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Spatial Dynamics of Competing Model in a Periodic Unstirred Chemostat with Delay

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Abstract In this paper, we propose two species competition model in a chemostat that uses a nonlocal delayed chemostat model of a single species feeding on a periodically varying input nutrient. By the theory of semigroup, the existence and uniqueness of solution of the system is obtained. Furthermore, we show the competitive exclusion principle for the model, and the sufficient conditions of the coexistence of the system is established.

Keywords chemostat; nonlocal delay; competition exclusion principle; coexistence

MR(2020) Subject Classification 35K55; 35K57; 37N25; 92C17

1. Introduction

The chemostat is a piece of laboratory apparatus which plays an important role in microbiology. In ecology the chemostat is a model of a simple lake, but in chemical engineering it also serves as a laboratory model of a bio-reactor used to manufacture products with genetically altered organisms. The chemostat has many applications in the commercial production of microorganisms [1] and as a model for waste water treatment [2]. Hence, chemostat models have attracted much attention of both biologists and mathematicians. Analytic work on the chemostat models can be found in [3–5] and references therein.

Recently, Pu, Jiang and Wang [6] discussed a nonlocal delayed chemostat model of a single species feeding on a periodically varying input. Motivated by [6], we will study the following single resource-two competing populations model:

$$\begin{cases} \frac{\partial R(t,x)}{\partial t} = \delta\frac{\partial R^2}{\partial x^2} - \nu\frac{\partial R}{\partial x} - q_1 f_1(R)u_1 - q_2 f_2(R)u_2, & t>0, x\in(0,L),\\ \frac{\partial u_1(t,x)}{\partial t} = \delta\frac{\partial^2 u_1}{\partial x^2} - \nu\frac{\partial u_1}{\partial x} - \mu_0 u_1 + & \\ \qquad q_1\int_0^L \Gamma(\tau_1,x,y) f_1(R(t-\tau_1,y))u_1(t-\tau_1,y)\mathrm{d}y, & t>0, x\in(0,L),\\ \frac{\partial u_2(t,x)}{\partial t} = \delta\frac{\partial^2 u_2}{\partial x^2} - \nu\frac{\partial u_2}{\partial x} - \mu_0 u_2 + & \\ \qquad q_2\int_0^L \Gamma(\tau_2,x,y) f_2(R(t-\tau_2,y))u_2(t-\tau_2,y)\mathrm{d}y, & t>0, x\in(0,L), \end{cases} \tag{1.1}$$

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with boundary conditions

$$\begin{cases} \nu R(t,0) - \delta \frac{\partial R}{\partial x}(t,0) = \nu R^{(0)}(t), \frac{\partial R}{\partial x}(t,L) = 0, & t > 0, \\ \nu u_1(t,0) - \delta \frac{\partial u_1}{\partial x}(t,0) = \frac{\partial u_1}{\partial x}(t,L) = 0, & t > 0, \\ \nu u_2(t,0) - \delta \frac{\partial u_2}{\partial x}(t,0) = \frac{\partial u_2}{\partial x}(t,L) = 0, & t > 0, \end{cases} \tag{1.2}$$

and initial conditions

$$\begin{cases} R(0,x) = R^0(x) \geq 0, & x \in (0,L), \\ u_i(0,x) = u_i^0(x) \geq 0, & x \in (0,L), \end{cases} \tag{1.3}$$

where q_i $(i = 1,2)$ is the constant nutrient quota for species, $R(t,x)$ and $u_i(t,x)$ $(i = 1,2)$ denote the concentration of the nutrient and two population of microorganisms at time t, position x; the constants δ and ν are the diffusion coefficient and advection coefficient, respectively; μ_0 is the death rate of species; $\tau_i > 0$ $(i = 1,2)$ is the time delay. The nonlinear function $f_i(R)$ $(i = 1,2)$ describes the nutrient uptake rate and the growth rate of the organism u_i $(i = 1,2)$ at nutrient concentration R. We assume $f_i(R)$ $(i = 1,2)$ satisfies

$$f_i(0) = 0, \ f_i'(R) > 0, \ \forall R \geq 0, \ f_i(\cdot) \in C^2(0,+\infty). \tag{1.4}$$

A usual example is the Monod function

$$f_i(R) = \frac{\mu_i R}{K_i + R}, \quad \forall R \geq 0, \tag{1.5}$$

where the constants $\mu_i > 0$ $(i = 1,2)$ and $K_i > 0$ $(i = 1,2)$ are, respectively, the maximum growth rate and half-saturation coefficient. The inflow nutrient concentration $R^{(0)}(t)$ satisfies

(H) $R^{(0)}(\cdot) \in C^2((-\tau,\infty),\mathbb{R})$, $R^{(0)}(t) \geq 0$ $(\not\equiv 0)$ and $R^{(0)}(t+\omega) = R^{(0)}(t)$ for some $\omega > 0$.

For the convenience of discussion, we let $\tau_1 = \tau_2 = \tau$.

The purpose of this paper is to study the global dynamics of system (1.1)–(1.3) and the rest of this paper is organized as follows. In Section 2, we first present some preliminaries, then investigate the existence and uniqueness of the global solution of system (1.1)–(1.3). In Section 3, we show the competitive exclusion principle for system (1.1)–(1.3). In Section 4, we obtain the sufficient conditions of the coexistence of system (1.1)–(1.3). Finally, we give a brief discussion of the paper in Section 5.

2. Existence and uniqueness of solution

Let $\mathbb{X} = C([0,L],\mathbb{R}^3)$ be the Banach space with the usual supremum norm $\|\cdot\|_{\mathbb{X}}$. Then $\mathbb{X}^+ = C([0,L],\mathbb{R}^3_+)$ is the positive cone of $\mathbb{X}$. For $\tau \geq 0$, define $C_\tau = C([-\tau,0],\mathbb{X})$ with the norm $\|\phi\| = \max_{\theta\in[-\tau,0]} \|\phi(\theta)\|_{\mathbb{X}}$, $\forall \phi \in C_\tau$. Then C_τ is a Banach space and $C_\tau^+ = C([-\tau,0],\mathbb{X}^+)$ is the positive cone of C_τ. Let $\hat{\mathbf{u}}$ denote the inclusion $\mathbb{X} \to C_\tau$ by $\mathbf{u} \to \hat{\mathbf{u}}$, $\hat{\mathbf{u}}(\theta) = \mathbf{u}$, $\theta \in [-\tau,0]$. Given a function $\mathbf{u}(t) : [-\tau,\sigma) \to \mathbb{X}$ $(\sigma > 0)$, define $\mathbf{u}_t \in C_\tau$ by $\mathbf{u}_t(\theta) = \mathbf{u}(t+\theta)$, $\theta \in [-\tau,0]$.

The idea is to view the system (1.1)–(1.3) as the abstract ordinary differential equation in $\mathbb{X}^+$ and so-called mild solutions can be obtained for any given initial data. Let $T(t)$ be the positive, non-expansive, analytic semigroup on $C([0,L],\mathbb{R})$ (see [7, Chapter 7]) such that $z = T(t)z_0$

satisfies the linear initial value problem

$$\begin{cases} \frac{\partial z}{\partial t} = \delta \frac{\partial^2 z}{\partial x^2} - \nu \frac{\partial z}{\partial x}, & t > 0,\ 0 < x < L, \\ \nu z(t,0) - \delta \frac{\partial z}{\partial x}(t,0) = \frac{\partial z}{\partial x}(t,L) = 0, & t > 0, \\ z(0,x) = z_0(x), & 0 < x < L. \end{cases} \tag{2.1}$$

Let $V(t,s)$ $(t > s)$ be the evolution operator on $C([0,L],\mathbb{R})$ (see [8, Chapter II]) such that $v = V(t,s)v_0$ satisfies the linear system with nonhomogeneous, periodic boundary conditions, with start time s, given by

$$\begin{cases} \frac{\partial v}{\partial t} = \delta \frac{\partial^2 v}{\partial x^2} - \nu \frac{\partial v}{\partial x}, & t > s,\ 0 < x < L, \\ \nu v(t,0) - \delta \frac{\partial v}{\partial x}(t,0) = vR^{(0)}(t),\ \ \frac{\partial v}{\partial x}(t,L) = 0, & t > s, \\ v(s,x) = v_0(x), & 0 < x < L. \end{cases} \tag{2.2}$$

