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Reliability and Controllability of the System Consisting of a Robot and Its Associated Safety Mechanism

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Abstract This paper mainly studies the system composed of robot and its safety mechanism. By using functional analysis method, the partial differential equation of the original system is transformed into the abstract Cauchy problem in Banach space. The transient and steady-state availability of the system can be obtained by algebraic theory and C_0 semi-group theory, and the system is proved to be reliable and zero-state controllable by using transformation variables. Finally, Maple software is used to simulate the system transient reliability and steady-state availability.

Keywords Cauchy problem; transient reliability; steady availability; numerical simulation

MR(2010) Subject Classification 35R09; 34K30; 47G20

1. Introduction

With the development of science and technology, domestic and foreign scholars pay more and more attention to robot research [1]. In recent years, outstanding progress has been made in the research and application of robot technology in China, including industrial robot [2, 3], medical and rehabilitation robot [4], Mobile robot [5], android, etc. Therefore, the safety of the robot and its associated mechanisms must be guaranteed [6, 7]. For this reason, many researchers have studied the safety, stability [8,9], reliability [10,11] and accuracy [12] of various robots. Dhillon and Fashandi established the system model [13] which was composed of the safety mechanism connected to the robot. Under the assumption that the only nonnegative solution exists and the limit of the solution also exists, the steady-state solution of the model was studied by Laplace transformation, and the state probability of the system was finally obtained under Laplace transform. Guo [14] and Gupur [15] proved that the system solution and the non-time-dependent solution of this kind of model are unique by using the theory of integral equation and the theory of strongly continuous operator semi-group. Wang et al. [16] gave the semi-discrete model of the system by using the approximation of elementary functions. Pang et al. [17] analyzed the distribution of operator spectral points and obtained the asymptotic stability condition of the system. Guo and Xu [18] proved that the system had a unique non-negative dynamic dependent

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solution by using the theory of strongly continuous operator semi-groups, and asymptotically converges to a stable solution under the condition of the corresponding space norm meaning.

On the basis of [18], this paper selects the appropriate state space in a system of safety mechanisms connected to the robot, defines the operator and corresponding norm, and limits the failure rate and repair rate of the system, transforms the system partial differential equation into ordinary differential equation by using functional analysis theory, and finally reaches the abstract Cauchy problem. Furthermore, the transient reliability and steady-state availability of the system are obtained and the reliability and controllability of the zero state are proved.

2. Mechanism of the system

In this paper, we consider the following models described by differential integral equations [17, 18]

$$\frac{\mathrm{d}p_0(t)}{\mathrm{d}t} = -(\lambda_1 + \lambda_4 + \lambda_{\bar{5}})p_0(t) + \mu_1 p_1(t) + \sum_{i=2}^{5} \int_0^\infty p_i(x,t)\mu_i(x)\mathrm{d}x, \tag{2.1}$$

$$\frac{\mathrm{d}p_1(t)}{\mathrm{d}t} = \lambda_1 p_0(t) - (\lambda_2 + \lambda_3 + \mu_1)p_1(t), \tag{2.2}$$

$$\frac{\mathrm{d}p_{\bar{5}}(t)}{\mathrm{d}t} = \lambda_{\bar{5}} p_0(t) - \lambda_5 p_{\bar{5}}(t), \tag{2.3}$$

$$\frac{\partial p_i(x,t)}{\partial x} + \frac{\partial p_i(x,t)}{\partial t} = -\mu_i(x)p_i(x,t), \quad i = 2,3,4,5, \tag{2.4}$$

$$p_2(0,t) = \lambda_2 p_1(t), \quad p_3(0,t) = \lambda_3 p_1(t), \tag{2.5}$$

$$p_4(0,t) = \lambda_4 p_0(t), \quad p_5(0,t) = \lambda_5 p_{\bar{5}}(t), \tag{2.6}$$

$$p_0(0) = 1, \quad p_1(0) = p_{\bar{5}}(0) = p_i(x,0) = 0, \quad i = 2,3,4,5, \tag{2.7}$$

