ldots, N$; $j = 1, \ldots, N$, we denote

$$\begin{aligned} &\bar{\mu} = \max_{t\in[0,\omega]_T} |\mu(t)|, \quad D_{ij} = \max_{t\in[0,\omega]_T} |D_{ij}(t)|, \quad D_{ij}^{\tau} = \max_{t\in[0,\omega]_T} |D_{ij}^{\tau}(t)|, \\ &\bar{D}_{ij} = \max_{t\in[0,\omega]_T} |\bar{D}_{ij}(t)|, \quad B_i = \max_{t\in[0,\omega]_T} |B_i(t)|, \quad E_i = \max_{t\in[0,\omega]_T} |E_i(t)|. \end{aligned}$$

Throughout this paper, we make the following assumptions:

(H_1) $\alpha_i(t)$,$c_i(t)$, $D_{ij}(t)$, $D_{ij}^{\tau}(t)$, $\bar{D}_{ij}(t)$, $B_i(t)$, $E_i(t)$, $\tau_{ij}(t)$, $I_i(t)$, $J_i(t)$ are continuous ω-periodic functions with $\omega > 0$, and there exist positive numbers $\underline{\alpha}_i$, $\overline{\alpha}_i$, $\underline{c}_i$, $\bar{c}_i$ such that $\underline{\alpha}_i \le \alpha_i(\cdot) \le \overline{\alpha}_i$, $\underline{c}_i \le c_i(\cdot) \le \bar{c}_i$, for $i, j = 1, \ldots, N$.

(H_2) The delay kernels $K_{ij}(s) : [0, +\infty) \to [0, +\infty)$ are continuous integral functions, and satisfy

$$\int_0^{+\infty} K_{ij}(s)\Delta s = 1, \quad \int_0^{+\infty} K_{ij}(s)e_\eta(s,t)\Delta s < \infty, \quad \text{for } \ i,j = 1,\dots,N. \tag{1.2}$$

(H_3) The functions $f_i \in C(R, R)$ are Lipschitz functions, that is, there exist positive constants $k_i > 0$, such that for all $x, y \in R$

$$|f_i(x) - f_i(y)| \le k_i|x - y|. \tag{1.3}$$

2. Preliminary

In this part, some useful definitions and lemmas are introduced.

Definition 2.1 ([18]) *A time scale T is an arbitrary nonempty closed subset of the real set R with the topology and ordering inherited from R. The graininess of the time scale T is determined by the formula $\mu(t) = \sigma(t) - t$, $\sigma(t) = \inf\{s \in T, s > t\}$.*

Definition 2.2 ([18]) *For $f : T \to R$ and $t \in T^k$, if for any $\varepsilon > 0$ there is an N-neighborhood of t such that*

$$|f(\sigma(t)) - f(s) - f^\Delta(t)(\sigma(t) - s)| \le \varepsilon|\sigma(t) - s|, \quad \textit{for all} \ \ s \in N,$$

then we call $f^\Delta(t)$ the Delta derivative of f at t.

Definition 2.3 ([18]) *For $s, t \in T$, if p is a regressive function, then we define the exponential function $e_p(t,s)$ by*

$$e_p(t,s) = \exp\Big(\int_s^t \xi_{\mu(\tau)}(p(\tau))\Delta\tau\Big), \quad \xi_h(z) = \begin{cases} \dfrac{\mathrm{Log}(1+zh)}{h}, & h \ne 0 \\ z, & h = 0 \end{cases}.$$

Definition 2.4 *The periodic solution $Z^*(t) = (x_1^*(t), x_2^*(t), \dots, x_N^*(t), S_1^*(t), \dots, S_N^*(t))^T$ of system (1) is said to be globally exponentially stable if there exists a positive constant ε and $N = N(\varepsilon) > 1$ such that all solutions $Z(t) = (x_1(t), x_2(t), \dots, x_N(t), S_1(t), \dots, S_N(t))^T$ of system (1) satisfy*

$$\begin{aligned}&\sum_{i=1}^N |x_i(t) - x_i^*(t)| + \sum_{i=1}^N |S_i(t) - S_i^*(t)| \\ &\quad \le N(\varepsilon)e_{\ominus\varepsilon}(t,0)\Big(\sum_{i=1}^N \sup_{u\in(-\infty,0]_T} |x_i(u) - x_i^*(u)| + \sum_{i=1}^N \sup_{u\in(-\infty,0]_T} |S_i(u) - S_i^*(u)|\Big), \quad \textit{for} \ \ t \in T^+.\end{aligned}$$

Lemma 2.5 ([18]) *If $p, q \in R$, $t, r, s \in T$, then*

- *(i)* $e_0(t,s) \equiv 1$ *and* $e_p(t,t) \equiv 1$;
- *(ii)* $e_p(t,s) = \frac{1}{e_p(s,t)} = e_{\ominus p}(s,t)$;
- *(iii)* $e_p(t,s)e_p(s,r) = e_p(t,r)$;
- *(iv)* $e_p^\Delta(t,t_0) = p(t)e_p(t,t_0)$;

(v) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t, s)$.