Due to the time periodicity of the inhomogeneity in the boundary condition, $R^0(t+\omega) = R^0(t)$, it follows from [9] that

$$V(t+\omega, s+\omega) = V(t,s), \quad \forall t > s.$$

Define $\mathbf{F} = (F_1, F_2, F_3) : C_\tau^+ \to C_\tau^+$ by

$$\begin{aligned} F_1(\phi_1,\phi_2,\phi_3) &:= -q_1 f_1(\phi_1(0,\cdot))\phi_2(0,\cdot) - q_2 f_2(\phi_1(0,\cdot))\phi_3(0,\cdot), \\ F_2(\phi_1,\phi_2,\phi_3) &:= q_1 \int_0^L \Gamma(\tau,x,y) f_1(\phi_1(-\tau,y))\phi_2(-\tau,y)\mathrm{d}y, \\ F_3(\phi_1,\phi_2,\phi_3) &:= q_2 \int_0^L \Gamma(\tau,x,y) f_2(\phi_1(-\tau,y))\phi_3(-\tau,y)\mathrm{d}y, \end{aligned}$$

where $x \in [0,L]$, $\phi = (\phi_1,\phi_2,\phi_3) \in C_\tau^+$. Set $\mathbf{u} = (R, u_1, u_2)$, then

$$\begin{cases} R(t) = V(t,0)R^0 + \displaystyle\int_0^t T(t-s)F_1(\mathbf{u}(s))\mathrm{d}s, \\ u_1(t) = e^{-\mu_0 t}T(t)u_1^0 + \displaystyle\int_0^t e^{-\mu_0(t-s)}T(t-s)F_2(\mathbf{u}(s))\mathrm{d}s, \\ u_2(t) = e^{-\mu_0 t}T(t)u_2^0 + \displaystyle\int_0^t e^{-\mu_0(t-s)}T(t-s)F_3(\mathbf{u}(s))\mathrm{d}s. \end{cases}$$

So, system (1.1)–(1.3) can be expressed as

$$\mathbf{u}(t,\phi) = \mathbf{U}(t,0)\phi(0) + \int_0^t \mathbf{T}(t-s)\mathbf{F}(s,\mathbf{u}(s))\mathrm{d}s, \quad t \ge 0,\ \phi \in C_\tau^+,$$

where,

$$\mathbf{U}(t,s) = \begin{pmatrix} V(t,s) & 0 & 0 \\ 0 & e^{-\mu_0 t}T(t-s) & 0 \\ 0 & 0 & e^{-\mu_0 t}T(t-s) \end{pmatrix},$$

$$\mathbf{T}(t-s) = \begin{pmatrix} T(t-s) & 0 & 0 \\ 0 & e^{-\mu_0(t-s)}T(t-s) & 0 \\ 0 & 0 & e^{-\mu_0(t-s)}T(t-s) \end{pmatrix}.$$

To show the global existence of solutions of (1.1)–(1.3), we first consider the following differential equation:

$$\begin{cases} \frac{\partial \hat{R}(t,x)}{\partial t} = \delta \frac{\partial^2 \hat{R}}{\partial x^2} - \nu \frac{\partial \hat{R}}{\partial x}, & t > 0,\ 0 < x < L, \\ \nu \hat{R}(t,0) - \delta \frac{\partial \hat{R}}{\partial x}(t,0) = \nu R^{(0)}(t),\ \frac{\partial \hat{R}}{\partial x}(t,L) = 0, & t > 0 \end{cases} \tag{2.3}$$

with the initial condition $\hat{R}(0,x) = \hat{R}^0(x) \geq 0$, $0 < x < L$. The following result is concerned with the dynamics of (2.3).

Lemma 2.1 ([10]) *The system (2.3) admits a unique positive ω-periodic solution $R^*(t,x)$, and for any $\hat{R}_0(x) \in C([0,L],\mathbb{R})$, the unique solution $\hat{R}(t,x)$ of (2.3) with $\hat{R}(0,x) = \hat{R}_0(x)$ satisfies*

$$\lim_{t\to\infty} (\hat{R}(t,x) - R^*(t,x)) = 0$$

uniformly for $x \in [0,L]$.

The following result shows that solutions of system (1.1)–(1.3) exist globally on $[0,\infty)$.

Theorem 2.2 *For every initial data $\phi \in C_\tau^+$, system (1.1)–(1.3) has a unique solution $\mathbf{u}(t,\phi)$ on $[0,\infty)$ with $\mathbf{u}_0 = \phi$. Furthermore, system (1.1)–(1.3) generates an ω-periodic semiflow $\Phi_t := \mathbf{u}_t(\cdot) : C_\tau^+ \to C_\tau^+$, i.e., $\Phi_t(\phi)(s,x) = \mathbf{u}(t+s,x;\phi), \forall \phi \in C_\tau^+, t \geq 0, s \in [-\tau,0], x \in [0,L]$, and Φ_t has a global compact attractor in C_τ^+.*

Proof Firstly, we show the local existence of the unique mild solution. Clearly, $\mathbf{F}$ is locally Lipschitz continuous. In view of (1.4) and (1.5), there exists $M_i > 0$ such that $0 \leq f_i(R) \leq M_i R$ $(i = 1,2)$, $\forall R \geq 0$. For any $\phi \in C_\tau^+$ and $h \geq 0$, we have

$$\phi(0,x) + h\mathbf{F}(\phi)(x) = \begin{pmatrix} \phi_1(0,x) + h[-q_1 f_1(\phi_1(0,x))\phi_2(0,x) - q_2 f_2(\phi_1(0,x))\phi_3(0,x)] \\ \phi_2(0,x) + hq_1 \int_0^L [\Gamma(\tau,x,y) f_1(\phi_1(-\tau,y))\phi_2(-\tau,y)]\mathrm{d}y \\ \phi_3(0,x) + hq_2 \int_0^L [\Gamma(\tau,x,y) f_2(\phi_1(-\tau,y))\phi_3(-\tau,y)]\mathrm{d}y \end{pmatrix}$$

$$\geq \begin{pmatrix} \phi_1(0,x)[1 - h(q_1 \frac{\mu_1}{K_1}\phi_2(0,x) + q_2 \frac{\mu_2}{K_2}\phi_3(0,x))] \\ \phi_2(0,x) \\ \phi_3(0,x) \end{pmatrix}, \quad t \geq 0, x \in (0,L).$$

The above inequality implies that $\phi(0,x) + h\mathbf{F}(\phi)(x) \in \mathbb{X}^+$ if h is sufficiently small. Therefore,

$$\lim_{h\to 0^+} \frac{1}{h} \mathrm{dist}(\phi(0,\cdot) + h\mathbf{F}(\phi), \mathbb{X}^+) = 0, \quad \forall \phi \in C_\tau^+. \tag{2.4}$$

By [11, Corollary 4], it then follows that for every $\phi \in C_\tau^+$, system (1.1)–(1.3) has a unique noncontinuable mild solution $\mathbf{u}(t,x;\phi)$ with $\mathbf{u}_0(\cdot,\cdot;\phi) = \phi$ and $\mathbf{u}(t,\cdot;\phi) \in \mathbb{X}^+$ for any t on its maximal interval of existence $[0,\sigma_\phi)$, where $\sigma_\phi \leq \infty$. Moreover, by the analyticity of $\mathbf{U}(t,s), s,t \in \mathbb{R}, s < t$, $\mathbf{u}(t,x;\phi)$ is a classical solution of (1.1)–(1.3) when $t > \tau$.