where $p_0(t)$ refers to the probability that the robot and its associated safety mechanism work normally; $p_1(t)$ refers to the probability that the robot works with the faulty safety mechanism; $p_{\bar{5}}(t)$ refers to the probability of a crash caused by a common fault; $p_i(x,t)$ refers to the probability of repair time x when the status remains in i at time $t, i = 2,3,4,5$; $\mu_i(x)$ refers to the repair rate of the system in the fault state $i, i = 2,3,4,5$; λ_1 refers to the failure rate of system safety mechanisms; λ_2 refers to the failure rate of occasional robot failures; λ_3 refers to the failure rate of robot safety mechanism; λ_4 refers to the robot failure rate; λ_5 refers to robot failure rate due to common failures; $\lambda_{\bar{5}}$ refers to the crashes caused by common failures; μ_1 refers to the stationary repair rate for states 1 to 0. The state transition diagram of the system is shown in Figure 1.

In Figure 1, state 0 indicates that the robot and its associated safety mechanism work normally; state 1 indicates that the safety mechanism fails and the robot works normally; state 2 indicates that the robot fails due to occasional failure; state 3 indicates that the robot fails due to safety failure; state 4 indicates that the safety mechanism works normally and the robot fails; and state 5 indicates that the robot fails due to common cause fault.

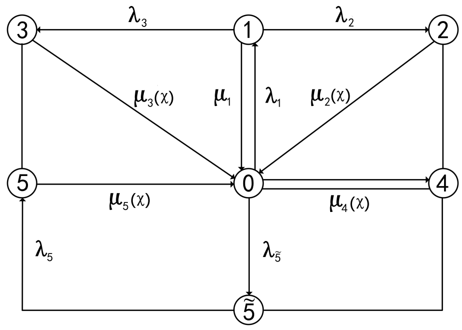

Figure 1 System model consisting of a robot and its associated safety mechanism

3. System reliability

The reliability of the system is an important part of the repairable system model.

Definition 3.1 ([19]) *$p_0(t)$ expresses the probability that the two components are in good condition and work normally while $t=0$. The magnitude of the probability determines the probability that the system will work normally, which affects the reliability of the system. $p_0(t)$ is called the transient reliability of the system (2.1)–(2.7).*

Definition 3.2 ([19]) *If $\lim_{t\to\infty} p_0(t) = p_0^*$, then p_0^* is the steady-state reliability of system (2.1)–(2.7).*

Definition 3.3 ([19]) *If $p_0(t) \geq p_0^*$, then system (2.1)–(2.7) has reliability.*

Theorem 3.4 *System (2.1)–(2.7) has reliability when both the failure rate and repair rate are constant.*

Proof For system (2.1)–(2.7), introduce the following operators B

$$B = \mathrm{diag}(-b_0,\ -b_1,\ \lambda_5,\ -\frac{\mathrm{d}}{\mathrm{d}x} - \mu_i(x)),\quad i = 2,3,4,5$$

where

$$b_0 = \lambda_1 + \lambda_4 + \lambda_{\tilde{5}},\quad b_1 = \lambda_2 + \lambda_3 + \mu_1.$$

To simplify system (2.1)–(2.7), we define the matrix

$$\hat{B} = \begin{pmatrix} 0 & \hat{B}_1 \\ \hat{B}_2 & 0 \end{pmatrix}, \hat{B}_1 = \Big(\mu_1,\ 0, \int_0^\infty \mu_i(x)\mathrm{d}x\Big)\ (i = 2,3,4,5),\ \hat{B}_2 = (\lambda_1, \lambda_{\tilde{5}}, 0, 0, 0, 0,)^T.$$

Let us take state space as

$$X = \Big\{p = (p_0, p_1, p_{\tilde{5}}, p_2, \ldots, p_5)^T \in R^3 \times (L^1[0,\infty))^4 | \|p\| = |p_0| + |p_1| + |p_{\tilde{5}}| + \sum_{i=2}^{5} \|p_i\|_{L^1_{[0,\infty)}}\Big\}.$$

Obviously, X is a Banach space [18].

