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Existence of Entire Solutions for Semilinear Elliptic Problems with Convection Terms

Hongtao XUE
Mathematics and Physics Teaching Department, Yantai Nanshan University, Shandong 265713*, P. R. China*

Abstract By a sub-supersolution method and a perturbed argument, we show the existence of entire solutions for the semilinear elliptic problem $-\Delta u + a(x)|\nabla u|^q = \lambda b(x)g(u)$, $u > 0$, $x \in \mathbb{R}^N$, $\lim_{|x|\to\infty} u(x) = 0$, where $q \in (1, 2]$, $\lambda > 0$, a and b are locally Hölder continuous, $a \geq 0$, $b > 0$, $\forall x \in \mathbb{R}^N$, and $g \in C^1((0,\infty),(0,\infty))$ which may be both possibly singular at zero and strongly unbounded at infinity.

Keywords semilinear elliptic equation; entire solution; convection term; existence

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1. Introduction and the main result

In this paper, we are concerned with the existence of entire solutions for the following semilinear elliptic problem

$$-\Delta u + a(x)|\nabla u|^q = \lambda b(x)g(u), \quad u > 0,\ x \in \mathbb{R}^N,\ \lim_{|x|\to\infty} u(x) = 0, \tag{1.1}$$

where $q \in (1, 2]$, $\lambda > 0$, $a(x) \in C^{\alpha}_{\rm loc}(\mathbb{R}^N)$ for some $\alpha \in (0, 1)$ is non-negative, $b(x)$ satisfies

(b$_1$) $b \in C^{\alpha}_{\rm loc}(\mathbb{R}^N)$ and $b(x) > 0$, $\forall x \in \mathbb{R}^N$,

(b$_2$) the linear problem

$$-\Delta u = b(x), \quad u > 0,\ x \in \mathbb{R}^N,\ \lim_{|x|\to\infty} u(x) = 0 \tag{1.2}$$

has a unique solution $w \in C^{2+\alpha}_{\rm loc}(\mathbb{R}^N)$, and the nonlinearity $g \in C^1((0,\infty),(0,\infty))$ may be both possibly singular at zero and strongly unbounded at infinity.

Set $g_0 := \lim_{s\to 0} g(s)/s$, $g_\infty := \lim_{s\to\infty} g(s)/s$, where $g_0 \in (0,\infty]$, $g_\infty \in [0,\infty]$.

Problem (1.1) arises from many branches of mathematics and applied mathematics. Concerning with entire solutions for semilinear elliptic problems, there is by now a broad literature and we refer the readers to [1–19] and the references cited therein. But we note that in most works, monotonicity on $g(s)$ or $g(s)/s$ is required to some extent.

Recently, the author showed in [13] that the problem

$$-\Delta u + a(x)|\nabla u|^q = b(x)g(u), \quad u > 0,\ x \in \mathbb{R}^N,\ \lim_{|x|\to\infty} u(x) = 0, \tag{1.3}$$

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E-mail address: hongtaoxue@126.com

admits a entire solution when $g_0 = \infty$, $g_\infty = 0$, where no monotonicity is required. And in [14], where the nonlinearity is not necessarily separable, the author extended the above results to the following problem

$$-\Delta u + a(x)|\nabla u|^q = f(x,u), \quad u > 0,\ x \in \mathbb{R}^N,\ \lim_{|x|\to\infty} u(x) = 0, \tag{1.4}$$

under the following conditions

(f_1) $f(x,s)$ is locally Hölder continuous on $\mathbb{R}^N \times (0,\infty)$ and continuously differentiable in the variable s,

(f_2) $f(x,s) \le b(x)g(s)$ for all $(x,s) \in \mathbb{R}^N \times (0,\infty)$, where b satisfies (b_1) and (b_2), g satisfies $\limsup_{s\to\infty} g(s)/s < 1/C_0$, where w is the solution of problem (1.2) and $C_0 := \max_{x\in\mathbb{R}^N} w(x)$,

(f_3) There exists $s_0 > 0$ such that $f(x,s) \ge a(x)h(s)$ for all $(x,s) \in \mathbb{R}^N \times (0,s_0)$, where $a : \mathbb{R}^N \to (0,\infty)$ is locally Hölder continuous, $h : (0,s_0) \to (0,\infty)$ is continuous, and $\lim_{s\to 0} h(s)/s = \infty$.

