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Inclusion Relationships for p-Valent Analytic Functions Involving the Dziok-Srivastava Operator

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Abstract In this paper, we use the methods of differential subordination and the properties of convolution to investigate the class $W_p(\mathcal{H}(a_i, b_j); \phi)$ of multivalent analytic functions, which is defined by the Dziok-Srivastava operator $\mathcal{H}(a_1, \ldots, a_q; b_1, \ldots, b_s)$. Some inclusion properties for this class are obtained.

Keywords analytic functions; subordination; Hadmard product (or convolution); Dziok-Srivastava operator.

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1. Introduction

Let $\mathcal{A}_p$ denote the class of functions f of the form

$$f(z) = z^p + \sum_{k=1}^{\infty} a_{k+p} z^{k+p}, \quad p \in \mathbb{N} = \{1, 2, \ldots\}, \tag{1.1}$$

which are analytic in the open unit disk $\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$. Also, let $\mathcal{A}_1 = \mathcal{A}$.

Let $f, g \in \mathcal{A}_p$, where f is given by (1.1) and g is defined by

$$g(z) = z^p + \sum_{k=1}^{\infty} b_{k+p} z^{k+p}.$$

Then the Hadmard product (or convolution) $f * g$ of the functions f and g is defined by

$$(f * g)(z) = z^p + \sum_{k=1}^{\infty} a_{k+p} b_{k+p} z^{k+p} = (g * f)(z).$$

For two functions f and g, analytic in $\mathbb{U}$, we say that the function f is subordinate to g in $\mathbb{U}$, if there exists a Schwarz function ω, which is analytic in $\mathbb{U}$ with

$$\omega(0) = 0 \text{ and } |\omega(z)| < 1, \quad z \in \mathbb{U},$$

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such that

$$f(z) = g(\omega(z)), \quad z \in \mathbb{U}.$$

We denote this subordination by $f(z) \prec g(z)$. Furthermore, if the function g is univalent in $\mathbb{U}$, then we have the following equivalence [3, 12, 19]:

$$f(z) \prec g(z) \ (z \in \mathbb{U}) \Longleftrightarrow f(0) = g(0) \text{ and } f(\mathbb{U}) \subset g(\mathbb{U}).$$

Let M be the class of functions $\phi(z)$ which are analytic and univalent in $\mathbb{U}$ and for which $\phi(\mathbb{U})$ is convex with $\phi(0) = 1$ and $\mathrm{Re}[\phi(z)] > 0$ for $z \in \mathbb{U}$.

By making use of the principle of subordination between analytic functions, Ma and Minda [11] introduced the subclasses $\mathcal{S}_p^*(\phi)$ and $\mathcal{K}_p(\phi)$ of the class $\mathcal{A}_p$ for $p \in \mathbb{N}$ and $\phi \in M$, which are defined by

$$\mathcal{S}_p^*(\phi) = \big\{f \in \mathcal{A}_p : \frac{zf'(z)}{pf(z)} \prec \phi(z) \text{ in } \mathbb{U}\big\}$$

and

$$\mathcal{K}_p(\phi) = \big\{f \in \mathcal{A}_p : \frac{1}{p} + \frac{zf''(z)}{pf'(z)} \prec \phi(z) \text{ in } \mathbb{U}\big\}.$$

In its special case when

$$p = 1 \text{ and } \phi(z) = \frac{1 + Az}{1 + Bz}, \quad -1 \leq B < A \leq 1,$$

we obtain the classes

$$\mathcal{S}^*(A, B) = \mathcal{S}_1^*[\frac{1 + Az}{1 + Bz}] = \big\{f \in \mathcal{A} : \frac{zf'(z)}{f(z)} \prec \frac{1 + Az}{1 + Bz} \quad (z \in \mathbb{U})\big\}$$

and

$$\mathcal{K}(A, B) = \mathcal{K}_1[\frac{1 + Az}{1 + Bz}] = \big\{f \in \mathcal{A} : 1 + \frac{zf''(z)}{f'(z)} \prec \frac{1 + Az}{1 + Bz} \quad (z \in \mathbb{U})\big\},$$

which were introduced by Janowski [10]. Further, for $A = 1$ and $B = -1$, the above classes reduce to the well-known classes $\mathcal{S}^*$ and $\mathcal{K}$ of starlike and convex functions in $\mathbb{U}$, respectively.