Next, we use similar arguments to those in [12] to prove the ultimate boundedness of solutions. Note that the first equation in (1.1)–(1.3) is dominated by (2.3), and it follows from Lemma 2.1 that (2.3) admits a unique positive ω-periodic solution $R^*(t,x)$ which is globally asymptotically stable in $C([0,L],\mathbb{R})$, the parabolic comparison theorem implies that $R(t,x)$ is bounded on

$[0,\sigma_\phi)$. Then there is a constant $B_0>0$ such that for any $\phi\in C_\tau^+$, there exists a positive integer $l_1=l_1(\phi)>0$ satisfying $R(t,x;\phi)\le B_0$ for all $t\ge l_1\omega$ and $x\in[0,L]$.

For any given $\phi\in C_\tau^+$, let $(R(t,x),u_1(t,x),u_2(t,x)):=(R(t,\phi)(x),u_1(t,\phi)(x),u_2(t,\phi)(x))$, $t\ge 0, x\in(0,L)$. Set

$$\bar{R}(t)=\int_0^L R(t,x)\mathrm{d}x,\ \bar{u}_i(t)=\int_0^L u_i(t,x)\mathrm{d}x,\quad i=1,2.$$

Integrating the first equation of (1.1)–(1.3) on $(0,L)$, by the Greens formula, we obtain

$$\begin{aligned}\frac{\mathrm{d}\bar{R}}{\mathrm{d}t}&=\int_0^L(\delta\frac{\partial^2R}{\partial x^2}-\nu\frac{\partial R}{\partial x})\mathrm{d}x-\sum_{i=1}^2 q_i\int_0^L f_i(R(t,x))u_i(t,x)\mathrm{d}x\\&=\big(\delta\frac{\partial R}{\partial x}-\nu R\big)(t,L)-\big(\delta\frac{\partial R}{\partial x}-\nu R\big)(t,0)-\sum_{i=1}^2 q_i\int_0^L f_i(R(t,x))u_i(t,x)\mathrm{d}x\\&=\nu R^{(0)}(t)-\nu R(t,L)-\sum_{i=1}^2 q_i\int_0^L f_i(R(t,x))u_i(t,x)\mathrm{d}x\\&\le\nu R^{(0)}(t)-\sum_{i=1}^2 q_i\int_0^L f_i(R(t,x))u_i(t,x)\mathrm{d}x,\quad t>0,\end{aligned}$$

that is,

$$\sum_{i=1}^2 q_i\int_0^L f_i(R(t,x))u_i(t,x)\mathrm{d}x\le[\nu R^{(0)}(t)-\frac{\mathrm{d}\bar{R}}{\mathrm{d}t}],\quad t>0.$$

By the property of the fundamental solutions, there exists $k_0>0$ such that

$$\begin{aligned}&q_i\int_0^L\Gamma(\tau,x,y)f_i(R(t-\tau,y))u_i(t-\tau,y)\mathrm{d}y\le k_0\cdot q_i\int_0^L f_i(R(t-\tau,y))u_i(t-\tau,y)\mathrm{d}y\\&\le k_0\big[\nu R^{(0)}(t-\tau)-\frac{\mathrm{d}\bar{R}(t-\tau)}{\mathrm{d}t}\big].\end{aligned}$$

Integrating the second and third equation of (1.1)–(1.3) on $(0,L)$ yields

$$\begin{aligned}\frac{\mathrm{d}\bar{u}_i}{\mathrm{d}t}&=\int_0^L(\delta\frac{\partial^2u_i}{\partial x^2}-\nu\frac{\partial u_i}{\partial x})\mathrm{d}x-\mu_0\int_0^L u_i(t,x)\mathrm{d}x+\\&\quad q_i\int_0^L\int_0^L\Gamma(\tau,x,y)f_i(R(t-\tau,y))u_i(t-\tau,y)\mathrm{d}y\mathrm{d}x\\&=-\nu u_i(t,L)-\mu_0\bar{u}_i(t)+q_i\int_0^L\int_0^L\Gamma(\tau,x,y)f_i(R(t-\tau,y))u_i(t-\tau,y)\mathrm{d}y\mathrm{d}x,\\&\le-\nu u_i(t,L)-\mu_0\bar{u}_i(t)+k_0\int_0^L\big[\nu R^{(0)}(t-\tau)-\frac{\mathrm{d}\bar{R}(t-\tau)}{\mathrm{d}t}\big]\mathrm{d}x\\&\le-\mu_0\bar{u}_i(t)+k_0\int_0^L\big[\nu R^{(0)}(t-\tau)-\frac{\mathrm{d}\bar{R}(t-\tau)}{\mathrm{d}t}\big]\mathrm{d}x\\&=-\mu_0\bar{u}_i(t)+k_0L\big[\nu R^{(0)}(t-\tau)-\frac{\mathrm{d}\bar{R}(t-\tau)}{\mathrm{d}t}\big]\\&\le-\mu_0\bar{u}_i(t)+k_1-k_2\frac{\mathrm{d}\bar{R}(t-\tau)}{\mathrm{d}t},\quad t\ge l_1\omega+\tau,\end{aligned}$$

where $k_1=k_0L\nu\cdot\max_{t\in[0,\omega]}\{R^{(0)}(t-\tau)\}$ and $k_2=k_0L$.

On the other hand, since

$$e^{\mu_0 t}\frac{\mathrm{d}\bar{u}_i(t)}{\mathrm{d}t} = \frac{\mathrm{d}[e^{\mu_0 t}\bar{u}_i(t)]}{\mathrm{d}t} - \mu_0 e^{\mu_0 t}\bar{u}_i(t),$$

then

$$\frac{\mathrm{d}[e^{\mu_0 t}\bar{u}_i(t)]}{\mathrm{d}t} \le k_1 e^{\mu_0 t} - k_2 e^{\mu_0 t}\frac{\mathrm{d}\bar{R}(t-\tau)}{\mathrm{d}t}, \quad t \ge l_1\omega + \tau. \tag{2.5}$$

Integrating (2.5) by parts over $[l_1\omega + \tau, t]$, we can find a positive number k_3, independent of ϕ, and a positive number $k_4 = k_4(\phi)$, dependent on ϕ, such that

$$\bar{u}_i(t) \le k_4(\phi)e^{-\mu_0 t} + k_3, \quad t \ge l_1\omega + \tau.$$

Since $\Gamma(\tau, \cdot, \cdot)$ and $R(\cdot, \cdot)$ are bounded, it follows from the second equation in (1.1)–(1.3) that

$$\begin{aligned}
\frac{\partial u_i(t,x)}{\partial t} &= \delta\frac{\partial^2 u_i}{\partial x^2} - \nu\frac{\partial u_i}{\partial x} - \mu_0 u_i(t,x) + q_i\int_0^L \Gamma(\tau,x,y)f_i(R(t-\tau,y))u_i(t-\tau,y)\mathrm{d}y\\
&\le \delta\frac{\partial^2 u_i}{\partial x^2} - \nu\frac{\partial u_i}{\partial x} - \mu_0 u_i(t,x) + M_i\int_0^L u_i(t-\tau,y)\mathrm{d}y\\
&= \delta\frac{\partial^2 u_i}{\partial x^2} - \nu\frac{\partial u_i}{\partial x} - \mu_0 u_i(t,x) + M_i\bar{u}_i(t-\tau),
\end{aligned}$$

with some constant $M_i > 0$ $(i = 1,2)$. By the standard parabolic comparison theorem, there exists a positive number B_i $(i = 1,2)$, independent of the initial value ϕ, and $l_2 = l_2(\phi) > l_1(\phi)$ such that $u_i(t,x;\phi) \le B_i$ $(i = 1,2)$ for any $t \ge l_2\omega + \tau$ and $x \in [0,L]$. Therefore, we have $\sigma_\phi = \infty$ for each $\phi \in C_\tau^+$.

Define the solution semiflow $\Phi_t = \mathbf{u}_t(\cdot) : C_\tau^+ \to C_\tau^+$, $t \ge 0$. It is easy to see that $\{\Phi_t\}_{t\ge 0}$ is an ω-periodic semiflow on C_τ^+. By the above arguments, we conclude that $\Phi_t : C_\tau^+ \to C_\tau^+$ is point dissipative. Moreover, $\Phi_t : C_\tau^+ \to C_\tau^+$ is compact for each $t > \tau$ by [13, Theorem 2.2.6]. Then, by [14, Theorem 3.4.8], $\Phi_t = \mathbf{u}_t(\cdot) : C_\tau^+ \to C_\tau^+, t \ge 0$ has a global compact attractor. □

3. Global dynamics

In this section, we study the global dynamics of system (1.1)–(1.3).