3. Global exponential stability of periodic solution

In this section, a suitable Lyapunov functional is constructed to study the global exponential stability of periodic solution of system (1.1).

Theorem 3.1 *Assume that assumptions (H_1)–(H_3) hold. Further, assume that*

(H_4) For $i = 1, 2, \ldots, N$;

$$\begin{aligned} &P_i = B_i - \underline{c}_i < 0, \\ &Q_i = -\underline{\alpha}_i + k_i E_i + \sum_{j=1}^{N} \big(D_{ji}k_i + \frac{1}{1-\tau} D_{ji}^{\tau} k_i + \bar{D}_{ji} k_i\big) < 0. \end{aligned} \tag{3.1}$$

Then the ω-periodic solution of system (1.1) is globally exponentially stable.

Proof Assume that the system (1.1) has an ω-periodic solution $Z^*(t) = \big(x_1^*(t), \ldots, x_N^*(t), S_1^*(t), \ldots, S_N^*(t)\big)^T$. Suppose that $Z(t) = (x_1(t), x_2(t), \ldots, x_N(t), S_1(t), \ldots, S_N(t))^T$ is an arbitrary solution of system (1.1). By (4), there exists a small positive constant η, such that

$$\begin{aligned} &\eta - \underline{c}_i + (1 + \eta\bar{\mu})B_i < 0, \\ &\eta - \underline{\alpha}_i + (1 + \eta\bar{\mu})k_i E_i + (1 + \eta\bar{\mu}) \sum_{j=1}^{N} \big(D_{ji}k_i + \frac{1}{1-\tau} D_{ji}^{\tau} k_i + \bar{D}_{ji} k_i\big) < 0. \end{aligned} \tag{3.2}$$

Consider the following Lyapunov functional

$$V(t) = \sum_{i=1}^{N} \big(V_{i1}(t) + V_{i2}(t) + V_{i3}(t) + V_{i4}(t)\big), \tag{3.3}$$

where

$$\begin{aligned} &V_{i1}(t) = |x_i(t) - x_i^*(t)| e_\eta(t, 0), \\ &V_{i2}(t) = \frac{1}{1-\tau} \sum_{j=1}^{N} D_{ij}^{\tau} k_j \int_{t-\tau_{ij}(t)}^{t} |x_j(s) - x_j^*(s)| e_\eta(\sigma(s + \tau_{ij}), 0)\Delta s, \\ &V_{i3}(t) = \sum_{j=1}^{N} \bar{D}_{ij} k_j \int_0^{+\infty} \int_{t-u}^{t} K_{ij}(u) |x_j(s) - x_j^*(s)| e_\eta(\sigma(s + u), 0)\Delta s \Delta u, \\ &V_{i4}(t) = |S_i(t) - S_i^*(t)| e_\eta(t, 0). \end{aligned}$$

Calculating the right upper derivatives $D^+V_{im}^{\Delta}(t)$ of $V_{im}(t)$, for $m = 1, 2, 3, 4$, we have

$$\begin{aligned} D^+V_{i1}^{\Delta}(t) =& \eta e_\eta(t, 0)|x_i(t) - x_i^*(t)| + e_\eta(\sigma(t), 0) D^+|x_i(t) - x_i^*(t)|^{\Delta} \\ \le& \eta e_\eta(t, 0)|x_i(t) - x_i^*(t)| + e_\eta(\sigma(t), 0)\Big\{ -\underline{\alpha}_i |x_i(t) - x_i^*(t)| + \\ & \sum_{j=1}^{N} D_{ij} k_j |x_j(t) - x_j^*(t)| + \sum_{j=1}^{N} D_{ij}^{\tau} k_j \big|x_j(t - \tau_{ij}(t)) - x_j^*(t - \tau_{ij}(t))\big| + \end{aligned}$$

$$
\begin{aligned}
&\sum_{j=1}^{N}\bar{D}_{ij}k_j\int_0^{+\infty}K_{ij}(u)|x_j(t-u)-x_j^*(t-u)|\Delta u+\\
&B_i|S_i(t)-S_i^*(t)|\Big\},
\end{aligned}
$$