The domain of operator B is

$D(B) = \{p = (p_0, p_1, p_{\bar{5}}, p_2, \ldots, p_5)^T \in X | \frac{\mathrm{d}p_i(x)}{\mathrm{d}x} \in L^1[0, \infty)$, $p_i(x)$ is an absolutely continuous function, $i = 2, 3, 4, 5$ and $p_2(0) = \lambda_2 p_1,\ p_3(0) = \lambda_3 p_1,\ p_4(0) = \lambda_4 p_0\ , p_5(0) = \lambda_5\ p_{\bar{5}}\}$.

By the above simplification, the system (2.1)–(2.7) can be described as an abstract Cauchy problem in Banach space.

$$\begin{cases} \frac{\mathrm{d}p(t)}{\mathrm{d}t} = (B + \hat{B})p(t), & t \geq 0, \\ p(0) = (1, 0, 0, 0, 0, 0, 0). \end{cases} \tag{3.1}$$

According to the physical meaning of the system and the methods in [20, 21], the failure rate and repair rate of the system are limited, and the model is simplified, let

$$\lambda_i = \lambda_{\bar{5}} = \lambda, \quad i = 1, 2, 3, 4, 5$$

$$\mu_1 = \mu_j(x) = \mu, \quad j = 2, 3, 4, 5$$

and

$$\int_0^\infty p_i(x, t)\mathrm{d}x = p_i(t), \quad i = 2, 3, 4, 5.$$

According to the actual physical background of system (2.1)–(2.7), it follows that

$$\sum_{i=0}^{5} p_i(t) = 1.$$

Therefore, (2.1)–(2.3) can be reduced to

$$\frac{\mathrm{d}p_0(t)}{\mathrm{d}t} = -(3\lambda + \mu)p_0(t) + \mu, \tag{3.2}$$

$$\frac{\mathrm{d}p_1(t)}{\mathrm{d}t} = \lambda p_0(t) - (2\lambda + \mu)p_1(t), \tag{3.3}$$

$$\frac{\mathrm{d}p_{\bar{5}}(t)}{\mathrm{d}t} = \lambda p_0(t) - \lambda p_{\bar{5}}(t). \tag{3.4}$$

Here, the coefficient matrix of equations (3.2)–(3.4) is

$$A = \begin{pmatrix} -(3\lambda + \mu) & 0 & 0 \\ \lambda & -(2\lambda + \mu) & 0 \\ \lambda & 0 & -\lambda \end{pmatrix}.$$

Thus, equations (3.2)–(3.4) can be reduced to

$$\begin{cases} \frac{\mathrm{d}\hat{p}(t)}{\mathrm{d}t} = A\hat{p}(t) + \vec{\mu}, \\ \hat{p}(0) = (1, 0, 0)^T \end{cases} \tag{3.5}$$

where

$$\hat{p}(t) = (p_0(t), p_1(t), p_{\bar{5}}(t))^T, \quad \vec{\mu} = (\mu, 0, 0)^T.$$

Next, use the theories of differential equations and C_0 semi-group [19] to find the solution of the system (3.5).

(i) Find all the eigenvalues of the matrix A.

For the matrix A, solve the characteristic equation

$$\det(rE-A)=\begin{vmatrix} r+(3\lambda+\mu) & 0 & 0 \\ -\lambda & r+(2\lambda+\mu) & 0 \\ -\lambda & 0 & r+\lambda \end{vmatrix}=0.$$

The eigenvalues are

$$r_1=-(3\lambda+\mu),\ r_2=-2\lambda-\mu,\ r_3=-\lambda.$$

(ii) For e^{At}, from [22], we obtain

$$\mathrm{e}^{At}=q_1(t)Q_0+q_2(t)Q_1+q_3(t)Q_2, \tag{3.6}$$

where

$$Q_0=E,\ Q_1=A-r_1E,\ Q_2=(A-r_2E)Q_1 \tag{3.7}$$

and

$$q_1(t)=\mathrm{e}^{r_1t},\ q_2(t)=\int_0^t \mathrm{e}^{r_2(t-s)}q_1(s)\mathrm{d}s,\ q_3(t)=\int_0^t \mathrm{e}^{r_3(t-s)}q_2(s)\mathrm{d}s.$$