Consider the following periodic and delayed reaction diffusion system:

$$\begin{cases}
\frac{\partial w_i(t,x)}{\partial t} = \delta\frac{\partial^2 w_i}{\partial x^2} - \nu\frac{\partial w_i}{\partial x} - \mu_0 w_i(t,x) + \\
\qquad q_i\int_0^L \Gamma(\tau,x,y)\frac{\mu_i R^*(t-\tau,y)}{K_i + R^*(t-\tau,y)}w_i(t-\tau,y)\mathrm{d}y, & t>0, x\in(0,L),\\
\nu w_i(t,0) - \delta\frac{\partial w_i}{\partial x}(t,0) = \frac{\partial w_i}{\partial x}(t,L) = 0, & t>0.
\end{cases} \tag{3.1}$$

Let $\mathbb{Y} = C([0,L],\mathbb{R})$ and $\mathbb{Y}^+ = C([0,L],\mathbb{R}^+)$. For $\tau \ge 0$, define $\mathcal{E} = C([-\tau,0],\mathbb{Y})$ with the norm $\|\varphi\| = \max_{\theta\in[-\tau,0]}\|\varphi(\theta)\|_{\mathbb{Y}}$, $\forall\varphi\in\mathcal{E}$, and $\mathcal{E}^+ = C([-\tau,0],\mathbb{Y}^+)$. Then $(\mathcal{E},\mathcal{E}^+)$ is a strongly ordered Banach space. Define the Poincaré map of (3.1) $P^{(i)} : \mathcal{E}\to\mathcal{E}$ by $P^{(i)}(\varphi) = w_{i\omega}(\varphi)$ for any $\varphi\in\mathcal{E}$, where $w_{i\omega}(\varphi)(s,x) = w_i(\omega+s,x,\varphi), (s,x)\in[-\tau,0]\times[0,L]$, and $w_{i\omega}$ is the solution of (3.1). Let $r_0^{(i)} = r(P^{(i)})$ be the spectral radius of $P^{(i)}$ $(i = 1,2)$, and $\lambda_i = -\frac{\ln r_0^{(i)}}{\omega}$. Then, by [15, Lemma 3.1], there exists a ω-periodic solution $v_i^*(t,x)$ such that $e^{-\lambda t}v_i^*(t,x)$ is a solution of (3.1).

Theorem 3.1 *Let $(R(t,x,\phi), u_1(t,x,\phi), u_2(t,x,\phi))$ be the solution of system (1.1)–(1.3) with initial value ϕ, for $\forall \phi = (\phi_1, \phi_2, \phi_3) \in C_\tau^+$. Then the following statements hold:*

(i) If $r_0^{(1)} > 1$ and $r_0^{(2)} < 1$, then for $\forall \phi \in C_\tau^+, \phi_2(0,.) \not\equiv 0$, and there exists a $\xi_1 > 0$ such that

$$\liminf_{t\to\infty}(R(t,x,\phi), u_1(t,x,\phi)) \geq (\xi_1, \xi_1), \lim_{t\to\infty} u_2(t,x,\phi) = 0$$

uniformly for any $x \in [0,L]$;

(ii) If $r_0^{(1)} < 1$ and $r_0^{(2)} > 1$, then for $\forall \phi \in C_\tau^+, \phi_3(0,.) \not\equiv 0$, and there exists a $\xi_2 > 0$ such that

$$\liminf_{t\to\infty}(R(t,x,\phi), u_2(t,x,\phi)) \geq (\xi_2, \xi_2), \lim_{t\to\infty} u_1(t,x,\phi) = 0$$

uniformly for any $x \in [0,L]$.

Proof (i) For any given $\phi \in C_\tau^+$, let $U(t,\phi)(x) = (R(t,x,\phi), u_1(t,x,\phi), u_2(t,x,\phi))$, $R(t,x) \geq 0$ and $u_i(t,x) \geq 0$, $t \geq 0$, $x \in [0,L]$, $i = 1,2$. For any $\epsilon > 0$, we consider the following periodic time-delayed nonlocal equation

$$\begin{cases} \frac{\partial v^\epsilon(t,x)}{\partial t} = \delta \frac{\partial^2 v^\epsilon(t,x)}{\partial x^2} - \nu \frac{\partial v^\epsilon(t,x)}{\partial x} - \mu_0 v^\epsilon(t,x) + \\ \qquad q_2 \int_0^L \Gamma(\tau,x,y) \frac{\mu_2(R^*(t-\tau,y)+\epsilon)}{K_2+R^*(t-\tau,y)+\epsilon} v^\epsilon(t-\tau,y)\mathrm{d}y, & t>0, x\in(0,L), \\ \nu v^\epsilon(t,0) - \delta \frac{\partial v^\epsilon}{\partial x}(t,0) = \frac{\partial v^\epsilon}{\partial x}(t,L) = 0, & t>0, \\ v^\epsilon(s,x) = \varphi(s,x), \varphi \in \mathcal{E}^+, & s\in[-\tau,0], x\in(0,L). \end{cases} \tag{3.2}$$

Define the Poincaré map of (3.2) $P_\epsilon^{(2)}: \mathcal{E} \to \mathcal{E}$ by

$$P_\epsilon^{(2)}(\varphi) = v_\omega^\epsilon(\varphi), \quad \forall \varphi \in \mathcal{E},$$

where

$$v_\omega^\epsilon(\varphi)(s,x) = v^\epsilon(\omega+s,x,\varphi), \quad \forall (s,x) \in [-\tau,0]\times[0,L],$$

and $v^\epsilon(t,x,\varphi)$ is the solution of (3.1) with initial value $v^\epsilon(s,x) = \varphi(s,x)$, $\forall (s,x) \in [-\tau,0]\times(0,L)$. Let $r_\epsilon^{(2)} = r(P_\epsilon^{(2)})$ be the spectral radius of $P_\epsilon^{(2)}$. Thus, we can conclude from $r_0^{(2)} < 1$ that there exists a sufficient small positive number ϵ_1 such that $r_\epsilon^{(2)} < 1$ for $\forall \epsilon \in [0,\epsilon_1)$. Fix $\epsilon \in (0,\epsilon_1)$, we have $\lambda_2^\epsilon = -\frac{\ln r_\epsilon^{(2)}}{\omega} > 0$. According to [16, Lemma 3.1], there exists an ω-periodic function $v_{2\epsilon}^*(t,x)$ such that $v^\epsilon(t,x) = e^{-\lambda_2^\epsilon t} v_{2\epsilon}^*(t,x)$ is a solution of (3.2). In particular, $v_\epsilon^*(t,x) > 0$ for $\forall t \in \mathbb{R}$ and $x \in [0,L]$.

