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A New Characterization of Simple K_4-Groups

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Abstract In this paper, we characterize some simple K_4-groups only by using the group order and largest element orders, where a simple K_4-group is a simple group of order containing exactly four distinct primes.

Keywords finite group; the largest element order; the second largest element order; simple K_4-group; characterization

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1. Introduction

Shi put forward the approach to characterizing a finite group by using the group order and the set of element-orders in the 1980's. At present, this characterization for finite simple groups was finished (some results can be seen in [1–8]). To weaken the quantitative condition, He and Chen began to characterize a finite group only by using the group order and the largest element order in 2009, and proved that simple K_3-groups, sporadic simple groups, some alternating groups, and some simple groups of Lie Type can be uniquely determined by the group order and largest element orders [9–17]. To continue this work, in this paper, we characterize some simple K_4-groups via the group order and largest element orders.

The groups mentioned in this paper are all finite groups, the number in bracket "()" behind a group is the order of the group, e.g., $L_2(7)(2^3\cdot 3\cdot 7)$ means that $L_2(7)$ is of order $2^3\cdot 3\cdot 7$. We use $\pi_e(G)$ to denote the set of orders of elements in G, $k_1(G)$ and $k_2(G)$ to denote the largest element order and second largest element order of G respectively, and $\pi(G)$ is the set of all prime divisors of $|G|$. Let $\Gamma(G)$ denote the prime graph of G and $t(G)$ is the number of connected components of $\Gamma(G)$. We denote by $\{\pi_i, i=1,\ldots,t(G)\}$ the sets of vertex of the connected components of the prime graph, and if the order of G is even, denote by π_1 the component containing 2 (see [18]).

2. Preliminary results

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Lemma 2.1 ([19]) *Suppose that G is a simple K_4-group. Then G is isomorphic to one of the following groups:*

(1) A_7, A_8, A_9, A_{10}, M_{11}, M_{12}, J_2, $Sz(8)$, $Sz(32)$, $L_3(4)$, $L_3(5)$, $L_3(7)$, $L_3(8)$, $L_3(17)$, $L_4(3)$, $S_4(4)$, $S_4(5)$, $S_4(7)$, $S_4(9)$, $S_6(2)$, $O_8^+(2)$, $G_2(3)$, $U_3(4)$, $U_3(5)$, $U_3(7)$, $U_3(8)$, $U_3(9)$, $U_4(3)$, $U_5(2)$, ${}^3D_4(2)$, ${}^2F_4(2)'$;

(2) $L_2(q)$, *where q is a prime power satisfying* $q(q^2-1)=(2,q-1)2^{n_1}\cdot 3^{n_2}\cdot p^{n_3}\cdot r^{n_4}$, *where* n_i $(1\le i\le 4)$ *are positive integers and p,r are distinct primes.*

Lemma 2.2 *Suppose that G has more than one prime graph components. Then one of the following holds:*

(1) G is a Frobenius group or a 2-Frobenius group;

(2) G has a normal series $1\trianglelefteq H\trianglelefteq K\trianglelefteq G$, such that H and G/K are π_1-groups and K/H a non-abelian simple group, where π_1 is the prime graph component containing 2, H is a nilpotent group, and $|G/K|\mid|\mathrm{Out}(K/H)|$.

Proof The lemma follows from Theorem A and Lemma 3 in [18]. □

Lemma 2.3 $\pi_e(S_4(7))=\{2,3,4,5,6,7,8,12,14,21,24,25,28,42,56\}$, $\pi_e(S_4(9))=\{2,3,4,5,6,8,9,10,12,15,20,24,30,40,41\}$. *And therefore* $k_1(S_4(7))=56$, $k_2(S_4(7))=42$, $k_1(S_4(9))=41$, $k_2(S_4(9))=40$.

Proof The lemma follows from Corollary 2 in [20]. □

Lemma 2.4 $\pi_e(L_3(17))=\{2,3,4,6,8,9,12,16,17,18,24,32,34,36,48,68,72,96,136,144,272,288,307\}$, *and therefore,* $k_1(L_3(17))=307$, $k_2(L_3(17))=288$.