We refer the readers to the paper [14] for details.

In this paper, we continue to improve the earlier results about the existence of entire solutions for problem (1.1), where the case $g_0 \in (0,\infty]$, $g_\infty \in [0,\infty]$ is treated. Our main result is summarized in the following theorem.

Theorem 1.1 *Let $q \in (1,2]$, $\lambda > 0$. Assume that $a : \mathbb{R}^N \to [0,\infty)$ is locally Hölder continuous, and b satisfies (b_1)–(b_2), $g \in C^1((0,\infty),(0,\infty))$. Then problem (1.1) has at least one solution $u \in C^{2+\alpha}_{\rm loc}(\mathbb{R}^N)$, if one of the following two conditions*

(i) $0 < \frac{2\beta\xi_1(b,B)}{\lambda} < g_0 \le \infty, 0 \le g_\infty < \infty, 0 < \lambda < \Lambda_0$,

(ii) $g_0 = g_\infty = \infty, 0 < \lambda \le \Lambda_1$,

holds, where $\beta := q/(q-1)$, B is the unit ball of $\mathbb{R}^N$,

$$\xi_1(b,B) := \inf_{\{u\in W_0^{1,2}(B),u\neq 0\}} \frac{\int_B |\nabla u|^2 \mathrm{d}x}{\int_B b(x)|u|^2 \mathrm{d}x}. \tag{1.5}$$

Remark 1.2 Λ_0, Λ_1 will be shown in the proof of Theorem 2.2.

The paper is organized as follows. In Section 2, we provide a suitable super-solution for problem (1.1). In Section 3, we show the existence of positive solutions in bounded domain. In Section 4, we prove Theorem 1.1.

2. Super-solutions decaying to zero

Consider the differential inequality problem

$$-\Delta v > \lambda b(x)g(v), \quad v > 0,\ x \in \mathbb{R}^N,\ \lim_{|x|\to\infty} v(x) = 0. \tag{2.1}$$

Obviously, any solution of problem (2.1) is a super-solution of problem (1.1).

First we recall the following auxiliary result.

Lemma 2.1 ([15, Lemma 2.1]) *Assume $g \in C^1((0,\infty),(0,\infty))$ with $0 < g_0 \le \infty$ and $0 \le g_\infty < \infty$. Then there exists a function $\Gamma_g(s)$ such that* *(i)* $\Gamma_g(s) \in C^1((0,\infty),(0,\infty))$;

(ii) $g(s)/s \le \Gamma_g(s),\ \forall s > 0$;

(iii) $\Gamma_g(s)$ *is non-increasing on* $(0,\infty)$;

(iv) $\lim_{s\to\infty} \Gamma_g(s) = g_\infty$.

The result below will provide a suitable super-solution for problem (1.1).

Theorem 2.2 *Let* $q \in (1,2]$, $\lambda > 0$. *Assume that* $a : \mathbb{R}^N \to [0,\infty)$ *is locally Hölder continuous, and b satisfies* (b_1)*–*(b_2), $g \in C^1((0,\infty),(0,\infty))$. *Then, there exists a function v satisfying problem (2.1), if one of the following two conditions*

(i) $0 < g_0 \le \infty, 0 \le g_\infty < \infty, 0 < \lambda < \Lambda_0$,

(ii) $g_0 = g_\infty = \infty, 0 < \lambda \le \Lambda_1$,

holds.

Proof of Theorem 2.2

Proof of (i) Define

$$\overline{g}_1(t) = \int_0^t \frac{s}{s\Gamma_g(s)+1}\mathrm{d}s,\ t \ge 0,$$

it follows that

(i) $\overline{g}_1(s)/s$ is non-decreasing; (ii) $\lim_{s\to\infty} \overline{g}_1(s)/s = 1/g_\infty$; (iii) $\lim_{s\to 0} \overline{g}_1(s)/s = 0$.

