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Composition Operators from Hardy-Orlicz Spaces to Bloch-Orlicz Type Spaces

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Abstract Let φ be an analytic self-map of $\mathbb{D}$. The composition operator C_φ is the operator defined on $H(\mathbb{D})$ by $C_\varphi(f) = f \circ \varphi$. In this paper, we investigate the boundedness and compactness of the composition operator C_φ from Hardy-Orlicz spaces to Bloch-Orlicz type spaces.

Keywords composition operator; Hardy-Orlicz space; Bloch-Orlicz space

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1. Introduction

Let $\mathbb{D}$ be the unit disk in the complex plane $\mathbb{C}$ and $H(\mathbb{D})$ the space of all holomorphic functions on $\mathbb{D}$. A function $f \in H(\mathbb{D})$ is called a μ-Bloch function, if

$$\|f\|_\mu := \sup_{z\in\mathbb{D}} \mu(z)|f'(z)| < \infty,$$

denoted as $f \in \mathcal{B}^\mu$, where μ is a bounded continuous positive function on $\mathbb{D}$. And $\mathcal{B}^\mu$ is a Banach space with the norm $\|f\|_{\mathcal{B}^\mu} := |f(0)| + \|f\|_\mu$.

Recently, the Bloch-Orlicz type space was introduced by Ramos Fernández in [1] using Young's functions. More precisely, let $\psi : [0, +\infty) \to [0, +\infty)$ be a strictly increasing convex function such that $\psi(0) = 0$ and $\lim_{t\to+\infty} \psi(t) = +\infty$. The Bloch-Orlicz type space associated with the function ψ, denoted by $\mathcal{B}^\psi$, is the class of all functions $f \in H(\mathbb{D})$ such that

$$\sup_{z\in\mathbb{D}}(1 - |z|^2)\psi(\lambda|f'(z)|) < \infty,$$

for some $\lambda > 0$ depending on f. The Minknowki's functional

$$\|f\|_\psi = \inf\{k > 0 : S_\psi(\frac{f'}{k}) \le 1\}$$

defines a seminorm for $\mathcal{B}^\psi$, which, in this case, is known as Luxemburg's seminorm, where

$$S_\psi(f) := \sup_{z\in\mathbb{D}}(1 - |z|^2)\psi(|f(z)|).$$

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Moreover, $\mathcal{B}^\psi$ is a Banach space with the norm

$$\|f\|_{\mathcal{B}^\psi} = |f(0)| + \|f\|_\psi.$$

Also, Ramos Fernández in [1] got that the Bloch-Orlicz type space was isometrically equal to μ-Bloch space, where

$$\mu(z) = \frac{1}{\psi^{-1}(\frac{1}{1-|z|^2})}, \quad z \in \mathbb{D}. \tag{1.1}$$

Then, for $f \in \mathcal{B}^\psi$

$$\|f\|_{\mathcal{B}^\psi} = |f(0)| + \sup_{z\in\mathbb{D}} \mu(z)|f'(z)|.$$

It is obvious to see that if $\psi(t) = t^p$ with $p \geq 1$, then the space $\mathcal{B}^\psi$ coincides with the α-Bloch space $\mathcal{B}^\alpha$ (see [2]), where $\alpha = 1/p$. Also, if $\psi(t) = t\log(1+t)$, then $\mathcal{B}^\psi$ coincides with the log-Bloch space [3,4].