Note that $R(t,x)$ satisfies

$$\begin{cases} \frac{\partial R(t,x)}{\partial t} \leq \delta \frac{\partial^2 R(t,x)}{\partial x^2} - \nu \frac{\partial R(t,x)}{\partial x}, & t>0, x\in(0,L), \\ \nu R(t,0) - \delta \frac{\partial R}{\partial x}(t,0) = \nu R^{(0)}(t), \frac{\partial R}{\partial x}(t,L) = 0, & t>0. \end{cases} \tag{3.3}$$

By Theorem 2.2 and the parabolic comparison principle, it follows that there exists an integer $k > 0$ such that $R(t,x,\phi) \leq R^*(t,x) + \epsilon$, $\forall t \geq k\omega,\ x \in [0,L], \epsilon > 0$. Therefore, for all $t \geq k\omega$,

$x \in [0, L]$, $u_2(t, x)$ satisfies

$$\begin{cases} \frac{\partial u_2(t,x)}{\partial t} \le \delta \frac{\partial^2 u_2(t,x)}{\partial x^2} - \nu \frac{\partial u_2(t,x)}{\partial x} - \mu_0 u_2(t,x) + \\ \qquad q_2 \int_0^L \Gamma(\tau, x, y) \frac{\mu_2 R^*(t-\tau,y)}{K_2 + R^*(t-\tau,y)} u_2(t-\tau, y) \mathrm{d}y, & t > k\omega, x \in (0, L), \\ \nu u_2(t,0) - \delta \frac{\partial u_2}{\partial x}(t,0) = \frac{\partial u_2}{\partial x}(t, L) = 0, & t > k\omega. \end{cases} \tag{3.4}$$

Given $\phi \in C_\tau^+$, since $u_2(t, x, \phi)$ is globally bounded, then there exists $\alpha > 0$, such that $u_2 \le \alpha e^{-\lambda_2^\epsilon t} v_{2\epsilon}^*(t, x), \forall t \in [k\omega, k\omega + \tau], x \in [0, L]$. Similar to the proof of [16, Theorem 3.3], we have

$$\lim_{t\to\infty} u_2(t, x, \phi) = 0$$

uniformly for $x \in [0, L]$.

Fix $\phi = (\phi_1, \phi_2, \phi_3) \in C_\tau^+$, and $\phi_2(0, .) \not\equiv 0$. Therefore, we can regard $u_2(t, x)$ as $\mathbb{R}^+ \times [0, L]$ a fixed function. Hence, $(R(t, x), u_1(t, x))$ satisfies the following nonlocal delayed reaction-diffusion system:

$$\begin{cases} \frac{\partial R(t,x)}{\partial t} = \delta \frac{\partial^2 R}{\partial x^2} - \nu \frac{\partial R}{\partial x} - q_1 f_1(R) u_1 - q_2 f_2(R) u_2, & t > 0, x \in (0, L), \\ \frac{\partial u_1(t,x)}{\partial t} = \delta \frac{\partial^2 u_1(t,x)}{\partial x^2} - \nu \frac{\partial u_1(t,x)}{\partial x} - \mu_0 u_1(t,x) + \\ \qquad q_1 \int_0^L \Gamma(\tau, x, y) \frac{\mu_1 R(t-\tau,y)}{K_1 + R(t-\tau,y)} u_1(t-\tau, y) \mathrm{d}y, & t > 0, x \in (0, L), \\ \nu R(t,0) - \delta \frac{\partial R}{\partial x}(t,0) = \nu R^{(0)}(t), \frac{\partial R}{\partial x}(t, L) = 0, & t > 0, \\ \nu u_1(t,0) - \delta \frac{\partial u_1}{\partial x}(t,0) = \frac{\partial u_1}{\partial x}(t, L) = 0, & t > 0. \end{cases} \tag{3.5}$$

Notice that $\lim_{t\to\infty} u_2(t, \cdot, \phi) = 0$. Hence, Eqs. (1.1)–(1.3) are asymptotic to the following system:

$$\begin{cases} \frac{\partial R(t,x)}{\partial t} = \delta \frac{\partial^2 R}{\partial x^2} - \nu \frac{\partial R}{\partial x} - q_1 f_1(R) u_1, & t > 0, x \in (0, L), \\ \frac{\partial u_1(t,x)}{\partial t} = \delta \frac{\partial^2 u_1(t,x)}{\partial x^2} - \nu \frac{\partial u_1(t,x)}{\partial x} - \mu_0 u_1(t,x) + \\ \qquad q_1 \int_0^L \Gamma(\tau, x, y) \frac{\mu_1 R(t-\tau,y)}{K_1 + R(t-\tau,y)} u_1(t-\tau, y) \mathrm{d}y, & t > 0, x \in (0, L), \\ \nu R(t,0) - \delta \frac{\partial R}{\partial x}(t,0) = \nu R^{(0)}(t), \frac{\partial R}{\partial x}(t, L) = 0, & t > 0, \\ \nu u_1(t,0) - \delta \frac{\partial u_1}{\partial x}(t,0) = \frac{\partial u_1}{\partial x}(t, L) = 0, & t > 0. \end{cases} \tag{3.6}$$

It then follows from [16] that $\Psi_t^{(1)}(\varphi) = (R_t(\varphi), u_{1t}(\varphi))$ is an ω-periodic semiflow on C_τ^+, where $C_\tau^+ := C([-\tau, 0], \mathbb{X}^+)$, and $(R(t, x, \varphi), u_1(t, x, \varphi))$ is the solution semiflow of system (3.6) with initial value $\varphi = (\phi_1, \phi_2) \in C_\tau^+$. Since $r_0^{(1)} > 1$, according to [16, Theorems 2.2 and 3.3 (ii)], it is easy to see that $\Psi_\omega^{(1)}$ has a positive global attractor $\mathcal{A}_0$. By the theory of internal chain transitive sets, and the arguments similar to [16, Theorem 3.3 (i)] and the conclusion of [16, Theorem 3.3 (ii)], it follows that there exists a $\xi_1 > 0$ such that

$$\liminf_{t\to\infty} (R(t, x, \phi), u_1(t, x, \phi)) \ge (\xi_1, \xi_1), \quad \lim_{t\to\infty} u_2(t, x, \phi) = 0$$

uniformly for all $x \in [0, L]$.

The proof of conclusion (ii) is completely similar to (i). □

4. Coexistence

In this section, we study the coexistence and uniform persistence of system (1.1)–(1.3)

$$\begin{cases} \frac{\partial w(t,x)}{\partial t} = \delta\frac{\partial^2 w(t,x)}{\partial x^2} - \nu\frac{\partial w(t,x)}{\partial x} - \mu_0 w(t,x) + \\ \qquad\qquad q_2\int_0^L \Gamma(\tau,x,y)\frac{\mu_2\eta_1}{K_2+\eta_1}w(t-\tau,y)\mathrm{d}y, & t>0, x\in(0,L), \\ \nu w(t,0) - \delta\frac{\partial w}{\partial x}(t,0) = \frac{\partial w}{\partial x}(t,L) = 0, & t>0, \\ w(s,x) = \varphi(s,x), & \varphi\in\mathcal{E}, s\in[-\tau,0], x\in[0,L]. \end{cases} \tag{4.1}$$

Define the Poincaré map of (1.1) $P^{(2)'}: \mathcal{E}\to\mathcal{E}$ by $P^{(2)'} = w_\omega(\varphi)$ for all $\varphi\in\mathcal{E}$, where $w_\omega(\varphi)(s,x) = w(\omega+s,x,\varphi)$ for all $(s,x)\in[-\tau,0]\times[0,L]$, and $w(t,x,\varphi)$ is the solution of (1.1) with $w(s,x)=\varphi(s,x)$. Let $\Lambda^{(2)} = r(P^{(2)'})$ be the spectral radius of $\Lambda^{(2)}$, it follows from [6] that $\Lambda^{(2)}$ is a simple eigenvalue of $P^{(2)'}$ having a strongly positive eigenvector $\bar\varphi_2$.

Similarly, we consider the following system:

$$\begin{cases} \frac{\partial w(t,x)}{\partial t} = \delta\frac{\partial^2 w(t,x)}{\partial x^2} - \nu\frac{\partial w(t,x)}{\partial x} - \mu_0 w(t,x) + \\ \qquad\qquad q_1\int_0^L \Gamma(\tau,x,y)\frac{\mu_1\eta_2}{K_1+\eta_2}w(t-\tau,y)\mathrm{d}y, & t>0, x\in(0,L), \\ \nu w(t,0) - \delta\frac{\partial w}{\partial x}(t,0) = \frac{\partial w}{\partial x}(t,L) = 0, & t>0, \\ w(s,x) = \varphi(s,x), & \varphi\in\mathcal{E}, s\in[-\tau,0], x\in[0,L]. \end{cases} \tag{4.2}$$

Define the Poincaré map of (4.2) $P^{(1)'}: \mathcal{E}\to\mathcal{E}$ by $P^{(1)'} = w_\omega(\varphi)$ for all $\varphi\in\mathcal{E}$, where $w_\omega(\varphi)(s,x) = w(\omega+s,x,\varphi)$ for all $(s,x)\in[-\tau,0]\times[0,L]$, and $w(t,x,\varphi)$ is the solution of (4.2) with $w(s,x)=\varphi(s,x)$. Let $\Lambda^{(1)} = r(P^{(1)'})$ be the spectral radius of $\Lambda^{(1)}$, it follows from [6] that $\Lambda^{(1)}$ is a simple eigenvalue of $P^{(1)'}$ having a strongly positive eigenvector $\bar\varphi_1$. By the comparison principle and monotonicity of spectral radius of positive operators, it is easy to see that $r_0^{(i)} \geq \Lambda^{(i)}$ $(i=1,2)$ combined with the Eqs. (3.1), (4.1) and (4.2).