Proof The lemma follows from Corollary 3 in [21]. □

Lemma 2.5 *Let G be a simple K_4-group, except that $L_2(q)$. Then $|G|$, $k_1(G)$ and $k_2(G)$ are as in Table 1:*

G	$\|G\|$	$k_1(G)$	$k_2(G)$
A_7	$2^3\cdot 3^2\cdot 5\cdot 7$	7	6
A_8	$2^6\cdot 3^2\cdot 5\cdot 7$	15	7
A_9	$2^6\cdot 3^4\cdot 5\cdot 7$	15	12
A_{10}	$2^7\cdot 3^4\cdot 5^2\cdot 7$	21	15
M_{11}	$2^4\cdot 3^2\cdot 5\cdot 11$	11	8
M_{12}	$2^6\cdot 3^3\cdot 5\cdot 11$	11	10
J_2	$2^7\cdot 3^3\cdot 5^2\cdot 7$	15	12
$Sz(8)$	$2^6\cdot 5\cdot 7\cdot 13$	24	21
$Sz(32)$	$2^{10}\cdot 5^2\cdot 31\cdot 41$	20	15

G	$\|G\|$	$k_1(G)$	$k_2(G)$
$L_3(4)$	$2^6 \cdot 3^2 \cdot 5 \cdot 7$	7	5
$L_3(5)$	$2^5 \cdot 3 \cdot 5^3 \cdot 31$	31	24
$L_3(7)$	$2^5 \cdot 3^2 \cdot 7^3 \cdot 19$	19	16
$L_3(8)$	$2^9 \cdot 3^2 \cdot 7^2 \cdot 73$	73	63
$L_3(17)$	$2^9 \cdot 3^2 \cdot 17^3 \cdot 307$	307	288
$L_4(3)$	$2^7 \cdot 3^6 \cdot 5 \cdot 13$	20	13
$S_4(4)$	$2^8 \cdot 3^2 \cdot 5^2 \cdot 17$	17	15
$S_4(5)$	$2^6 \cdot 3^2 \cdot 5^4 \cdot 13$	30	20
$S_4(7)$	$2^8 \cdot 3^2 \cdot 5^2 \cdot 7^4$	56	42
$S_4(9)$	$2^8 \cdot 3^8 \cdot 5^2 \cdot 41$	41	40
$S_6(2)$	$2^9 \cdot 3^4 \cdot 5 \cdot 7$	15	12
$O_8^+(2)$	$2^{12} \cdot 3^5 \cdot 5^2 \cdot 7$	15	12
$G_2(3)$	$2^6 \cdot 3^6 \cdot 7 \cdot 13$	13	12
$U_3(4)$	$2^6 \cdot 3 \cdot 5^2 \cdot 13$	15	13
$U_3(5)$	$2^4 \cdot 3^2 \cdot 5^3 \cdot 7$	10	8
$U_3(7)$	$2^7 \cdot 3 \cdot 7^3 \cdot 43$	56	48
$U_3(8)$	$2^9 \cdot 3^4 \cdot 7 \cdot 19$	21	19
$U_3(9)$	$2^5 \cdot 3^6 \cdot 5^2 \cdot 73$	80	73
$U_4(3)$	$2^7 \cdot 3^6 \cdot 5 \cdot 7$	12	9
$U_5(2)$	$2^{10} \cdot 3^5 \cdot 5 \cdot 11$	18	15
${}^3D_4(2)$	$2^{12} \cdot 3^4 \cdot 7^2 \cdot 13$	28	21
${}^2F_4(2)'$	$2^{11} \cdot 3^3 \cdot 5^2 \cdot 13$	16	13

Table 1 The $|G|$,$k_1(G)$ and $k_2(G)$ of simple K_4-groups, except that $L_2(q)$

Proof The lemma follows from [22], Lemmas 2.3 and 2.4. □

3. Main results

In [17], we discussed the simple K_4-groups of part (II) in Lemma 2.1, and proved that simple K_4-groups of type $L_2(p)$ can be uniquely determined only by the group order and largest element order, where p is a prime but not $2^n - 1$. In this paper, we will try to discuss the simple K_4-groups of part (I) in Lemma 2.1.