The Hardy-Orlicz space $H^\psi(\mathbb{D}) = H^\psi$ is the space of all $f \in H(\mathbb{D})$ such that

$$\|f\|_{H^\psi} := \sup_{0<r<1} \int_{\partial\mathbb{D}} \psi(|f(r\xi)|)\mathrm{d}\sigma(\xi) < \infty,$$

where $\partial\mathbb{D}$ is the boundary of the unit disk $\mathbb{D}$ and σ is the normalized Lebesgue measure on $\partial\mathbb{D}$. On H^ψ is defined the next quasi-norm

$$\|f\|_{H^\psi} = \sup_{0<r<1} \|f_r\|_{L^\psi},$$

where $f_r(\xi) = f(r\xi)$, $0 \leq r < 1$, $\xi \in \partial\mathbb{D}$ and $\|g\|_{L^\psi}$ is the Luxembourg quasi-norm defined by

$$\|g\|_{L^\psi} := \inf\{\lambda > 0 : \int_{\partial\mathbb{D}} \psi(\frac{|g(\xi)|}{\lambda})\mathrm{d}\sigma(\xi) \leq 1\}.$$

If $\psi(t) = t^p$ with $p > 0$, then H^ψ is the classical Hardy space H^p (see [5]), consisting of all $f \in H(\mathbb{D})$ such that

$$\|f\|_{H^p}^p = \sup_{0<r<1} \int_{\partial\mathbb{D}} |f(r\xi)|^p \mathrm{d}\sigma(\xi) < \infty.$$

Let φ be an analytic self-map of $\mathbb{D}$, and the composition operator C_φ be the operator defined on $H(\mathbb{D})$ by

$$C_\varphi(f)(z) := (f \circ \varphi)(z) = f(\varphi(z)).$$

The function φ is called the symbol of C_φ. Composition operators between various spaces of holomorphic functions on different domains have been studied by numerous authors [1,4,6–12] and the references therein. This paper is devoted to characterizing the boundedness and compactness of composition operators from Hardy-Orlicz spaces to Bloch-Orlicz type spaces.

Throughout this paper, we will use the letter C to denote a generic positive constant that can change its value at each occurrence. The notation $a \preceq b$ means that there is a positive constant C such that $a \leq Cb$. If both $a \preceq b$ and $b \preceq a$ hold, then one says that $a \simeq b$.

2. Auxiliary results

Here we quote some auxiliary results which will be used in the proofs of the main results in this paper.

Proposition 2.1 *For every $f \in H^\psi$ and $z \in \mathbb{D}$, we have*

$$|f(z)| \le \psi^{-1}(\frac{2}{1-|z|})\|f\|_{H^\psi}. \tag{2.1}$$

Proof Since f is analytic, employing [13, Corollary 4.5] to the functions $f_r(z)$, we get for $z \in \mathbb{D}$

$$|f(rz)| \le \int_{\partial\mathbb{D}} P(z,\zeta)|f_r(\zeta)|\mathrm{d}\sigma(\zeta), \tag{2.2}$$

where

$$P(z,\zeta) = \frac{1-|z|^2}{|1-z\bar{\zeta}|^2}$$

is the invariant Poison Kernel [13].

Using the inequality

$$P(z,\zeta) \le \frac{2}{1-|z|}$$

and applying Jensen's inequality to (2.2) with f_r replaced by $f_r/\|f_r\|_{H^\psi}$, we obtain

$$\begin{aligned}\psi(\frac{|f(rz)|}{\|f_r\|_{H^\psi}}) &\le \int_{\partial\mathbb{D}} P(z,\zeta)\psi(\frac{|f_r(\zeta)|}{\|f_r\|_{H^\psi}})\mathrm{d}\sigma(\zeta)\\ &\le \frac{2}{1-|z|}\int_{\partial\mathbb{D}}\psi(\frac{|f_r(\zeta)|}{\|f_r\|_{H^\psi}})\mathrm{d}\sigma(\zeta)\\ &\le \frac{2}{1-|z|}. \end{aligned} \tag{2.3}$$

From (2.3) we obtain

$$|f(rz)| \le \psi^{-1}(\frac{2}{1-|z|})\|f_r\|_{H^\psi},$$

letting $r \to 1^-$, then inequality (2.1) follows. The proof is completed. □

Proposition 2.2 *For every $f \in H^\psi$ and $z \in \mathbb{D}$, we have*

$$|f'(z)| \le \frac{1}{1-|z|^2}\psi^{-1}(\frac{2}{1-|z|})\|f\|_{H^\psi}.$$

Proof By [13, Proposition 4.2], we have

$$f(z) = \int_{\partial\mathbb{D}} P(z,\zeta)f(\zeta)\mathrm{d}\sigma(\zeta). \tag{2.4}$$