By substituting the eigenvalues in the integrals, we obtain

$$q_1(t)=\mathrm{e}^{-(3\lambda+\mu)t}, q_2(t)=\frac{\mathrm{e}^{r_2t}-\mathrm{e}^{r_1t}}{r_2-r_1}=\frac{\mathrm{e}^{-(2\lambda+\mu)t}-\mathrm{e}^{-(3\lambda+\mu)t}}{\lambda}, \tag{3.8}$$

$$q_3(t)=\frac{\mathrm{e}^{r_3t}-\mathrm{e}^{r_2t}}{(r_2-r_1)(r_3-r_2)}-\frac{\mathrm{e}^{r_3t}-\mathrm{e}^{r_1t}}{(r_2-r_1)(r_3-r_1)}=\frac{\mathrm{e}^{-(2\lambda+\mu)t}}{\lambda(\lambda+\mu)}+\frac{\mathrm{e}^{-(3\lambda+\mu)t}}{\lambda(2\lambda+\mu)}+\frac{\mathrm{e}^{-\lambda t}}{(\lambda+\mu)(2\lambda+\mu)}. \tag{3.9}$$

Substituting (3.7)–(3.9) into (3.6), it follows that

$$\mathrm{e}^{At}=\begin{pmatrix} \mathrm{e}^{-(3\lambda+\mu)t} & 0 & 0 \\ \mathrm{e}^{-(2\lambda+\mu)t}-\mathrm{e}^{-(3\lambda+\mu)t} & \mathrm{e}^{-(2\lambda+\mu)t} & 0 \\ \frac{\lambda}{2\lambda+\mu}(\mathrm{e}^{-\lambda t}-\mathrm{e}^{-(3\lambda+\mu)t}) & 0 & \mathrm{e}^{-\lambda t} \end{pmatrix}. \tag{3.10}$$

(iii) From the knowledge of linear algebra, it is easy to find the inverse matrix A^{-1} of A

$$A^{-1}=\begin{pmatrix} -\frac{1}{3\lambda+\mu} & 0 & 0 \\ -\frac{\lambda}{(3\lambda+\mu)(2\lambda+\mu)} & -\frac{1}{2\lambda+\mu} & 0 \\ -\frac{1}{3\lambda+\mu} & 0 & -\frac{1}{\lambda} \end{pmatrix}. \tag{3.11}$$

(iv) From [22], the solution of (3.5) is given by

$$\hat{p}(t)=\mathrm{e}^{At}\hat{p}(0)+\int_0^t \mathrm{e}^{A(t-s)}\mu\mathrm{d}s=\mathrm{e}^{At}\hat{p}(0)-A^{-1}(E-\mathrm{e}^{At})\mu. \tag{3.12}$$

It follows from (3.10)–(3.12) that

$$\hat{p}(t)=\begin{pmatrix} \frac{\mu}{3\lambda+\mu}+\frac{3\lambda}{3\lambda+\mu}\mathrm{e}^{-(3\lambda+\mu)t} \\ \frac{\lambda\mu}{(3\lambda+\mu)(2\lambda+\mu)}+\frac{2\lambda}{(2\lambda+\mu)}\mathrm{e}^{-(2\lambda+\mu)t}-\frac{3\lambda}{3\lambda+\mu}\mathrm{e}^{-(3\lambda+\mu)t} \\ \frac{\mu}{3\lambda+\mu}+\frac{\lambda-\mu}{2\lambda+\mu}\mathrm{e}^{-\lambda t}-\frac{3\lambda^2}{(2\lambda+\mu)(3\lambda+\mu)}\mathrm{e}^{-(3\lambda+\mu)t} \end{pmatrix}.$$

Then, the transient reliability of system (2.1)–(2.7) is

$$p_0(t)=\frac{\mu}{3\lambda+\mu}+\frac{3\lambda}{3\lambda+\mu}\mathrm{e}^{-(3\lambda+\mu)t}. \tag{3.13}$$

Thus, from Definition 3.2, the steady-state reliability of system (2.1)–(2.7) is

$$p_0^* = \lim_{t\to\infty} p_0(t) = \frac{\mu}{3\lambda+\mu}. \tag{3.14}$$

To sum up the above discussion, we have $p_0(t) \geq p_0^*$. As a result, based on Definition 3.3, system (2.1)–(2.7) has reliability. □