Before proving the main result on the threshold dynamics of system (1.1)–(1.3), we need the following lemma

Lemma 4.1 *Suppose* $(R(t,x),u_1(t,x),u_2(t,x))$ *is the solution of system (1.1)–(1.3) with initial value* $\phi=(\phi_1,\phi_2,\phi_3)\in C_\tau^+$.

(i) If there exists some $t_0\geq 0$ *such that* $u_i(t_0,\cdot,\phi)\not\equiv 0, i=1,2$, *then* $u_i(t,x,\phi)>0$ *for all* $t>t_0$ *and* $x\in[0,L]$;

(ii) For any $\phi\in C_\tau^+$, *we have* $R(t,\cdot,\phi)>0, \forall t>0$ *and*

$$\liminf_{t\to\infty} R(t,\cdot,\phi)\geq\eta$$

uniformly for any $x\in[0,L]$, *where* η *is a positive constant.*

The proof of Lemma 4.1 is completely similar to [16], we omit it.

Theorem 4.2 *Let* $(R(t,x,\phi),u_1(t,x,\phi),u_2(t,x,\phi))$ *be the solution of (1.1)–(1.3) with the initial value* $\phi=(\phi_1,\phi_2,\phi_3)\in C_\tau^+$. *If* $\Lambda^{(i)}>1$, $i=1,2$, *then system (1.1)–(1.3) admits at least one* $\omega-$*periodic solution* $(R(t,x,\phi),u_1(t,x,\phi),u_2(t,x,\phi))$, *and there exists* $\xi>0$ *such that for any*

$\phi \in C_\tau^+$ with $\phi_2(0,\cdot) \not\equiv 0$ and $\phi_3(0,\cdot) \not\equiv 0$, we have

$$\liminf_{t\to\infty}(R(t,x,\phi), u_1(t,x,\phi), u_2(t,x,\phi)) \geq (\xi,\xi,\xi)$$

uniformly for all $x \in [0,L]$.

Proof Retrospect

$$\begin{cases} \frac{\partial \tilde{R}(t,x)}{\partial t} = \delta \frac{\partial \tilde{R}^2}{\partial x^2} - \nu \frac{\partial \tilde{R}}{\partial x} - q_1 f_1(\tilde{R})\tilde{u}_1, & t>0, x\in(0,L), \\ \frac{\partial \tilde{u}_1(t,x)}{\partial t} = \delta \frac{\partial^2 \tilde{u}_1}{\partial x^2} - \nu \frac{\partial \tilde{u}_1}{\partial x} - \mu_0 \tilde{u}_1 + & \\ \qquad q_1 \int_0^L \Gamma(\tau,x,y) \frac{\mu_1 \tilde{R}(t-\tau,y)}{K_1+\tilde{R}(t-\tau,y)} \tilde{u}_1(t-\tau,y)\mathrm{d}y, & t>0, x\in(0,L), \\ \nu \tilde{R}(t,0) - \delta \frac{\partial \tilde{R}}{\partial x}(t,0) = \nu \tilde{R}^{(0)}(t), \frac{\partial \tilde{R}}{\partial x}(t,L) = 0, & t>0, \\ \nu \tilde{u}_1(t,0) - \delta \frac{\partial \tilde{u}_1}{\partial x}(t,0) = \frac{\partial \tilde{u}_1}{\partial x}(t,L) = 0, & t>0. \end{cases} \tag{4.3}$$

According to the analysis of [16], it is easy to see that system (4.3) generates an ω-periodic semiflow $\Psi_t^{(1)}$, for $\forall \varphi = (\varphi_1,\varphi_2) = (\phi_1,\phi_2) \in C_\tau^+$, where

$$\Psi_t^{(1)}(\varphi)(x) = (\tilde{R}(t,x,\varphi), \tilde{u}_1(t,x,\varphi)), \quad \forall t>0,\ x\in(0,L).$$

It then follows from $r_0^{(1)} > 1$ and [16, Theorem 3.3 (ii)] that there exists a positive constant η_1 such that

$$\liminf_{t\to\infty}(\tilde{R}(t,x,\varphi), \tilde{u}_1(t,x,\varphi)) \geq (\eta_1,\eta_1) \tag{4.4}$$

uniformly for all $x\in[0,L]$. Moreover, $\Psi_{n_0\omega}^{(1)} : C_\tau^+ \to C_\tau^+$ has a positive global attractor $\mathcal{A}_0^{(1)}$. Let $\mathfrak{B}_0^{(1)} = \cup_{t\in[0,n_0\omega]} \Psi_t^{(1)} \mathcal{A}_0^{(1)}$, then $\mathfrak{B}_0^{(1)} \subset \mathrm{Int}(C_\tau^{(+)})$, for $\forall \varphi \in C_\tau^+$, $\varphi_2(0,\cdot) \not\equiv 0$, we have

$$\lim_{t\to\infty} d(\Psi_t^{(1)}(\varphi), \mathfrak{B}_0^{(1)}) = 0.$$

According to (4.4), it is easy to see that

$$(\mathfrak{X}_1^{(1)}(s,x), \mathfrak{X}_2^{(1)}(s,x)) \geq (\eta_1,\eta_1), \quad \forall (\mathfrak{X}_1^{(1)}, \mathfrak{X}_2^{(1)}) \in \mathfrak{B}_0^{(1)},\ (s,x)\in[-\tau,0]\times[0,L].$$

It is not hard to find out that $(\tilde{R}(t,x,\mathfrak{X}^{(1)}), \tilde{u}_1(t,x,\mathfrak{X}^{(1)}))$ is defined for $\forall \mathfrak{X}^{(1)} := (\mathfrak{X}_1^{(1)}, \mathfrak{X}_2^{(1)}) \in \mathfrak{B}_0^{(1)}$, then for $\forall t\in\mathbb{R}$, $(\tilde{R}(t+,\cdot,\mathfrak{X}^{(1)}), \tilde{u}_1(t+,\cdot,\mathfrak{X}^{(1)})) = \Psi_t^{(1)}(\mathfrak{X}^{(1)}(\cdot,\cdot)) \in \mathfrak{B}_0^{(1)}$.