For parameters $a_i \in \mathbb{C}$ $(i = 1, 2, \ldots, q)$ and $b_j \in \mathbb{C}\backslash\mathbb{Z}_0^-$ $(\mathbb{Z}_0^- = 0, -1, -2, \ldots;\ j = 1, 2, \ldots, s)$, the generalized hypergeometric function ${}_qF_s(a_1, \ldots, a_q; b_1, \ldots, b_s; z)$ is defined by

$${}_qF_s(a_1, \ldots, a_q; b_1, \ldots, b_s; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k \cdots (a_q)_k}{(b_1)_k \cdots (b_s)_k} \frac{z^k}{k!},$$

$$q \leq s + 1;\ q, s \in \mathbb{N}_0 = \mathbb{N} \cup \{0\};\ z \in \mathbb{U},$$

where $(\lambda)_k$ denotes the Pochhammer symbol defined, in terms of Gamma function, by

$$(\lambda)_k = \frac{\Gamma(\lambda + k)}{\Gamma(\lambda)} = \begin{cases} 1, & k = 0;\ \lambda \in \mathbb{C} \setminus \{0\}, \\ \lambda(\lambda + 1) \cdots (\lambda + k - 1), & k \in \mathbb{N};\ \lambda \in \mathbb{C}. \end{cases}$$

Dziok and Srivastava in [6] (see also [7, 8]) considered a linear operator

$$\mathcal{H}(a_1, \ldots, a_q; b_1, \ldots, b_s) : \mathcal{A}_p \longrightarrow \mathcal{A}_p,$$

defined by the Hadamard product

$$\mathcal{H}(a_1, \ldots, a_q; b_1, \ldots, b_s)f(z) = [z^p \cdot {}_qF_s(a_1, \ldots, a_q; b_1, \ldots, b_s; z)] * f(z)$$

$$= z^p + \sum_{k=1}^{\infty} \frac{(a_1)_k \cdots (a_q)_k}{(b_1)_k \cdots (b_s)_k} \frac{a_{k+p}}{k!} z^{k+p}, \tag{1.2}$$

where $f \in \mathcal{A}_p$ is given by (1.1).

The Dziok-Srivastava operator $\mathcal{H}(a_1, \ldots, a_q; b_1, \ldots, b_s)$ includes various linear operators, which were considered in earlier works, such as (for example) the linear operators introduced by Hohlov [9], Carlson and Shaffer [2], Bernardi [1], Ruschewyh [13] and Srivastava and Owa [18].

For the sake of simplicity, we denote

$$\mathcal{H}(a_i, b_j)f(z) = \mathcal{H}(a_1, \ldots, a_i, \ldots, a_q; b_1, \ldots, b_j, \ldots, b_s)f(z),$$

$$\mathcal{H}(a_i')f(z) = \mathcal{H}(a_i', b_j)f(z) = \mathcal{H}(a_1, \ldots, a_i', \ldots, a_q; b_1, \ldots, b_j, \ldots, b_s)f(z), \tag{1.3}$$

and

$$\mathcal{H}(b_j')f(z) = \mathcal{H}(a_i, b_j')f(z) = \mathcal{H}(a_1, \ldots, a_i, \ldots, a_q; b_1, \ldots, b_j', \ldots, b_s)f(z). \tag{1.4}$$

Definition 1.1 *Let $z \in \mathbb{U}$, $p \in \mathbb{N}$ and $\phi \in M$. We denote by $W_p(\mathcal{H}(a_i, b_j); \phi)$ the subclass of functions $f \in \mathcal{A}_p$ of the form (1.1) which satisfy the following condition*

$$\frac{z[\mathcal{H}(a_i, b_j)f(z)]'}{p\mathcal{H}(a_i, b_j)f(z)} \prec \phi(z).$$

In particular, when $\phi(z) = \frac{1+Az}{1+Bz}$ $(-1 \le B < A \le 1)$, we write

$$W_p(\mathcal{H}(a_i, b_j); A, B) = W_p\big(\mathcal{H}(a_i, b_j); \frac{1+Az}{1+Bz}\big).$$

Remark 1.1 (i) For positive real numbers $a_1, \ldots, a_q; b_1, \ldots, b_s$ and for $0 \le B \le 1$ and $-B \le A < B$, the class $W_p(\mathcal{H}(a_i, b_j); A, B) = V_p(a_i; A, B)$ was investigated by Sokol [16].