Set $\Lambda_0 := \frac{1}{C_0 g_\infty}$, where $C_0 = \max\limits_{x\in\mathbb{R}^N} w(x)$, w is the solution of problem (1.2). For any $\lambda \in (0,\Lambda_0)$, there exists a positive constant μ such that

$$\frac{1}{\mu}\int_0^\mu \frac{t}{t\Gamma_g(t)+1}\mathrm{d}t = \lambda C_0.$$

Now, we define a function v by

$$\lambda w(x) = \frac{1}{\mu}\int_0^{v(x)} \frac{t}{t\Gamma_g(t)+1}\mathrm{d}t. \tag{2.2}$$

Then, $0 < v(x) \le \mu$, and $\lim\limits_{|x|\to\infty} v(x) = 0$.

Differentiating (2.2), we have

$$\mu\lambda\Delta w(x) = \frac{v}{v\Gamma_g(v)+1}\Delta v + \frac{\mathrm{d}}{\mathrm{d}v}\left(\frac{v}{v\Gamma_g(v)+1}\right)|\nabla v|^2,$$
$$-\mu\lambda\Delta w(x) \le \frac{v}{v\Gamma_g(v)+1}(-\Delta v).$$

So,

$$-\Delta v \ge \lambda\mu b(x)\left(\Gamma_g(v) + \frac{1}{v}\right) \ge \lambda b(x) v\left(\Gamma_g(v) + \frac{1}{v}\right). \tag{2.3}$$

By Lemma 2.1(ii), we have $-\Delta v > \lambda b(x) g(v)$.

Proof of (ii) There is some $m > 0$ such that

$$\inf_{s>0} \frac{g(s)}{s} = \frac{g(m)}{m} := I_m \in [0,\infty).$$

We define

$$g^*(s) := \begin{cases} g(s), & 0 < s \le m, \\ I_m s, & s > m. \end{cases}$$

Notice that $g^* \in C^1$ and satisfies

(i) $\lim_{s\to 0} g^*(s)/s = \infty$; (ii) $\lim_{s\to\infty} g^*(s)/s = I_m \in [0,\infty)$.

Moreover, any solution of

$$-\Delta v > \lambda b(x)g^*(v), \quad v>0,\ x\in\mathbb{R}^N,\ \lim_{|x|\to\infty} v(x)=0,\ v\le m, \tag{2.4}$$

is also a super-solution of problem (1.1).

Next, we show that problem (2.4) has at least one solution.

Define

$$\overline{g}_2(t)=\int_0^t \frac{s}{s\Gamma_{g^*}(s)+1}\mathrm{d}s,\ t\ge 0,$$

it follows that

(i) $\overline{g}_2(s)/s$ is non-decreasing; (ii) $\lim_{s\to\infty}\overline{g}_2(s)/s = 1/I_m$; (iii) $\lim_{s\to 0}\overline{g}_2(s)/s=0$.

Notice that $\frac{\overline{g}_2(s)}{s}$ is non-decreasing. And set $\Lambda_1 := \frac{\overline{g}_2(m)}{C_0 m}$, where $C_0=\max_{x\in\mathbb{R}^N} w(x)$, w is the solution of problem (1.2).

For any $\lambda\in(0,\Lambda_1]$, there exists $\mu\in(0,m]$ such that

$$\frac{1}{\mu}\int_0^\mu \frac{t}{t\Gamma_{g^*}(t)+1}\mathrm{d}t=\lambda C_0.$$

Now, we define a function v by

$$\lambda w(x)=\frac{1}{\mu}\int_0^{v(x)} \frac{t}{t\Gamma_{g^*}(t)+1}\mathrm{d}t. \tag{2.5}$$

Then, $0<v(x)\le\mu\le m$, and $\lim_{|x|\to\infty} v(x)=0$. The remaining part of the proof follows as in the proof of (i).

The proof of Theorem 2.2 is completed. □

3. Positive solutions on bounded domains

Consider the following problem

$$-\triangle u + a(x)|\nabla u|^q = \lambda b(x)g(u), \quad u>0,\ x\in\Omega,\ u|_{\partial\Omega}=0, \tag{3.1}$$

where Ω is a bounded domain with smooth boundary $\partial\Omega$.