Hence, $(\tilde{R}(t,x,\mathfrak{X}^{(1)}), \tilde{u}_1(t,x,\mathfrak{X}^{(1)}), 0)$ is a solution of system (1.1), define

$$\begin{aligned} B_0^{(1)'} &= \{(\mathfrak{X}_1^{(1)}, \mathfrak{X}_2^{(1)}) \in C_\tau^+ : (\mathfrak{X}_1^{(1)}(\cdot,\cdot), \mathfrak{X}_2^{(1)}(\cdot,\cdot)) \\ &= (\tilde{R}(t+,\cdot,\mathfrak{X}^{(1)}), \tilde{u}_1(t+,\cdot,\mathfrak{X}^{(1)})), \forall t\in\mathbb{R}, \mathfrak{X}^{(1)} \in \mathfrak{B}_0^{(1)}\}. \end{aligned}$$

For

$$\begin{cases} \frac{\partial \hat{R}(t,x)}{\partial t} = \delta \frac{\partial \hat{R}^2}{\partial x^2} - \nu \frac{\partial \hat{R}}{\partial x} - q_2 f_2(\hat{R})\hat{u}_2, & t>0, x\in(0,L), \\ \frac{\partial \hat{u}_2(t,x)}{\partial t} = \delta \frac{\partial^2 \hat{u}_2}{\partial x^2} - \nu \frac{\partial \hat{u}_2}{\partial x} - \mu_0 \hat{u}_2 + & \\ \qquad q_2 \int_0^L \Gamma(\tau,x,y) \frac{\mu_2 \hat{R}(t-\tau,y)}{K_2+\hat{R}(t-\tau,y)} \hat{u}_2(t-\tau,y)\mathrm{d}y, & t>0, x\in(0,L), \\ \nu \hat{R}(t,0) - \delta \frac{\partial \hat{R}}{\partial x}(t,0) = \nu \hat{R}^{(0)}(t), \frac{\partial \hat{R}}{\partial x}(t,L) = 0, & t>0, \\ \nu \hat{u}_2(t,0) - \delta \frac{\partial \hat{u}_2}{\partial x}(t,0) = \frac{\partial \hat{u}_2}{\partial x}(t,L) = 0, & t>0. \end{cases} \tag{4.5}$$

Similar to the previous arguments, we have

$$\begin{aligned} B_0^{(2)'} &= \{(\mathfrak{X}_1^{(2)}, \mathfrak{X}_2^{(2)}) \in C_\tau^+ : (\mathfrak{X}_1^{(2)}(\cdot,\cdot), \mathfrak{X}_2^{(2)}(\cdot,\cdot)) \\ &= (\hat{R}(t+,\cdot,\mathfrak{X}^{(2)}), \hat{u}_2(t+,\cdot,\mathfrak{X}^{(2)})), \forall t \in \mathbb{R}, \mathfrak{X}^{(2)} \in \mathfrak{B}_0^{(2)}\}. \end{aligned}$$

Let $\mathbf{Z}_0 = \{\phi \in C_\tau^+ : \phi_i(0,\cdot) \not\equiv 0, j = 2,3\}$ and $\partial\mathbf{Z}_0 := C_\tau^+ \setminus \mathbf{Z}_0 = \{\phi \in C_\tau^+ : \phi_2(0,\cdot) \equiv 0$, or $\phi_3(0,\cdot) \equiv 0\}$. Define $M_\partial := \{\phi \in \partial\mathbf{Z}_0 : \Psi_\omega^k(\phi) \in \partial\mathbf{Z}_0, k \in \mathbb{N}\}$. Set $E_0 := \{(R^*(0,\cdot), \hat{0}, \hat{0})\}$, $E_1 := \{(\phi_1, \phi_2, \hat{0}) : (\phi_1, \phi_2) \in \mathfrak{B}_0^{(1)'}\}$, $E_2 := \{(\phi_1, \hat{0}, \phi_3) : (\phi_1, \phi_3) \in \mathfrak{B}_0^{(2)'}\}$, and $\tilde{J}(\phi)$ be the omega limit set of the orbit $\gamma^+(\phi) := \{\Psi_t(\phi) : \forall t \geq 0, \phi \in C_\tau^+\}$. It then follows from Lemma (4.1) that $\Psi_t(\mathbf{Z}_0) \subset \mathbf{Z}_0$ for $\forall t \geq 0$.

We further have the following claim.

Claim 1. $\cup_{\phi \in M_\partial} \tilde{J}(\phi) = E_0 \cup E_1 \cup E_2, \forall \phi \in M_\partial$.

Let $\phi \in M_\partial$. By the definition of M_∂, we have $(R(t+\cdot,\cdot,\phi), u_1(t+\cdot,\cdot,\phi), u_2(t+\cdot,\cdot,\phi)) = \Psi_t \in \partial\mathbf{Z}_0$, $\forall t \geq 0$. It then follows from Lemma 4.1 that we have the following cases:

(i) $u_1(t,x,\phi) = u_2(t,x,\phi) \equiv 0$;

(ii) $u_1(t,x,\phi) \equiv 0$;

(iii) $u_2(t,x,\phi) \equiv 0$.

For the first situation, it follows from [10, Proposition 2.1] that

$$\lim_{t\to\infty} (R(t,\cdot,\phi) - R^*(0,\cdot)) = 0.$$

For the second situation, we have

$$(R(\cdot,\cdot,\phi), u_1(\cdot,\cdot,\phi)) = (\tilde{R}(\cdot,\cdot,\varphi), \tilde{u}_1(\cdot,\cdot,\varphi))$$

and

$$\varphi(s,x) = (\phi_1(s,x), \phi_2(s,x)), \quad \forall (s,x) \in [-\tau,0] \times [0,L].$$

Case (iii) is similar to case (ii). Hence, $\cup_{\phi \in M_\partial} \tilde{J}(\phi) = E_0 \cup E_1 \cup E_2$.

Consider the following time-periodic parabolic system:

$$\begin{cases} \frac{\partial u^{\tilde{\rho}}(t,x)}{\partial t} = \delta \frac{\partial^2 u^{\tilde{\rho}}(t,x)}{\partial x^2} - \nu \frac{\partial u^{\tilde{\rho}}(t,x)}{\partial x} - \mu_0 u^{\tilde{\rho}}(t,x) + \\ \qquad q \int_0^L \Gamma(\tau,x,y) \frac{\mu(R^*(t-\tau,y)-\tilde{\rho})}{K+R^*(t-\tau,y)-\tilde{\rho}} u^{\tilde{\rho}}(t-\tau,y)\mathrm{d}y, & (t,x) \in (0,\infty) \times (0,L), \\ \nu u^{\tilde{\rho}} - \delta \frac{\partial u^{\tilde{\rho}}}{\partial x}(t,0) = \frac{\partial u^{\tilde{\rho}}}{\partial x}(t,L) = 0, & t > 0, \\ u^{\tilde{\rho}}(s,x) = \varphi(s,x), \varphi \in \mathcal{E}, & s \in [-\tau,0], x \in [0,L]. \end{cases} \tag{4.6}$$

Define the Poincaré map of (4.6) $P_{\tilde{\rho}} : \mathcal{E} \to \mathcal{E}, P_{\tilde{\rho}}(\varphi) = u_\omega^{\tilde{\rho}}(\varphi)$, where $P_\omega^{\tilde{\rho}}(\varphi)(s,x) = u^{\tilde{\rho}}(\omega+s,x;\varphi)$ for $(s,x) \in [-\tau,0] \times [0,L]$, and $u^{\tilde{\rho}}(t,x;\varphi)$ is the solution of (4.6) with $u^{\tilde{\rho}}(s,x) = \varphi(s,x)$ for all $s \in [-\tau,0], x \in [0,L]$. Since $r_0 > 1$, there exists a sufficiently small positive number ρ_1 such that $r_{\tilde{\rho}} = r(P_{\tilde{\rho}}) > 1$ for all $\tilde{\rho} \in [0,\tilde{\rho}_1)$, where $r(P_{\tilde{\rho}})$ is the spectral radius of $P_{\tilde{\rho}}$. Fix a $\tilde{\rho'} \in (0,\tilde{\rho}_1)$. By the continuous dependence of solutions on the initial value, there exists $\tilde{\rho}_0 \in (0,\tilde{\rho}_1)$ such that

$$\| (R(t,x,\psi), u_1(t,x,\phi), u_2(t,x,\phi)) - (R^*(t,x),0,0) \| < \tilde{\rho}', \quad \forall t \in [0,\omega],\ x \in [0,L], \tag{4.7}$$

if $|\psi(s,x) - (R^*(s,x),0,0)| < \tilde{\rho}_0, \forall s \in [-\tau,0], x \in [0,L]$.