$$
\begin{aligned}
D^+V_{i2}^{\Delta}(t)=&\frac{1}{1-\tau}\sum_{j=1}^{N}D_{ij}^{\tau}k_j\Big\{|x_j(t)-x_j^*(t)|e_\eta(\sigma(t+\tau_{ij}),0)-(1-\tau_{ij}^{\Delta}(t))\\
&|x_j(t-\tau_{ij}(t))-x_j^*(t-\tau_{ij}(t))|e_\eta(\sigma(t-\tau_{ij}(t)+\tau_{ij}),0)\Big\}\\
\le&\sum_{j=1}^{N}D_{ij}^{\tau}k_j\Big\{\frac{1}{1-\tau}|x_j(t)-x_j^*(t)|e_\eta(\sigma(t+\tau_{ij}),0)-\\
&|x_j(t-\tau_{ij}(t))-x_j^*(t-\tau_{ij}(t))|e_\eta(\sigma(t),0)\Big\},\\
D^+V_{i3}^{\Delta}(t)=&\sum_{j=1}^{N}\bar{D}_{ij}k_j\int_0^{+\infty}K_{ij}(u)\Big\{|x_j(t)-x_j^*(t)|e_\eta(\sigma(t+u),0)-\\
&|x_j(t-u)-x_j^*(t-u)|e_\eta(\sigma(t),0)\Big\}\Delta u,\\
D^+V_{i4}^{\Delta}(t)=&\eta e_\eta(t,0)|S_i(t)-S_i^*(t)|+e_\eta(\sigma(t),0)D^+|S_i(t)-S_i^*(t)|^{\Delta}\\
\le&\Big[\eta e_\eta(t,0)-\underline{c}_i e_\eta(\sigma(t),0)\Big]|S_i(t)-S_i^*(t)|+\\
&k_iE_ie_\eta(\sigma(t),0)|x_i(t)-x_i^*(t)|.
\end{aligned}
$$

Noting that $1\le e_\eta(\sigma(t),0)\le(1+\eta\bar{\mu})e_\eta(t,0)$, in view of (3.2), we have

$$
\begin{aligned}
D^+V^{\Delta}(t)=&\sum_{i=1}^{N}\big(D^+V_{i1}^{\Delta}(t)+D^+V_{i2}^{\Delta}(t)+D^+V_{i3}^{\Delta}(t)+D^+V_{i4}^{\Delta}(t)\big)\\
\le&\sum_{i=1}^{N}\Big\{\big[\eta e_\eta(t,0)+(k_iE_i-\underline{\alpha}_i)e_\eta(\sigma(t),0)\big]|x_i(t)-x_i^*(t)|+\\
&\sum_{j=1}^{N}\big\{D_{ij}k_je_\eta(\sigma(t),0)+\frac{1}{1-\tau}D_{ij}^{\tau}k_je_\eta(\sigma(t+\tau_{ij}),0)+\\
&\bar{D}_{ij}k_j\int_0^{+\infty}K_{ij}(u)e_\eta(\sigma(t+u),0)\Delta u\big\}|x_j(t)-x_j^*(t)|\Big\}+\\
&\sum_{i=1}^{N}\big\{B_ie_\eta(\sigma(t),0)+\eta e_\eta(t,0)-\underline{c}_ie_\eta(\sigma(t),0)\big\}|S_i(t)-S_i^*(t)|\\
\le&\sum_{i=1}^{N}\Big\{\eta-\underline{\alpha}_i+(1+\eta\bar{\mu})k_i\big\{E_i+\sum_{j=1}^{N}\big(D_{ji}+\frac{1}{1-\tau}D_{ji}^{\tau}+\bar{D}_{ji}\big)\big\}\Big\}\\
&e_\eta(t,0)|x_i(t)-x_i^*(t)|+\\
&\sum_{i=1}^{N}\Big\{\eta-\underline{c}_i+(1+\eta\bar{\mu})B_i\Big\}e_\eta(t,0)|S_i(t)-S_i^*(t)|\\
\le&0,\quad t>0.
\end{aligned}
$$

It follows that $V(t)\le V(0)$ for $t>0$. Hence, $\sum_{i=1}^{N}(V_{i1}(t)+V_{i4}(t))\le V(0)$. Letting $A_i(t)=$