In this section, by a sub-supersolution method [16, Lemma 3], we show the existence of positive solutions for problem (3.1).

Let $\phi_1(b,\Omega)\in C^1(\bar\Omega)\cap C^{2+\alpha}(\Omega)$ be the first eigenfunction corresponding to the first eigenvalue $\xi_1(b,\Omega)$ of

$$-\Delta u=\xi b(x)u, \quad u>0,\ x\in\Omega,\ u|_{\partial\Omega}=0.$$

Notice that $\xi_1(b,\Omega)$ is given by an expression like (1.5).

For the convenience, we denote $|u|_\infty = \max_{x\in\bar\Omega}|u(x)|$, $|u|_0=\min_{x\in\bar\Omega}|u(x)|$, whenever $u\in C(\bar\Omega)$.

Theorem 3.1 *Let $q\in(1,2]$, $\lambda>0$. Assume that $a,b\in C^\alpha(\bar\Omega)$, and $a\ge 0, b>0$, $g\in$*

$C^1((0,\infty)\ (0,\infty))$. *Then problem (3.1) has at least one solution* $u \in C(\bar{\Omega}) \cap C^{2+\alpha}(\Omega)$, *if one of the following two conditions*

(i) $0 < \frac{2\beta\xi_1(b,\Omega)}{\lambda} < g_0 \le \infty, 0 \le g_\infty < \infty, 0 < \lambda < \Lambda_0$,

(ii) $g_0 = g_\infty = \infty, 0 < \lambda \le \Lambda_1$,

holds.

Proof of Theorem 3.1 In the course of the following proof, denote $\xi_1 = \xi_1(b,\Omega), \phi_1 = \phi_1(b,\Omega)$.

Proof of (i) Take $\lambda \in (0,\Lambda_0)$. Since $b > 0$ on $\bar{\Omega}$ and $g_0 > \frac{2\beta\xi_1}{\lambda}$, there is $\delta > 0$ such that

$$\frac{g(s)}{s} > \frac{2\beta\xi_1}{\lambda},\ s \in (0,\delta).$$

Let $\underline{u} = c_1\phi_1^\beta$, where $c_1 \in (0, \min\{1, \frac{\delta}{|\phi_1|_\infty^\beta}, (\frac{\xi_1|b|_0}{\beta^{q-1}|a|_\infty|\nabla\phi_1|_\infty^q})^{\frac{1}{q-1}})$. We claim that $\underline{u}$ is a sub-solution of problem (3.1).

In fact, since $c_1 \le (\frac{\xi_1|b|_0}{\beta^{q-1}|a|_\infty|\nabla\phi_1|_\infty^q})^{\frac{1}{q-1}}$, we have

$$a(x)\beta^q c_1^q \phi_1^{q(\beta-1)}|\nabla\phi_1|^q \le \beta\xi_1 c_1 b(x)\phi_1^\beta.$$

Then,

$$\begin{aligned} -\triangle\underline{u} + a(x)|\nabla\underline{u}|^q &= \beta\xi_1 c_1 b(x)\phi_1^\beta - c_1\beta(\beta-1)\phi_1^{\beta-2}|\nabla\phi_1|^2 + a(x)\beta^q c_1^q\phi_1^{q(\beta-1)}|\nabla\phi_1|^q \\ &\le \beta\xi_1 c_1 b(x)\phi_1^\beta + a(x)\beta^q c_1^q \phi_1^\beta|\nabla\phi_1|^q \\ &\le 2\beta\xi_1 c_1 b(x)\phi_1^\beta \le \lambda b(x) g(c_1\phi_1^\beta) \\ &\le \lambda b(x) g(\underline{u}). \end{aligned}$$

Let $\overline{u} = v$ be given as in Theorem 2.2(i). Then $\overline{u}$ is a super-solution of problem (3.1). We claim that $\underline{u} \le \overline{u}$.