(ii) For complex numbers $a_1, \ldots, a_q; b_1, \ldots, b_s$ and for $-1 \le B \le 0$ and $|A| < 1$ $(A \in \mathbb{C})$, the class $W_p(\mathcal{H}(a_i, b_j); A, B) = V_1^p(\mathcal{H}(a_i); A, B)$ was studied by Sokol [17]. Further, for $p = 1$, the class $V_1^1(\mathcal{H}(a_i); A, B) = V(a_i; A, B)$ was considered by Dziok and Srivastava [4].

In this paper, we aim to investigate some inclusion properties of the class $W_p(\mathcal{H}(a_i, b_j); \phi)$. Also, some results involving the special case $W_p(\mathcal{H}(a_i, b_j); A, B)$ $(-1 \le B < A \le 1)$ of this class are considered. The results obtained unify and extend some results of [5], [16] and [17].

2. Main results

The following lemmas will be required in our investigation.

Lemma 2.1 *Let $\mathcal{H}(a_i')(z), \mathcal{H}(a_i'')(z), \mathcal{H}(b_j')(z)$ and $\mathcal{H}(b_j'')(z)$ be defined by (1.2), (1.3) and (1.4). Then, for $p \in \mathbb{N}$, $i \in \{1, 2, \ldots, q\}$ and $j \in \{1, 2, \ldots, s\}$*

$$\mathcal{H}(a_i')(z) = \mathcal{H}(a_i'')(z) * \phi_p(a_i', a_i'')(z) \tag{2.1}$$

and

$$\mathcal{H}(b_j')(z) = \mathcal{H}(b_j'')(z) * \phi_p(b_j'', b_j')(z), \tag{2.2}$$

where

$$\phi_p(\alpha, \beta)(z) = \sum_{k=0}^{\infty} \frac{(\alpha)_k}{(\beta)_k} z^{k+p}.$$

Proof From (1.2) and (1.3), we have

$$\begin{aligned}\mathcal{H}(a_i')(z) &= \sum_{k=0}^{\infty} \frac{(a_1)_k \cdots (a_i')_k \cdots (a_q)_k}{(b_1)_k \cdots (b_s)_k} \frac{z^{k+p}}{k!} \\ &= \sum_{k=0}^{\infty} \frac{(a_1)_k \cdots (a_i'')_k \cdots (a_q)_k}{(b_1)_k \cdots (b_s)_k} \cdot \frac{(a_i')_k}{(a_i'')_k} \cdot \frac{z^{k+p}}{k!} \\ &= \mathcal{H}(a_i'')(z) * \phi_p(a_i', a_i'')(z)\end{aligned}$$

and the assertion (2.1) holds. Similarly, we can prove (2.2) by using (1.2) and (1.4). □

Lemma 2.2 ([15]) *Let $f \in \mathcal{K}$ and $g \in \mathcal{S}^*$. Then, for every analytic function h in $\mathbb{U}$,*

$$\frac{(f * hg)(\mathbb{U})}{(f * g)(\mathbb{U})} \subset \overline{\mathrm{co}}[h(\mathbb{U})],$$

where $\overline{\mathrm{co}}[h(\mathbb{U})]$ denotes the closed convex hull of $h(\mathbb{U})$.

Lemma 2.3 ([14]) *If either $0 < \alpha \leq \beta$ and $\beta \geq 2$ when α, β are real, or $\mathrm{Re}[\alpha + \beta] \geq 3$, $\mathrm{Re}[\alpha] \leq \mathrm{Re}[\beta]$ and $\mathrm{Im}[\alpha] = \mathrm{Im}[\beta]$ when α, β are complex, then the function*

$$\phi_1(\alpha, \beta)(z) = \sum_{k=0}^{\infty} \frac{(\alpha)_k}{(\beta)_k} z^{k+1}, \quad z \in \mathbb{U}$$

belongs to the class $\mathcal{K}$ of convex functions.

We begin by proving our first inclusion relationship given by Theorem 2.1 below.