Theorem 3.1 *Let G be a group and M be one of the following simple K_4-groups: A_7, A_9, A_{10}, J_2, M_{11}, M_{12}, $Sz(8)$, $Sz(32)$, $L_3(4)$, $L_3(7)$, $L_3(8)$, $S_6(2)$, $L_4(3)$, $S_4(4)$, $S_4(9)$, $O_8^+(2)$, $G_2(3)$, $U_3(4)$, $U_3(5)$, $U_3(7)$, $U_3(8)$, $U_3(9)$, $U_4(3)$, $U_5(2)$, ${}^3D_4(2)$, $L_3(17)$ and ${}^2F_4(2)'$. Then $G \cong M$ if and only if*

(1) $k_1(G) = k_1(M)$;

(ii) $|G| = |M|$.

Proof We only need to prove the sufficiency. If $M = A_7, A_9, A_{10}$, then the proof can be seen in [14]. If $M = M_{11}, M_{12}$, then the proof can be seen in [11]. If $M = Sz(8)$, $Sz(32)$, then the proof can be seen in [16]. If $M = L_3(4)$, $L_3(7)$, $L_3(8)$, $U_3(4)$, $U_3(5)$, $U_3(7)$, $U_3(8)$, $U_3(9)$, then the proof can be seen in [12]. And if $M = L_4(3)$, $S_4(4)$, $U_4(3)$, $G_2(3)$, ${}^2F_4(2)'$, then the proof can be seen in [13]. Therefore, we just need to consider the cases $M{=}J_2$, $S_4(9)$, $O_8^+(2)$, $S_6(2)$, ${}^3D_4(2)$, $U_5(2)$, $L_3(17)$. Now we will complete the proof through a case by case analysis.

Case 1 If $M{=}J_2$, then $G \cong M$.

In such case, $|G| = 2^6 \cdot 3^3 \cdot 5^2 \cdot 7$ and $k_1(G) = 15$. Firstly, we can assert that the G has a normal series $G \geq K \geq H \geq 1$, such that $\overline{K} = K/H$ is a non-abelian simple group, and $\{5, 7\} \subseteq \pi(\overline{K})$. In fact, let $G = G_0 > G_1 > \cdots > G_{k-1} > G_k = 1$ be a chief series of G. Then there must exist an integer i, such that $\{5, 7\} \cap \pi(G_i) \neq \Phi$, and $\{5, 7\} \cap \pi(G_{i+1}) = \Phi$. Let $K = G_i$, $H = G_{i+1}$. Then $G \geq K \geq H \geq 1$ is a normal series of G, and $\overline{K} = K/H$ is a minimal normal subgroup of $\overline{G} = G/H$. If $5 \in \pi(K)$, $7 \notin \pi(K)$, then $7 \in \pi(G/K)$. By Frattini's argument, we have $G = N_G(S_5)K$, where S_5 is a Sylow 5-subgroup of K. Therefore, we have $7 \in \pi(N_G(S_5))$, from which we know that $35 \in \pi_e(G)$, a contradiction. So $7 \in \pi(K)$. Similarly, we can prove that if $7 \in \pi(K)$, then $5 \in \pi(K)$. Thus we have $\{5, 7\} \subseteq \pi(K)$, and therefore $\{5, 7\} \subseteq \pi(\overline{K})$. Since $\overline{K}$ is the direct product of isomorphic simple groups, $\overline{K}$ is a non-abelian simple group. From [22] we can assume that $\overline{K}$ is isomorphic to one of the following simple groups: A_7 $(2^3 \cdot 3^2 \cdot 5 \cdot 7)$, $L_4(2)$ $(2^6 \cdot 3^2 \cdot 5 \cdot 7)$, $L_3(4)$ $(2^6 \cdot 3^2 \cdot 5 \cdot 7)$, A_8 $(2^6 \cdot 3^2 \cdot 5 \cdot 7)$ and $J_2(2^7 \cdot 3^3 \cdot 5^2 \cdot 7)$. We first suppose that $\overline{K}$ is isomorphic to $A_7, L_4(2), A_8, L_3(4)$. Since $G/C_G(\overline{K}) \lessapprox \mathrm{Aut}(\overline{K})$ and $|\mathrm{Aut}(\overline{K})| = |\mathrm{Out}(\overline{K})| \cdot |\overline{K}|$, we have $5 \mid |C_G(\overline{K})|$, which means that $35 \in \pi_e(G)$, a contradiction. Therefore, $\overline{K} \cong J_2$. In such case, $H = 1$, $K = G$, and thus $G \cong J_2$.