Indeed if we assume the contrary that there is $x_0 \in \Omega$ such that $\underline{u}(x_0) > \overline{u}(x_0)$, then $\sup_{x\in\Omega}(\ln(\underline{u}(x)) - \ln(\overline{u}(x))) > 0$. There is some $x_1 \in \Omega$ such that

$$\nabla(\ln(\underline{u}(x_1)) - \ln(\overline{u}(x_1))) = 0 \text{ and } \Delta(\ln(\underline{u}(x_1)) - \ln(\overline{u}(x_1))) \le 0.$$

By Lemma 2.1(ii)–(iii) and (2.3), we have

$$\begin{aligned} \Delta(\ln(\underline{u}(x_1)) - \ln(\overline{u}(x_1))) =& \frac{\Delta\underline{u}(x_1)}{\underline{u}(x_1)} - \frac{\Delta\overline{u}(x_1)}{\overline{u}(x_1)} \\ \ge& \frac{a(x_1)|\nabla\underline{u}(x_1)|^q}{\underline{u}(x_1)} - \lambda b(x_1)(\frac{g(\underline{u}(x_1))}{\underline{u}(x_1)} - (\Gamma_g(\overline{u}(x_1)) + \frac{1}{\overline{u}(x_1)})) > 0, \end{aligned}$$

which is a contradiction. So we can obtain that $\underline{u}(x) \le \bar{u}(x)$, $x \in \Omega$. It follows that problem (3.1) has at least one solution $u \in C^{2+\alpha}(\Omega) \cap C(\bar{\Omega})$ in the ordered interval $[\underline{u}, \bar{u}]$.

Proof of (ii) We notice that any solution of the problem

$$-\triangle u + a(x)|\nabla u|^q = \lambda b(x) g^*(u),\quad u > 0,\ x \in \Omega,\ u|_{\partial\Omega} = 0,\ u \le m, \tag{3.2}$$

where g^* was defined in the proof of Theorem 2.2, is a solution of problem (3.1). Since

$\lim_{s\to 0} g^*(s)/s = \infty$, there is $\delta > 0$ such that

$$g^*(s)/s > \frac{2\beta\xi_1}{\lambda}, \quad s \in (0, \delta).$$

Proceeding as in the proof of item (i), it follows that $\underline{u} = c_1\phi_1^\beta$ is a sub-solution of problem (3.1).

On the other hand, the function $\overline{u} = v$ with v given by Theorem 2.2(ii) is a super-solution of problem (3.1). Proceeding as in the proof of (i), we have $\underline{u} \le \overline{u}$ in Ω. Then problem (3.1) has at least one solution $u \in C^{2+\alpha}(\Omega) \cap C(\bar{\Omega})$ such that $u \in [\underline{u}, \bar{u}]$.

The proof of Theorem 3.1 is completed. □

4. Proof of Theorem 1.1

In this section, we prove Theorem 1.1.

Proof of (i) Take $\lambda \in (0, \Lambda_0)$ and consider the problem

$$-\triangle u_k + a(x)|\nabla u_k|^q = \lambda b(x)g(u_k), \quad u_k > 0, \ x \in B(0,k), \ u_k|_{\partial B(0,k)} = 0, \tag{4.1}$$

where $B(0,k) = \{x \in \mathbb{R}^N : |x| < k\}$, $k = 1, 2, 3, \ldots$.

Using the conditions of Theorem 1.1(i), we have

$$g_0 > \frac{2\beta\xi_1(a,B)}{\lambda} \ge \frac{2\beta\xi_1(a,B_k)}{\lambda}. \tag{4.2}$$

It follows by Theorem 3.1 that problem (4.1) has at least one solution $u_k \in C^{2+\alpha}(B(0,k)) \cap C(\bar{B}(0,k))$. Put $u_k(x) = 0$, $\forall |x| > k$. Then,

$$0 < \underline{u} \le u_k \le v \le \mu, \tag{4.3}$$

where $\underline{u}$ is the sub-solution corresponding to $\Omega = B_k$ and v is the function given by Theorem 2.2(i).