Theorem 2.1 *Let $p \in \mathbb{N}$ and $\phi \in M$ with*

$$\mathrm{Re}[\phi(z)] > 1 - \frac{1}{p}, \quad z \in \mathbb{U}. \tag{2.3}$$

If a_i', a_i'' satisfy either

$$a_i',\ a_i'' \text{ are real such that } 0 < a_i' \leq a_i'' \text{ and } a_i'' \geq 2, \tag{2.4}$$

or

$$a_i',\ a_i'' \text{ are complex such that } \mathrm{Re}[a_i' + a_i''] \geq 3,\ \mathrm{Re}[a_i'] \leq \mathrm{Re}[a_i''] \text{ and } \mathrm{Im}[a_i'] = \mathrm{Im}[a_i''], \tag{2.5}$$

then

$$W_p(\mathcal{H}(a_i''); \phi) \subset W_p(\mathcal{H}(a_i'); \phi).$$

Proof Let $f \in W_p(\mathcal{H}(a_i''); \phi)$. Then, by the definition of the class $W_p(\mathcal{H}(a_i''); \phi)$, we have

$$\frac{z[\mathcal{H}(a_i'')f(z)]'}{p\mathcal{H}(a_i'')f(z)} = \phi(\omega(z)), \tag{2.6}$$

where ϕ is convex univalent with $\mathrm{Re}[\phi(z)] > 0$ and $|\omega(z)| < 1$ in $\mathbb{U}$ with $\omega(0) = 0 = \phi(0) - 1$. Therefore,

$$\frac{z[z^{1-p}\mathcal{H}(a_i'')f(z)]'}{z^{1-p}\mathcal{H}(a_i'')f(z)} = p[\phi(\omega(z)) - 1] + 1 \prec \frac{1+z}{1-z}. \tag{2.7}$$

Applying (1.2), (2.1) and the properties of convolution, we obtain

$$\frac{z[\mathcal{H}(a_i')f(z)]'}{p\mathcal{H}(a_i')f(z)} = \frac{z[(\mathcal{H}(a_i') * f)(z)]'}{p(\mathcal{H}(a_i') * f)(z)} = \frac{z[(\mathcal{H}(a_i'') * \phi_p(a_i', a_i'') * f)(z)]'}{p(\mathcal{H}(a_i'') * \phi_p(a_i', a_i'') * f)(z)}$$

$$= \frac{\phi_p(a'_i, a''_i)(z) * z[(\mathcal{H}(a''_i) * f)(z)]'}{p\phi_p(a'_i, a''_i)(z) * (\mathcal{H}(a''_i) * f)(z)} = \frac{\phi_p(a'_i, a''_i)(z) * z[\mathcal{H}(a''_i)f(z)]'}{p\phi_p(a'_i, a''_i)(z) * \mathcal{H}(a''_i)f(z)}. \quad (2.8)$$

It follows from (2.3) and (2.7) that $z^{1-p}\mathcal{H}(a''_i)f(z) \in \mathcal{S}^*$. Also, by Lemma 2.3, we see that $z^{1-p}\phi_p(a'_i, a''_i)(z) \in \mathcal{K}$. Thus, in view of (2.8) and Lemma 2.2, we have

$$\frac{\left\{[z^{1-p}\phi_p(a'_i, a''_i)] * \phi(\omega)z^{1-p}\mathcal{H}(a''_i)f\right\}(\mathbb{U})}{\left\{[z^{1-p}\phi_p(a'_i, a''_i)] * z^{1-p}\mathcal{H}(a''_i)f\right\}(\mathbb{U})} \subset \overline{\mathrm{co}}\phi[\omega(\mathbb{U})] \subset \phi(\mathbb{U})$$

because ϕ is convex univalent function. By the definition of subordination, we know that (2.8) is subordinate to ϕ in $\mathbb{U}$, and so $f \in W_p(\mathcal{H}(a'_i); \phi)$. $\Box$

Theorem 2.2 *Let $p \in \mathbb{N}$ and $\phi \in M$ with (2.3) holding. If b'_j, b''_j satisfy either*

$$b'_j,\ b''_j \text{ are real such that } 0 < b''_j \le b'_j \text{ and } b'_j \ge 2, \quad (2.9)$$

or

$$b'_j,\ b''_j \text{ are complex such that } \mathrm{Re}[b'_j + b''_j] \ge 3,\ \mathrm{Re}[b''_j] \le \mathrm{Re}[b'_j] \text{ and } \mathrm{Im}[b'_j] = \mathrm{Im}[b''_j], \quad (2.10)$$

then

$$W_p(\mathcal{H}(b''_j); \phi) \subset W_p(\mathcal{H}(b'_j); \phi).$$

Proof Applying the same techniques as in the proof of Theorem 2.1, and using (1.2) and (2.2), we obtain the result asserted by Theorem 2.2. $\Box$

Taking

$$\phi(z) = \frac{1 + Az}{1 + Bz}, \quad -1 \le B < A \le 1;\ z \in \mathbb{U}$$

in Theorems 2.1 and 2.2, respectively, we have the following results.