Differentiating (2.4) yields

$$f'(z) = \int_{\partial\mathbb{D}} \frac{\bar{\zeta}f(\zeta)}{(1-z\bar{\zeta})^2}\mathrm{d}\sigma(\zeta).$$

Then

$$|f'(z)| \le \int_{\partial\mathbb{D}} \frac{|f(\zeta)|}{|1-z\bar{\zeta}|^2}\mathrm{d}\sigma(\zeta),$$

and hence

$$(1-|z|^2)|f'(z)| \le (1-|z|^2)\int_{\partial\mathbb{D}} \frac{|f(\zeta)|}{|1-z\bar{\zeta}|^2}\mathrm{d}\sigma(\zeta) = \int_{\partial\mathbb{D}} \frac{1-|z|^2}{|1-z\bar{\zeta}|^2}|f(\zeta)|\mathrm{d}\sigma(\zeta).$$

Applying Jensen's inequality, we obtain

$$\psi(\frac{(1-|z|^2)|f'(z)|}{\|f\|_{H^\psi}}) \leq \int_{\partial\mathbb{D}} \frac{1-|z|^2}{|1-z\bar{\zeta}|^2}\psi(\frac{|f(\zeta)|}{\|f\|_{H^\psi}})\mathrm{d}\sigma(\zeta)$$
$$\leq \frac{2}{1-|z|}.$$

That is,

$$|f'(z)| \leq \frac{1}{1-|z|^2}\psi^{-1}(\frac{2}{1-|z|})\|f\|_{H^\psi}.$$

The proof is completed. □

Lemma 2.3 *For each $a \in \mathbb{D}$, the function*

$$f_a(z) = \frac{1}{4}\psi^{-1}(\frac{2}{1-|a|})(\frac{1-|a|^2}{1-z\bar{a}})^2$$

belongs to H^ψ. Moreover

$$\sup_{a\in\mathbb{D}} \|f_a\|_{H^\psi} \leq 1.$$

Proof Using Jensen's inequality for the fact $\frac{1}{4}\frac{(1-|a|^2)^2}{|1-z\bar{a}|^2} \leq 1$, we have

$$\int_{\mathbb{D}} \psi(|f_a(r\zeta)|)\mathrm{d}\sigma(\zeta) = \int_{\mathbb{D}} \psi(\frac{1}{4}\psi^{-1}(\frac{2}{1-|a|})|\frac{1-|a|^2}{1-r\zeta\bar{a}}|^2)\mathrm{d}\sigma(\zeta)$$
$$\leq \frac{1}{4}\int_{\mathbb{D}} |\frac{1-|a|^2}{1-r\zeta\bar{a}}|^2 \frac{2}{1-|a|}\mathrm{d}\sigma(\zeta)$$
$$\leq \int_{\mathbb{D}} \frac{1-|a|^2}{|1-r\zeta\bar{a}|^2}\mathrm{d}\sigma(\zeta) \leq 1.$$

From this the lemma follows. The proof is completed. □

The following compactness criteria can be proved similar to [11, Proposition 3.11].

Lemma 2.4 *The bounded operator $T : H^\psi \to \mathcal{B}^\psi$ is compact if and only if for every bounded sequence $\{f_j\}_{j\in\mathbb{N}}$ in H^ψ which converges to zero uniformly on any compact subset of $\mathbb{D}$ as $j \to \infty$, it follows that*

$$\lim_{j\to\infty} \|Tf_j\|_{\mathcal{B}^\psi} = 0.$$

3. Boundedness and compactness

In this section, we characterize the boundedness and compactness of the operators $C_\varphi : H^\psi \to \mathcal{B}^\psi$.