Case 2 If $M{=}$ $O_8^+(2)$, then $G \cong M$.

In such case, $|G| = 2^{12} \cdot 3^5 \cdot 5^2 \cdot 7$ and $k_1(G) = 15$. By the similar arguments in Case 1, we know that G has a normal series $G \geq K \geq H \geq 1$, such that $\overline{K} = K/H$ is a non-abelian simple group, and $\{5,\ 7\} \subseteq \pi(\overline{K})$. From [22], we can assume that $\overline{K}$ is isomorphic to one of the following simple groups: A_7 $(2^3{\cdot}3^2{\cdot}5{\cdot}7)$, A_8 $(2^6{\cdot}3^2{\cdot}5{\cdot}7)$, $L_3(4)$ $(2^3{\cdot}3^2{\cdot}5{\cdot}7)$, A_9 $(2^6{\cdot}3^4{\cdot}5{\cdot}7)$, J_2 $(2^7{\cdot}3^3{\cdot}5^2{\cdot}7)$, $S_6(2)$ $(2^9 \cdot 3^4 \cdot 5 \cdot 7)$, A_{10} $(2^7 \cdot 3^4 \cdot 5^2 \cdot 7)$ and $O_8^+(2)$ $(2^{12} \cdot 3^5 \cdot 5^2 \cdot 7)$. Clearly, $G/C_G(\overline{K}) \lessapprox \mathrm{Aut}(\overline{K})$ and $|\mathrm{Aut}(\overline{K})| = |\mathrm{Out}(\overline{K})| \cdot |\overline{K}|$. If $\overline{K}$ is isomorphic to $A_7, A_8, L_3(4), A_9, S_6(2)$, then $5 \mid |C_G(\overline{K})|$, which means that $35 \in \pi_e(G)$, a contradiction. If $\overline{K}$ is isomorphic to J_2, A_{10}, then $3 \mid |C_G(\overline{K})|$. If $3 \nmid |H|$, then $\overline{G} = G/H$ has an element with order 21, a contradiction. Therefore, we assume that $3 \mid |H|$. Consider the action on H by an element of order 7. We get that there exists a Sylow 3-subgroup L of H fixed by this action. Since $|L| \mid 3^2$, we have $7 \nmid |\mathrm{Aut}(L)|$, which means that such action is trivial. So $21 \in \pi_e(G)$, a contradiction too. Therefore, $\overline{K} \cong O_8^+(2)$. In such case, $H = 1$, $K = G$, and thus $G \cong O_8^+(2)$.

Case 3 If $M{=}$ $S_6(2)$, then $G \cong M$.