Corollary 2.1 *Let $p \in \mathbb{N}$, $i \in \{1, 2, \ldots, q\}$ and*

$$\mathrm{Re}\Big(\frac{1 + Az}{1 + Bz}\Big) > 1 - \frac{1}{p}, \quad -1 \le B < A \le 1;\ z \in \mathbb{U}. \quad (2.11)$$

If a'_i and a''_i satisfy either (2.4) or (2.5), then

$$W_p(\mathcal{H}(a''_i); A, B) \subset W_p(\mathcal{H}(a'_i); A, B).$$

Corollary 2.2 *Let $p \in \mathbb{N}$, $j \in \{1, 2, \ldots, s\}$ and (2.11) hold. If b'_j and b''_j satisfy either (2.9) or (2.10), then*

$$W_p(\mathcal{H}(b''_j); A, B) \subset W_p(\mathcal{H}(b'_j); A, B).$$

Remark 2.1 We note that, in [4, 17] there are no results concerning inclusion relationships between the function classes with respect to the parameters $b_j \in \mathbb{C}\backslash\mathbb{Z}_0^-$ ($\mathbb{Z}_0^- = 0, -1, -2, \ldots;\ j = 1, 2, \ldots, s$). However, in this paper, we obtain some inclusion relationships with respect to the parameters b_j, see, for details, the above Theorem 2.2 and Corollary 2.2.

Next, we will show that the class $W_p(\mathcal{H}(a_i, b_j); \phi)$ is preserved under convolution with convex functions.

Theorem 2.3 *Let $p \in \mathbb{N}$, $g \in \mathcal{K}$ and $\phi \in M$ with (2.3) holding. Then*

$$f \in W_p(\mathcal{H}(a_i, b_j); \phi) \Rightarrow (z^{p-1}g) * f \in W_p(\mathcal{H}(a_i, b_j); \phi).$$

Proof Let $f \in W_p(\mathcal{H}(a_i, b_j); \phi)$ and $g \in \mathcal{K}$. Based on the same concept as the proof of Theorem 2.1, we have

$$\begin{aligned}\frac{z[\mathcal{H}(a_i, b_j)\left((z^{p-1}g) * f\right)(z)]'}{p[\mathcal{H}(a_i, b_j)\left((z^{p-1}g) * f\right)(z)]} &= \frac{(z^{p-1}g(z)) * z[\mathcal{H}(a_i, b_j)f(z)]'}{p(z^{p-1}g(z)) * \mathcal{H}(a_i, b_j)f(z)} \\ &= \frac{(z^{p-1}g(z)) * \phi(\omega)\mathcal{H}(a_i, b_j)f(z)}{(z^{p-1}g(z)) * \mathcal{H}(a_i, b_j)f(z)} \\ &= \frac{g(z) * \phi(\omega)z^{p-1}\mathcal{H}(a_i, b_j)f(z)}{g(z) * z^{p-1}\mathcal{H}(a_i, b_j)f(z)} \\ &\prec \phi(z), \quad z \in \mathbb{U},\end{aligned}$$

and so that $(z^{p-1}g) * f \in W_p(\mathcal{H}(a_i, b_j); \phi)$. $\square$

Corollary 2.3 *Let $p \in \mathbb{N}$ and $\phi \in M$ with (2.3) holding. Suppose also that*

$$h_1(z) = \sum_{k=1}^{\infty} \big(\frac{1+\xi}{k+\xi}\big) z^k, \quad \xi > -1;\ z \in \mathbb{U},$$

$$h_2(z) = \frac{1}{1-\varepsilon} \log\big[\frac{1-\varepsilon z}{1-z}\big], \quad \log 1 = 0;\ |\varepsilon| \le 1\ (\varepsilon \ne 1);\ z \in \mathbb{U},$$

and

$$h_3(z) = \sum_{k=1}^{\infty} \frac{z^k}{k} = -\log(1-z).$$

Then, for $\rho = 1, 2, 3$, we have

$$f \in W_p(\mathcal{H}(a_i, b_j); \phi) \Rightarrow (z^{p-1}h_\rho) * f \in W_p(\mathcal{H}(a_i, b_j); \phi).$$

Proof The function h_1 was shown to be convex by Ruschewyh [13], while h_2 and h_3 are well known to be convex in $\mathbb{U}$. Thus, the assertion follows from Theorem 2.3. $\square$

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