Theorem 3.1 *Let φ be an analytic self-map of $\mathbb{D}$. Then $C_\varphi : H^\psi \to \mathcal{B}^\psi$ is bounded if and only if*

$$M := \sup_{z\in\mathbb{D}} \frac{\mu(z)|\varphi'(z)|}{1-|\varphi(z)|^2}\psi^{-1}(\frac{2}{1-|\varphi(z)|}) < \infty. \tag{3.1}$$

Moreover

$$\|C_\varphi\|_{H^\psi\to\mathcal{B}^\psi} \simeq M. \tag{3.2}$$

Proof Suppose that the condition (3.1) holds. For an arbitrary $f \in H^{\psi}$, by Proposition 2.2, we have

$$\begin{aligned}\mu(z)|(C_{\varphi}f)'(z)| &= \mu(z)|f'(\varphi(z))|\cdot|\varphi'(z)|\\ &\le \mu(z)\psi^{-1}(\frac{2}{1-|\varphi(z)|})\frac{1}{1-|\varphi(z)|^2}|\varphi'(z)|\cdot\|f\|_{H^{\psi}}\\ &\le M\cdot\|f\|_{H^{\psi}}.\end{aligned}$$

Then $C_{\varphi}: H^{\psi}\to\mathcal{B}^{\psi}$ is bounded. Moreover, the above proof gets that

$$\|C_{\varphi}\|_{H^{\psi}\to\mathcal{B}^{\psi}} = \sup_{f\in H^{\psi}\setminus\{0\}}\frac{\|C_{\varphi}f\|_{\mathcal{B}^{\psi}}}{\|f\|_{H^{\psi}}} \preceq M. \tag{3.3}$$

Conversely, suppose that $C_{\varphi}: H^{\psi}\to\mathcal{B}^{\psi}$ is bounded. Then there is a positive constant C such that for any $f\in H^{\psi}$,

$$\|C_{\varphi}f\|_{\mathcal{B}^{\psi}} \le C\|f\|_{H^{\psi}}.$$

By Lemma 2.3, we have the following functions are uniformly bounded in H^{ψ}

$$f_a(z) = \frac{1}{4}\psi^{-1}(\frac{2}{1-|a|})(\frac{1-|a|^2}{1-z\bar{a}})^2. \tag{3.4}$$

Differentiating (3.4) we have

$$f_a'(z) = \frac{1}{2}\psi^{-1}(\frac{2}{1-|a|})\frac{\bar{a}(1-|a|^2)^2}{(1-z\bar{a})^3}.$$

We easily obtain that

$$\begin{aligned}I(z) &:= \mu(z)\psi^{-1}(\frac{2}{1-|\varphi(z)|})\frac{|\varphi(z)|\cdot|\varphi'(z)|}{1-|\varphi(z)|^2}\\ &\le \|C_{\varphi}f_a\|_{\mathcal{B}^{\psi}} \preceq \|C_{\varphi}\|_{H^{\psi}\to\mathcal{B}^{\psi}}.\end{aligned}$$

It follows that

$$\sup_{|\varphi(z)|>1/2}\frac{\mu(z)\cdot|\varphi'(z)|}{1-|\varphi(z)|^2}\psi^{-1}(\frac{2}{1-|\varphi(z)|}) \le \sup_{z\in\mathbb{D}} I(z) \preceq \|C_{\varphi}\|_{H^{\psi}\to\mathcal{B}^{\psi}} < \infty. \tag{3.5}$$

Let $f(z) = z\in H^{\psi}$. Applying the boundedness of $C_{\varphi}: H^{\psi}\to\mathcal{B}^{\psi}$, we have

$$\sup_{z\in\mathbb{D}}\mu(z)|\varphi'(z)| = \|C_{\varphi}f\|_{\mathcal{B}^{\psi}} \preceq \|C_{\varphi}\|_{H^{\psi}\to\mathcal{B}^{\psi}} < \infty.$$