4. Zero-stage controllability of the system

The controllability of a system is one of the basic characteristics of modern control system, so the controllability of the system, especially the zero-state controllability of the system $p_0(t)$, is an important content of research of repairable systems. When discussing the controllability $p_0(t)$, that is, to find an element that can be transferred to a specified state at a finite moment, where

$$U = \{\, \mu(x) |\, \mu(x) = (\mu_2(x), \mu_3(x)) \in L^\infty[0,\infty) \times L^\infty[0,\infty),\ 0 \leq \mu_i(x) < \infty \},$$

$$M = \sup_{x\in[0,\infty)} \mu_i(x) < \infty, \quad \int_0^\infty \mu_i(x)\mathrm{d}x = \infty, \quad i = 2, 3$$

here U is the admissible control set.

Theorem 4.1 *Assume that the failure rate is $\lambda_i = \lambda_{\tilde{5}} = \lambda$ $(i = 1,2,3,4,5)$, and the repair rate is $\mu_1 = \mu_j(x) = \mu$ $(j = 2,3,4,5)$, and η is the probability that the system reached at the time T when $T > 0$, and meets $\mathrm{e}^{-2\lambda T} < \eta < 1$, then there is a $\mu^* \in U$, such that $p_0(T) = \eta$.*

Proof From (3.13), we obtain

$$p_0(t) = \frac{\mu}{3\lambda+\mu} + \frac{3\lambda}{3\lambda+\mu}\mathrm{e}^{-(3\lambda+\mu)t},$$

$$p_0(T) = \frac{\mu}{3\lambda+\mu} + \frac{3\lambda}{3\lambda+\mu}\mathrm{e}^{-(3\lambda+\mu)T}. \tag{4.1}$$

Considering $p_0(T)$ in (4.1) as a function of variable μ, we can get

$$\frac{\mathrm{d}p_0(T)}{\mathrm{d}\mu} = \frac{3\lambda}{(3\lambda+\mu)^2} - \frac{3\lambda}{(3\lambda+\mu)^2}\mathrm{e}^{-(3\lambda+\mu)T}[1 + (3\lambda+\mu)T].$$

It follows form $\mathrm{e}^x > x + 1$, $x > 0$. That, $\mathrm{e}^{(3\lambda+\mu)T} > 1 + (3\lambda+\mu)T$. Thus,

$$\frac{\mathrm{d}p_0(T)}{\mathrm{d}\mu} \geq \frac{3\lambda}{(3\lambda+\mu)^2} - \frac{3\lambda}{(3\lambda+\mu)^2}\mathrm{e}^{-(3\lambda+\mu)T}\mathrm{e}^{(3\lambda+\mu)T} = 0.$$

This indicates that $p_0(T)$ is a monotonically increasing function of the variable μ, notice

$$\lim_{\mu\to 0} p_0(T) = \mathrm{e}^{-3\lambda T}, \quad \lim_{\mu\to\infty} p_0(T) = 1.$$

Therefore, for any η: $\mathrm{e}^{-3\lambda T} < \eta < 1$, based on the intermediate value theorem with the continuous function, there exists a $\mu^* \in U$, such that $p_0(T) = \eta$. □

Theorem 4.1 proves that the zero state $p_0(t)$ of the system is controllable.

5. Numerical simulation

5.1. Reliability of the system

Assume $\lambda = 0.5$ and $\mu = 0.75$. Supported by Maple mathematical software, the transient and steady-state reliability are presented by Figures 2 and 3.