$\sum_{i=1}^N |x_i(t)-x_i^*(t)|+\sum_{i=1}^N |S_i(t)-S_i^*(t)|$, we have

$$\begin{aligned}A_i(t)=&e_{\ominus\eta}(t,0)\sum_{i=1}^N(V_{i1}(t)+V_{i4}(t))\\ \le&e_{\ominus\eta}(t,0)\Bigg\{\sum_{i=1}^N\Big(|x_i(0)-x_i^*(0)|e_\eta(0,0)+|S_i(0)-S_i^*(0)|e_\eta(0,0)+\\ &\sum_{j=1}^N D_{ij}^\tau k_j\int_{-\tau_{ij}(0)}^0|x_j(s)-x_j^*(s)|e_\eta(\sigma(s+\tau_{ij}),0)\Delta s+\\ &\sum_{j=1}^N \bar{D}_{ij}k_j\int_0^{+\infty}\int_{-u}^0 K_{ij}(u)|x_j(s)-x_j^*(s)|e_\eta(\sigma(s+u),0)\Delta s\Delta u\Big)\Bigg\}\\ \le&M^*e_{\ominus\eta}(t,0)\Bigg\{\sum_{i=1}^N\sup_{u\in(-\infty,0]_T}|\phi_i(u)-x_i^*(u)|+\sum_{i=1}^N\sup_{u\in(-\infty,0]_T}|\psi_i(u)-S_i^*(u)|\Bigg\},\end{aligned}$$

where

$$\begin{aligned}M^*=&1+\sum_{j=1}^N D_{ij}^\tau k_j\int_{-\tau_{ij}(0)}^0 e_\eta(\sigma(s+\tau_{ij}),0)\Delta s+\\ &\sum_{j=1}^N\bar{D}_{ij}k_j\int_0^{+\infty}\int_{-u}^0 K_{ij}(u)e_\eta(\sigma(s+u),0)\Delta s\Delta u>1.\end{aligned}$$

By Definition 2.4, the periodic solution of system (1.1) is globally exponentially stable. This completes the proof. □

4. Examples

In system (1.1), we consider the following assumptions. For $i,j=1,2$;

$$\begin{bmatrix} f_1(x_1)\\ f_2(x_2)\end{bmatrix}=\frac{1}{2\pi e^{4\pi}}\begin{bmatrix}\arctan x_1\\ \arctan x_2\end{bmatrix},\ \begin{bmatrix}\alpha_1(t)\\ \alpha_2(t)\end{bmatrix}=\begin{bmatrix}3+2\cos\frac{\pi t}{3}\\ 2+\sin\frac{\pi t}{3}\end{bmatrix},\ \begin{bmatrix} I_1(t)\\ I_2(t)\end{bmatrix}=\begin{bmatrix}\sin\frac{\pi t}{3}\\ \cos\frac{\pi t}{3}\end{bmatrix}$$

$$\begin{bmatrix} B_1(t)\\ B_2(t)\end{bmatrix}=\frac{1}{2\pi e^{4\pi}}\begin{bmatrix}\sin\frac{\pi t}{3}\\ \cos\frac{\pi t}{3}\end{bmatrix},\ \begin{bmatrix} c_1(t)\\ c_2(t)\end{bmatrix}=\begin{bmatrix}2+\sin\frac{\pi t}{3}\\ 2+\cos\frac{\pi t}{3}\end{bmatrix},\ \begin{bmatrix}\tau_{11}(t)&\tau_{12}(t)\\ \tau_{21}(t)&\tau_{22}(t)\end{bmatrix}=\begin{bmatrix}1&2\\1&1\end{bmatrix}$$

$$\begin{bmatrix} D_{11}(t)&D_{12}(t)\\ D_{21}(t)&D_{22}(t)\end{bmatrix}=\begin{bmatrix} D_{11}^\tau(t)&D_{12}^\tau(t)\\ D_{21}^\tau(t)&D_{22}^\tau(t)\end{bmatrix}=\begin{bmatrix}\bar{D}_{11}(t)&\bar{D}_{12}(t)\\ \bar{D}_{21}(t)&\bar{D}_{22}(t)\end{bmatrix}=\begin{bmatrix}\frac{1}{24}\sin\frac{\pi t}{3}&\frac{1}{24}\sin\frac{\pi t}{3}\\ \frac{1}{24}\cos\frac{\pi t}{3}&\frac{1}{24}\cos\frac{\pi t}{3}\end{bmatrix}$$

$$\begin{bmatrix} E_1(t)\\ E_2(t)\end{bmatrix}=\begin{bmatrix}3+\sin\frac{\pi t}{3}\\ 3+\cos\frac{\pi t}{3}\end{bmatrix},\quad \begin{bmatrix} J_1(t)\\ J_2(t)\end{bmatrix}=\begin{bmatrix}\sin(\frac{\pi t-3}{3})\\ \cos\frac{\pi t}{3}\end{bmatrix}$$