Then

$$\sup_{0<|\varphi(z)|\le 1/2}\frac{\mu(z)\cdot|\varphi'(z)|}{1-|\varphi(z)|^2}\psi^{-1}(\frac{2}{1-|\varphi(z)|}) \le \frac{4}{3}\psi^{-1}(1)\sup_{z\in\mathbb{D}}\mu(z)|\varphi'(z)| < \infty. \tag{3.6}$$

From (3.5) and (3.6) we get that (3.1) holds. Moreover

$$M \preceq \|C_{\varphi}\|_{H^{\psi}\to\mathcal{B}^{\psi}}. \tag{3.7}$$

Therefore, from (3.3) and (3.7) the asymptotic expression (3.2) is obtained. The proof is completed. □

Theorem 3.2 *Let φ be an analytic self-map of $\mathbb{D}$. Then $C_{\varphi}: H^{\psi}\to\mathcal{B}^{\psi}$ is compact if and only if $\varphi\in\mathcal{B}^{\psi}$ and*

$$\lim_{|\varphi(z)|\to 1^-}\frac{\mu(z)|\varphi'(z)|}{1-|\varphi(z)|^2}\psi^{-1}(\frac{2}{1-|\varphi(z)|}) = 0. \tag{3.8}$$

Proof Suppose that $C_\varphi : H^\psi \to \mathcal{B}^\psi$ is compact. Then $C_\varphi : H^\psi \to \mathcal{B}^\psi$ is bounded, from the proof of Theorem 3.1 we have obtained that $\varphi \in \mathcal{B}^\psi$.

Consider a sequence $\{\varphi(z_j)\}_{j\in\mathbb{N}}$ in $\mathbb{D}$ such that $\lim_{j\to\infty} |\varphi(z_j)| = 1^-$. If such sequence does not exist, then (3.8) obviously holds. Using this sequence, we define the functions

$$f_j(z) = \frac{1}{4}\psi^{-1}(\frac{2}{1-|\varphi(z_j)|})(\frac{1-|\varphi(z_j)|^2}{1-z\overline{\varphi(z_j)}})^2, \quad j \in \mathbb{N}.$$

By Lemma 2.3 we know that the sequence $\{f_j\}_{j\in\mathbb{N}}$ is uniformly bounded in H^ψ. From the proof of [11, Theorem 3.6], it follows that the sequence $\{f_j\}_{j\in\mathbb{N}}$ uniformly converges to zero on any compact subset of $\mathbb{D}$ as $j \to \infty$. Hence, by Lemma 2.4

$$\lim_{j\to\infty} \|C_\varphi f_j\|_{\mathcal{B}^\psi} = 0.$$

From this, we have

$$\begin{aligned} &\frac{\mu(z_j)|\varphi'(z_j)|}{1-|\varphi(z_j)|^2}\psi^{-1}(\frac{2}{1-|\varphi(z_j)|}) \le \sup_{z\in\mathbb{D}} \frac{\mu(z)|\varphi'(z)|}{1-|\varphi(z)|^2}\psi^{-1}(\frac{2}{1-|\varphi(z)|}) \\ &\le \|C_\varphi f_j\|_{\mathcal{B}^\psi}. \end{aligned}$$

This implies that

$$\lim_{|\varphi(z)|\to 1^-} \frac{\mu(z)|\varphi'(z)|}{1-|\varphi(z)|^2}\psi^{-1}(\frac{2}{1-|\varphi(z)|}) = 0.$$

Now suppose that $\varphi \in \mathcal{B}^\psi$ and (3.8) holds. We first check that $C_\varphi : H^\psi \to \mathcal{B}^\psi$ is bounded. We observe that (3.8) implies that for every $\epsilon > 0$, there is a $0 < \delta < 1$ such that for any $z \in \mathbb{D}$ with $|\varphi(z)| > \delta$

$$\frac{\mu(z)|\varphi'(z)|}{1-|\varphi(z)|^2}\psi^{-1}(\frac{2}{1-|\varphi(z)|}) < \epsilon. \tag{3.9}$$