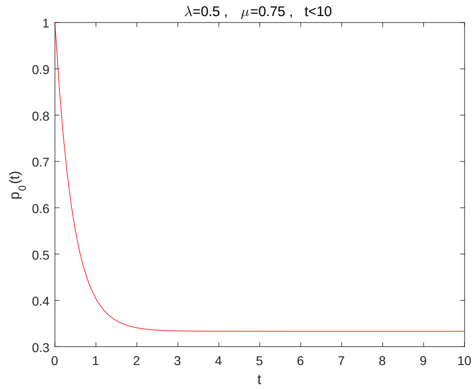

Figure 2 Transient availability of the system

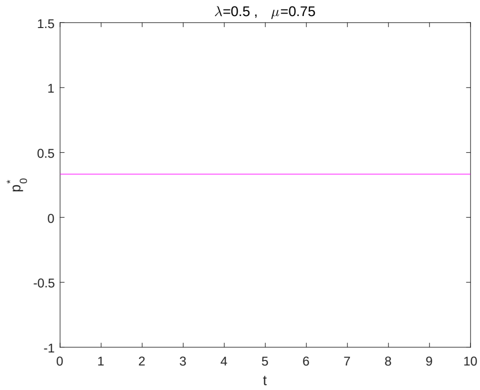

Figure 3 Steady-state availability of the system

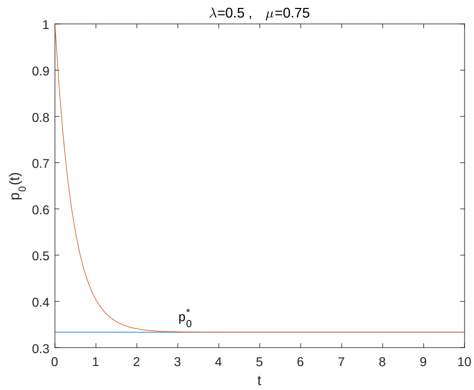

Figure 4 Transient availability $p_0(t)$ and steady-state availability p_0^* of the system

To compare transient availability $p_0(t)$ and steady state availability p_0^* of system, put $p_0(t)$ and p_0^* together, as shown in Figure 4 above.

Figure 4 shows that at any time t, the transient availability $p_0(t)$ is always above the state availability p_0^* of the system.

5.2. Controllability of the system

To testify through numerical mechanism, from formula (4.1), if $p_0(T)$ is set as the variable μ of $p_0(T, \mu)$, when $\lambda = 0.5$, $T = 50$, then Figure 5 shows that $p_0(T, \mu)$ is supported by Maple.

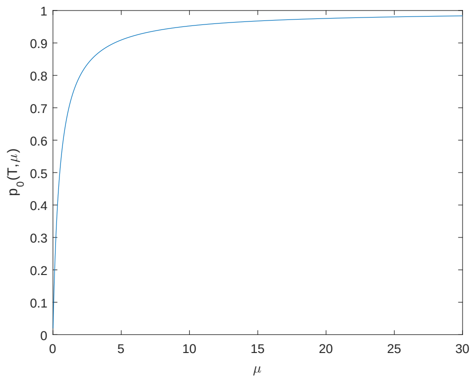

Figure 5 Simulation diagram of $p_0(T, \mu)$

From Figure 5, the following conclusion can be drawn:

(1) $p_0(T, \mu)$ is the monotone increasing function of μ.

(2) $\lim_{\mu\to\infty} p_0(T, \mu) = 1$ and $\lim_{\mu\to 0} p_0(T, \mu) = 0$.

(3) For any $\eta : 0 < \eta < 1$, there is a $\mu > 0$, such that $p_0(T, \mu) = \eta$.

Table 1 shows the relationship between η and μ, using mathematical software Maple.

η	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
μ	0.5000	0.8077	1.2273	1.8333	2.7857	4.5000	8.5000	28.5000

Table 1 Corresponding relation of η and μ

The above numerical calculation and simulation results show that when the failure rate and the repair rate are assumed to be constant, $p_0(T, \mu)$ can be regarded as a function of μ, which can make the state transfer to the desired state $p_0(t) = \eta$ at a limited time T $(T > 0)$, so the state $p_0(t)$ is controllable.

6. Conclusions

Through numerical experiments on the results of system reliability and controllability, simulation results in Figures 2-4 show that when the failure rate and repair rate are assumed to be constant the transient availability of the system meets the requirements of $p_0(t) > p_0^*$. Therefore, the system has reliability. The conclusion is consistent with Theorem 3.4. Although when $\mu \to 0$, we have $p_0(T,\mu)\to 0$. In fact, when $\lambda=0.5$, $T=50$, then $\lim\limits_{\mu\to 0}p_0(T,\mu)=\mathrm{e}^{-75}\neq 0$. However, as the absolute error value approaches to zero, it cannot affect the whole research. Similarly, using the same method, $p_1(t)$-$p_5(t)$ is also controllable and hence system (2.1)–(2.7) is controllable.

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