In such case, $|G| = 2^9 \cdot 3^4 \cdot 5 \cdot 7$ and $k_1(G) = 15$. By the similar arguments in Case 1, we know G has a normal series $G \geq K \geq H \geq 1$, such that $\overline{K} = K/H$ is a non-abelian simple group, and $\{5,7\} \subseteq \pi(\overline{K})$. From [22], we can assume that $\overline{K}$ is isomorphic to one of the following simple groups: A_7 $(2^3 \cdot 3^2 \cdot 5 \cdot 7)$, A_8 $(2^6 \cdot 3^2 \cdot 5 \cdot 7)$, $L_3(4)$ $(2^3 \cdot 3^2 \cdot 5 \cdot 7)$, A_9 $(2^6 \cdot 3^4 \cdot 5 \cdot 7)$ and $S_6(2)$ $(2^9 \cdot 3^4 \cdot 5 \cdot 7)$. Clearly, $G/C_G(\overline{K}) \lessapprox \mathrm{Aut}(\overline{K})$ and $|\mathrm{Aut}(\overline{K})| = |\mathrm{Out}(\overline{K})| \cdot |\overline{K}|$. If $\overline{K}$ is isomorphic to $A_7, A_8, L_3(4)$, then $3 \mid |C_G(\overline{K})|$, which means $21 \in \pi_e(G)$, a contradiction. If $\overline{K}$ is isomorphic to A_9, then $2 \mid |C_G(\overline{K})|$. As A_9 has an element with 15, G has an element of order 30, a contradiction. Therefore, $\overline{K} \cong S_6(2)$. In such case, $H = 1$, $K = G$, and thus $G \cong S_6(2)$.

Case 4 If $M=$ $S_4(9)$ or ${}^3D_4(2)$, then $G \cong M$.

The proof is similar to the above cases.

Case 5 If $M=$ $U_5(2)$, then $G \cong M$.

In such case, $|G| = 2^{10} \cdot 3^5 \cdot 5 \cdot 11$ and $k_1(G) = 18$. Since $k_1(G) = 18$, 11 is an isolated point in $\Gamma(G)$. If G is a Frobenius group with kernel K and complement H, then H is of order 11 as $|H|$ divides $|K| - 1$. Now H acts trivially on a Sylow 5-subgroup of K and so $55 \in \pi_e(G)$, which contradicts $k_1(G) = 18$. Suppose that G is a 2-Frobenius group with normal series $1 \le H \le K \le G$, where $|K/H| = 11$ and $|G/K| | 10$. In such case, $3 | |H|$. Consider the action on H by the element of order 11. One can see that K has a Sylow 3-subgroup L fixed by this action. Since $G = 2^{10} \cdot 3^5 \cdot 5 \cdot 11$, we have $|L| = 3^5$. Clearly, $\Omega_1(Z(L))$ is an elementary abelian 3-group. Because $k_1(G) = 18$, $|\Omega_1(Z(L))| \mid 3^4$. Consider the action on $\Omega_1(Z(L))$ by the element of order 11. We know such action is trivial for $11 \nmid |\mathrm{Aut}(\Omega_1(Z(L)))|$, which implies that $33 \in \pi_e(G)$, a contradiction. Therefore, by Lemma 2.2, G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$, such that H and G/K are π_1-groups and K/H a non-abelian simple group, where π_1 is the prime graph component containing 2, H is a nilpotent group, and $|G/K| \mid |\mathrm{Out}(K/H)|$. As $|G| = 2^{10} \cdot 3^5 \cdot 5 \cdot 11$, and 11 is an isolated point in $\Gamma(G)$, we have $\pi(H) \cup \pi(G/K) \subseteq \{2,3,5\}$ and $11 \in \pi(K/H)$. From [22], we can suppose that K/H is isomorphic to one of the following simple groups: $L_2(11)$ $(2^2 \cdot 3 \cdot 5 \cdot 11)$, M_{11} $(2^4 \cdot 3^2 \cdot 5 \cdot 11)$, M_{12} $(2^6 \cdot 3^3 \cdot 5 \cdot 11)$ and $U_5(2)$ $(2^{10} \cdot 3^5 \cdot 5 \cdot 11)$.

Suppose that $K/H \cong L_2(11)$, M_{11} or M_{12}. In such case, we can get that $3 \nmid |\mathrm{Out}(K/H)|$ and thus $3 \mid |H|$ by comparing the order of G. Let L be a Sylow 3-subgroup of H. We have $L \trianglelefteq G$ and $|L| \mid 3^4$. Clearly, $\Omega_1(Z(L))$ is an elementary abelian 3-group, and $|\Omega_1(Z(L))| \mid 3^4$. Consider the action on $\Omega_1(Z(L))$ by the element of order 11. Because $11 \nmid |\mathrm{Aut}(\Omega_1(Z(L)))|$, this action is trivial, which implies that $33 \in \pi_e(G)$, a contradiction. Therefore, we have $K/H \cong U_5(2)$. So $H = 1$, $K = G$, and therefore, $G \cong U_5(2)$.