Since for $z \in \mathbb{D}$ with $0 < |\varphi(z)| \le \delta$

$$\frac{\mu(z)|\varphi'(z)|}{1-|\varphi(z)|^2}\psi^{-1}(\frac{2}{1-|\varphi(z)|}) \le \|\varphi\|_{\mathcal{B}^\psi}\frac{1}{1-\delta^2}\psi^{-1}(\frac{2}{1-\delta}),$$

we have

$$\begin{aligned} &\sup_{z\in\mathbb{D}} \frac{\mu(z)|\varphi'(z)|}{1-|\varphi(z)|^2}\psi^{-1}(\frac{2}{1-|\varphi(z)|}) \\ &\le \sup_{0<|\varphi(z)|\le\delta} \frac{\mu(z)|\varphi'(z)|}{1-|\varphi(z)|^2}\psi^{-1}(\frac{2}{1-|\varphi(z)|}) + \sup_{|\varphi(z)|>\delta} \frac{\mu(z)|\varphi'(z)|}{1-|\varphi(z)|^2}\psi^{-1}(\frac{2}{1-|\varphi(z)|}) \\ &\le \|\varphi\|_{\mathcal{B}^\psi}\frac{1}{1-\delta^2}\psi^{-1}(\frac{2}{1-\delta}) + \epsilon. \end{aligned}$$

This proves that $C_\varphi : H^\psi \to \mathcal{B}^\psi$ is bounded.

By Lemma 2.4, in order to prove that $C_\varphi : H^\psi \to \mathcal{B}^\psi$ is compact, we just need to prove that if the sequence $\{f_j\}_{j\in\mathbb{N}}$ is uniformly bounded in H^ψ and uniformly converges to zero on any compact subset of $\mathbb{D}$ as $j \to \infty$, then

$$\lim_{j\to\infty} \|C_\varphi f_j\|_{\mathcal{B}^\psi} = 0.$$

For any $\epsilon > 0$ and the associated δ in (3.9), by using again that $\varphi \in \mathcal{B}^{\psi}$ and Lemma 2.3, we have

$$\begin{aligned}
\|C_{\varphi} f_j\|_{\mathcal{B}^{\psi}} &= \sup_{z\in\mathbb{D}} \mu(z)|f_j'(\varphi(z))| \cdot |\varphi'(z)| \\
&\le \sup_{0<|\varphi(z)|\le\delta} \mu(z)|f_j'(\varphi(z))| \cdot |\varphi'(z)| + \sup_{|\varphi(z)|>\delta} \frac{\mu(z)|\varphi'(z)|}{1-|\varphi(z)|^2}\psi^{-1}(\frac{2}{1-|\varphi(z)|})\|f_j\|_{H^{\psi}} \\
&\le \|\varphi\|_{\mathcal{B}^{\psi}} \sup_{0<|\varphi(z)|\le\delta} |f_j'(\varphi(z))| + \epsilon \sup_{j\in\mathbb{N}} \|f_j\|_{H^{\psi}} \\
&\to 0, \quad \text{as} \quad j \to \infty,
\end{aligned}$$

where we have used the fact that from $f_j \to 0$ as $j \to \infty$ uniformly on compact subsets of $\mathbb{D}$, it follows that $f_j' \to 0$ as $j \to \infty$ uniformly on compact subsets of $\mathbb{D}$. Hence

$$\lim_{j\to\infty} \|C_{\varphi} f_j\|_{\mathcal{B}^{\psi}} = 0,$$

which follows that $C_{\varphi} : H^{\psi} \to \mathcal{B}^{\psi}$ is compact. The proof is completed. □

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