$$\begin{bmatrix} K_{11}(t)&K_{12}(t)\\ K_{21}(t)&K_{22}(t)\end{bmatrix}=\begin{bmatrix}\frac{1}{2}e^{-2t}&\frac{1}{4}e^{-4t}\\ \frac{1}{8}e^{-8t}&\frac{1}{2}e^{-2t}\end{bmatrix},$$

thus

$$\begin{bmatrix} \underline{\alpha}_1 \\ \underline{\alpha}_2 \end{bmatrix} = \begin{bmatrix} \underline{c}_1 \\ \underline{c}_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \frac{1}{2\pi e^{4\pi}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \frac{1}{2\pi e^{4\pi}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} = \begin{bmatrix} D_{11}^{\tau} & D_{12}^{\tau} \\ D_{21}^{\tau} & D_{22}^{\tau} \end{bmatrix} = \begin{bmatrix} \bar{D}_{11} & \bar{D}_{12} \\ \bar{D}_{21} & \bar{D}_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{24} & \frac{1}{24} \\ \frac{1}{24} & \frac{1}{24} \end{bmatrix}$$

By Theorem 3.1, we know

$$P_1 = P_2 = \frac{1}{2\pi e^{4\pi}} - 1 < 0,$$
$$Q_1 = Q_2 = \frac{4}{2\pi e^{4\pi}} + \frac{1}{6\pi e^{4\pi}} - 1 = \frac{13}{6\pi e^{4\pi}} - 1 < 0.$$

All the conditions in Theorem 3.1 are satisfied. Therefore, the 6-periodic solution of the system (1.1) in example is globally exponentially stable (see Figures 1 and 2).

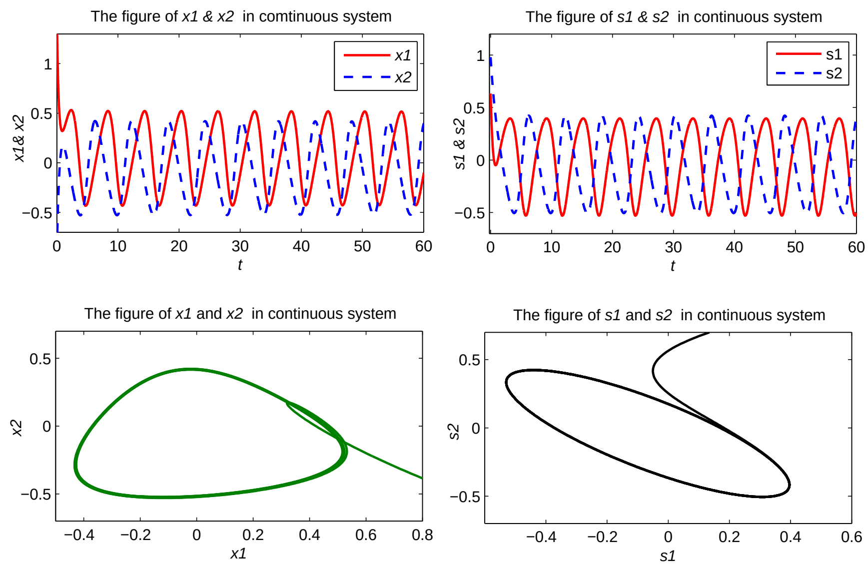

Figure 1 The behavior of periodic solution in continuous-system in Example

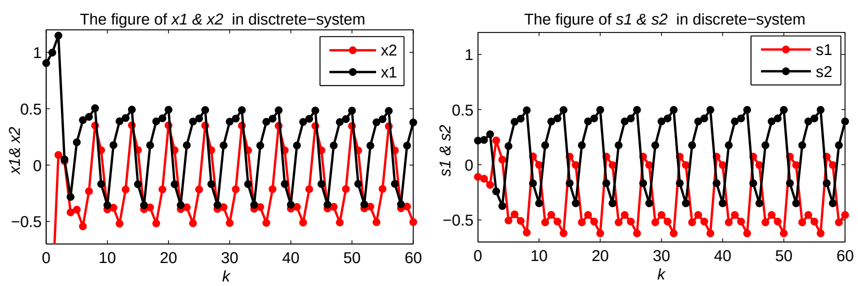

Figure 2 The behavior of periodic solution in discrete-system in Example

5. Conclusions

Without assuming the boundedness of the activation functions [15] and monotone of the variable $\alpha_i(t)$ [10], some conditions are obtained to ensure the global exponential stability of periodic solution for competitive neural networks with time-varying and distributed delays on time scales which unify the continuous and discrete situations. Therefore, our results for applications are more general than the previous results involved in competitive neural networks.

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