Case 6 If $M=$ $L_3(17)$, then $G \cong M$.

The proof is similar to Case 5.

The proof of Theorem 3.1 is completed. $\square$

Theorem 3.2 *Let G be a group and M be one of the following simple K_4-groups: A_8, $L_3(5)$ and $S_4(5)$. Then $G \cong M$ if and only if*

(1) $k_i(G) = k_i(M)$, where $i = 1, 2$;

(2) $|G| = |M|$.

Proof It is enough to prove the sufficiency. If $M = A_8$, then the proof can be seen in [14]. So we just need to consider the cases $M = L_3(5)$, $S_4(5)$. Because the proof is similar, we only consider the case $M = L_3(5)$. In this case, $|G| = 2^5 \cdot 3 \cdot 5^3 \cdot 31$, $k_1(G) = 31$ and $k_2(G) = 24$, and therefore, 31 is an isolated point in $\Gamma(G)$. If G is a Frobenius group with kernel K and complement H, then H is of order 31 as $|H|$ divides $|K|-1$. Now H acts trivially on the Sylow 3-subgroup of K and so $93 \in \pi_e(G)$, which contradicts $k_1(G) = 31$. Suppose that G is a 2-Frobenius group with normal series $1 \leq H \leq K \leq G$, where $|K/H| = 31$ and $|G/K| | 30$. In such case, $2 | |H|$. Consider the action on H by the element of order 11. We can get that there exists a Sylow 2-subgroup L of K fixed by this action. Since $G = 2^5 \cdot 3 \cdot 5^3 \cdot 31$, we have $|L| = 2^5$. Clearly, $\Omega_1(Z(L))$ is an elementary abelian 2-group. Because $k_2(G) = 24$, G has an element with order 8, and thus $|\Omega_1(Z(L))| \mid 2^3$. Consider the action on $\Omega_1(Z(L))$ by the element of order 31. We know such action is trivial for $31 \nmid |\mathrm{Aut}(\Omega_1(Z(L)))|$, which implies that $62 \in \pi_e(G)$, a contradiction. Therefore, by Lemma 2.2, we know that G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$, such that H and G/K are π_1-groups and K/H a non-abelian simple group, where π_1 is the prime graph component containing 2, H is a nilpotent group, and $|G/K| \mid |\mathrm{Out}(K/H)|$. Because $|G| = 2^5 \cdot 3 \cdot 5^3 \cdot 31$, and 31 is an isolated point of $\Gamma(G)$, we have $\pi(H) \cup \pi(G/K) \subseteq \{2, 3, 5\}$ and $31 \in \pi(K/H)$. From [22] we know that K/H is isomorphic to $L_2(31)(2^5 \cdot 3 \cdot 5 \cdot 31)$ or $L_3(5)(2^5 \cdot 3 \cdot 5^3 \cdot 31)$.

Suppose that K/H is isomorphic to $L_2(31)$. In this case, we know that $5 \nmid |\mathrm{Out}(K/H)|$, and thus $5 \mid |H|$. Let L be a Sylow 5-subgroup of H. We know that $L \trianglelefteq G$ and $|L| = 5^2$. Consider the action on L by the element of order 31. Clearly, this action is trivial. It implies that $155 \in \pi_e(G)$, which is a contradiction. Therefore, we have $K/H \cong L_3(5)$. So $H = 1$, $K = G$, and therefore, $G \cong L_3(5)$.

This completes the proof. □

As a corollary of preceding theorems, we have

Theorem 3.3 *Let G be one of the simple K_4-groups mentioned in part (I) in Lemma 2.1, except that $S_4(7)$. Then G can be uniquely determined by the order of G and $k_i(G)$, where $i \leq 2$.*

Remark 3.4 For simple K_4-group $S_4(7)$, we cannot judge whether their prime graphs are connected only by their largest element order and second largest element order, so we cannot characterize them in the way used in this paper.

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