Now, we need to estimate $\{u_k\}$. For any bounded $C^{2+\alpha}$-smooth domain $\Omega' \subset \mathbb{R}^N$, take Ω_1 and Ω_2 with $C^{2+\alpha}$-smooth boundaries, and K_1 large enough, such that

$$\Omega' \subset\subset \Omega_1 \subset\subset \Omega_2 \subset\subset B_k, \quad k \ge K_1.$$

Note that

$$u_k(x) \ge \underline{u}(x) > 0, \quad \forall x \in B(0, K_1), \tag{4.4}$$

when $B(0,K_1)$ is the substitution for Ω in the proof of Theorem 3.1.
Let

$$\rho_k(x) = \lambda b(x)g(u_k) - a(x)|\nabla u_k(x)|^q, \quad x \in \bar{B}(0, K_1).$$

Since $-\Delta u_k(x) = \rho_k(x)$, $x \in B(0,K_1)$, by the interior estimate theorem of Ladyzenskaja and Ural'tseva [20, Theorem 3.1, pp.266], we get a positive constant C_1 independent of k such that

$$\max_{x\in\bar{\Omega}_2} |\nabla u_k(x)| \le C_1 \max_{x\in\bar{B}(0,K_1)} u_k(x) \le C_1 \max_{x\in\bar{B}(0,K_1)} v(x), \quad \forall x \in B(0, K_1), \tag{4.5}$$

i.e., $|\nabla u_k(x)|$ is uniformly bounded on $\bar{\Omega}_2$. It follows that $\{\rho_k\}_{K_1}^{\infty}$ is uniformly bounded on $\bar{\Omega}_2$ and hence $\rho_k \in L^p(\Omega_2)$ for any $p > 1$. Since $-\Delta u_k(x) = \rho_k(x), x \in \Omega_2$, we see by [21, Theorem 9.11] that there exists a positive constant C_2 independent of k such that

$$||u_k||_{W^{2,p}(\Omega_1)} \le C_2(||\rho_k||_{L^p(\Omega_2)} + ||u_k||_{L^p(\Omega_2)}), \quad \forall k \ge K_1. \tag{4.6}$$

Taking $p > N$ such that $\alpha < 1 - N/p$ and applying Sobolev's embedding inequality, we see that $\{||u_k||_{C^{1+\alpha}(\bar{\Omega}_1)}\}_{K_1}^{\infty}$ is uniformly bounded. Therefore $\rho_k \in C^{\alpha}(\bar{\Omega}_1)$ and $\{||\rho_k||_{C^{\alpha}(\bar{\Omega}_1)}\}_{K_1}^{\infty}$ is uniformly bounded. It follows by Schauder's interior estimate theorem [21, Chapter 1, pp.2] that there exists a positive constant C_3 independent of k such that

$$\|u_k\|_{C^{2+\alpha}(\bar{\Omega}')} \le C_3(\|\rho_k\|_{C^{\alpha}(\bar{\Omega}_1)} + \|u_k\|_{C(\bar{\Omega}_1)}), \quad \forall k \ge K_1, \tag{4.7}$$

i.e., $\{\|u_k\|_{C^{2+\alpha}(\bar{\Omega}')}\}_{K_1}^{\infty}$ is uniformly bounded. Using Ascoli-Arzela's theorem and the diagonal sequential process, we see that $\{u_k\}_{K_1}^{\infty}$ has a subsequence that converges uniformly in the $C^2(\bar{\Omega}')$ norm to a function $u \in C^2(\bar{\Omega}')$ and u satisfies

$$-\Delta u + a(x)|\nabla u|^q = \lambda b(x)g(u), \quad x \in \bar{\Omega}'.$$

By (4.4), we obtain that $u > 0, \ \ \forall x \in \bar{\Omega}'$. Applying Schauder's regularity theorem, we see that $u \in C^{2+\alpha}(\bar{\Omega}')$. Since Ω' is arbitrary, we also see that $u \in C^{2+\alpha}_{\rm loc}(\mathbb{R}^N)$. It follows by (4.3) that $\lim_{|x|\to\infty} u(x) = 0$. Thus, a standard bootstrap argument (with the same details as in [22]) shows that u is one solution of problem (1.1).

Proof of (ii) Take $\lambda \in (0, \Lambda_1]$. By Theorem 3.1(ii), problem (4.1) admits a solution $u_k \in C^{2+\alpha}(B(0,k)) \cap C(\bar{B}(0,k))$ satisfying $0 < \underline{u} \le u_k \le v \le m$. The proof now follows as in the proof of (i). The proof of Theorem 1.1 is completed. □

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