Claim 2. E_0 is a uniform weak repeller for $\mathbf{Z}_0$ in the sense that

$$\limsup_{k\to\infty} \parallel \Psi_\omega^k(\psi) - M \parallel \geq \tilde{\rho}_0, \quad \forall \psi \in \mathbf{Z}_0, \ M = (R^*(0,\cdot), \hat{0}, \hat{0}).$$

Suppose, by contradiction, there exists $\psi_0 \in \mathbf{Z}_0$, such that

$$\limsup_{k\to\infty} \parallel \Psi_\omega^k(\psi) - M \parallel < \tilde{\rho}_0,$$

then there exists $k_0 \in \mathbb{N}$ such that $|R(k\omega+s,x,\psi_0) - R^*(k\omega+s,x)| < \tilde{\rho}_0$, $|u_1(k\omega+s,x,\psi_0)| < \tilde{\rho}_0$ and $|u_2(k\omega+s,x,\psi_0)| < \tilde{\rho}_0$ for all $k \geq k_0$, $s \in [-\tau, 0]$ and $x \in [0, L]$. In view of (4.7), it follows that $R(t,x,\psi_0) > R^*(t,x) - \tilde{\rho}'$, therefore,

$$0 < u_1(t,x,\psi_0) < \tilde{\rho}', \quad 0 < u_2(t,x,\psi_0) < \tilde{\rho}'. \tag{4.8}$$

for any $t > k_0\omega$ and $x \in [0, L]$. In particular, $u_i(t,x,\psi_0)$ $(i = 1,2)$ satisfies

$$\begin{cases} \frac{\partial u_i(t,x)}{\partial t} = \delta \frac{\partial^2 u_i(t,x)}{\partial x^2} - \nu \frac{\partial u_i(t,x)}{\partial x} - \mu_0 u_i(t,x) + \\ \qquad q_i \int_0^L \Gamma(\tau,x,y) \frac{\mu_i(R^*(t-\tau,y)-\tilde{\rho}')}{K_i + R^*(t-\tau,y) - \tilde{\rho}'} u_i(t-\tau,y)\mathrm{d}y, & t > 0, x \in (0,L), \\ \nu u_i(t,0) - \delta \frac{\partial u_i}{\partial x}(t,0) = \frac{\partial u_i}{\partial x}(t,L) = 0, & t > 0, \\ u_i(s,x) = \psi(s,x), & \psi \in \mathcal{E}, s \in [-\tau,0], x \in [0,L]. \end{cases} \tag{4.9}$$

Let $\tilde{\psi}' \in \mathcal{E}$ be the positive eigenfunction of $P_{\tilde{\rho}'}$ associated with $r_{\tilde{\rho}'}$. Since $u_i(t,x,\phi_0) > 0$ $(i = 1,2)$ for all $t > \tau$, $x \in [0, L]$, there exists a $\xi > 0$ such that

$$u_i((k_0+1)\omega + s, x, \phi_0) \geq \xi \tilde{\psi}', \quad \forall s \in [-\tau, 0], \ x \in [0, L], \ i = 1, 2.$$

By (4.9) and the comparison principle, we have

$$u_i(t,x,\phi_0) \geq \xi u^{\tilde{\rho}}(t - (k_0+1)\omega, x, \tilde{\psi}'), \quad \forall t \geq (k_0+1)\omega, \ x \in [0,L], \ i = 1,2.$$

Therefore, we have

$$u_i(k\omega, x, \phi) = \xi u^{\tilde{\rho}'}((k-k_0-1)\omega, x, \tilde{\psi}') = \xi (r_{\tilde{\rho}'})^{(k-k_0-1)} \tilde{\psi}'(0,x) \to +\infty,$$

as $k \to +\infty$, which contradicts (4.8). Hence the claim is true.

Consider the following nonlocal delayed reaction-diffusion system with parameter ρ $(0 < \rho < \eta_1)$:

$$\begin{cases} \frac{\partial u^\rho(t,x)}{\partial t} = \delta \frac{\partial^2 u^\rho(t,x)}{\partial x^2} - \nu \frac{\partial u^\rho(t,x)}{\partial x} - \mu_0 u^\rho(t,x) + \\ \qquad q_2 \int_0^L \Gamma(\tau,x,y) \frac{\mu_2(\eta_1 - \rho}{K_2 + \eta_1 - \rho} u^\rho(t-\tau,y)\mathrm{d}y, & t > 0, x \in (0,L), \\ \nu u^\rho(t,0) - \delta \frac{\partial u^\rho}{\partial x}(t,0) = \frac{\partial u^\rho}{\partial x}(t,L) = 0, & t > 0, \\ u^\rho(s,x) = \psi(s,x), & \psi \in \mathcal{E}, s \in [-\tau,0], x \in [0,L]. \end{cases} \tag{4.10}$$

Define the Poincaré map of (4.10) $P_\rho : \mathcal{E} \to \mathcal{E}$ by $P_\rho(\psi) = u_\omega^\rho(\psi)$ for all $\psi \in \mathcal{E}$, where $u_\omega^\rho(\psi)(s,x) = u^\rho(\omega + s, x, \psi)$ for all $(s,x) \in [-\tau, 0] \times [0, L]$, and $u^\rho(t,x,\psi)$ is the solution of (4.10) with $u^\rho(s,x) = \psi(s,x)$. Since $\Lambda^{(2)} > 1$, there exists a sufficiently small positive number ρ_1 such that $\Lambda^{(2)} = r(P_\rho) > 1$ for $\forall \rho \in [0, \rho_1)$, where $r(P_\rho)$ is the spectral radius of P_ρ. Fix $\rho_0 \in (0, \rho_1)$, we have the following claim:

Claim 3. E_1 is a uniform weak repeller for $\mathbf{Z}_0$, that is

$$\limsup_{k\to\infty} \inf_{\Upsilon\in E_1} \parallel \Psi_{k\omega}(\phi) - \Upsilon \parallel \geq \rho_0, \quad \forall\phi \in \mathbf{Z}_0.$$

Suppose, by contradiction, there exists $\phi_0 \in \mathbf{Z}_0$, such that

$$\limsup_{k\to\infty} \inf_{\Upsilon\in E_1} \parallel \Psi_{k\omega}(\phi) - \Upsilon \parallel < \rho_0,$$

then there exists a positive integer m, such that

$$\inf_{\Upsilon\in E_1} \parallel \Psi^k_\omega(\phi_0) - \Upsilon \parallel < \rho_0, \quad \forall k > m.$$

By the compactness of E_1, we can find a $\Upsilon^* \in E_1$, such that

$$\parallel \Psi^k_\omega(\phi_0) - \Upsilon^* \parallel \leq \rho_0, \quad \forall k > m,$$

therefore,

$$R(t+s, x, \phi_0) \geq \Upsilon^* - \rho_0 > \eta_1 - \rho_0, \quad \forall t > m\omega - \tau, \ x \in (0, L)$$

and

$$u_2(t, x, \phi_0) \leq \rho_0, \quad \forall t > m\omega, \quad x \in (0, L), \tag{4.11}$$

it then follows from the third equation of system (1.1)-(1.3) that $u_2(t, x, \phi_0), t > (m+1)\omega$ and $x \in (0, L)$ satisfies

$$\begin{cases} \frac{\partial u_2(t,x)}{\partial t} \geq \delta\frac{\partial^2 u_2(t,x)}{\partial x^2} - \nu\frac{\partial u_2(t,x)}{\partial x} - \mu_0 u_2(t,x) + \\ \qquad q_2 \int_0^L \Gamma(\tau, x, y)\frac{\mu_2(\eta_1-\rho_0}{K_2+\eta_1-\rho_0} u_2(t-\tau, y)\mathrm{d}y, & t > 0, x \in (0, L), \\ \nu u_2(t, 0) - \delta\frac{\partial u_2}{\partial x}(t, 0) = \frac{\partial u_2}{\partial x}(t, L) = 0, & t > 0. \end{cases} \tag{4.12}$$

Let $\psi_0 \in \mathcal{E}$ be the positive eigenfunction of $\Lambda^{(2)}_{\rho_0}$ associated with P_{ρ_0}. Since $u_2((m+1)\omega + s, x, \phi_0) \geq 0$ for all $t > \tau, x \in [0, L]$, there exists a $\varsigma_0 > 0$ such that

$$u_2((m+1)\omega + s, x, \phi_0) \geq \varsigma_0\bar{\psi}(s, x), \quad \forall (s, x) \in [-\tau, 0] \times [0, L].$$

By (4.12) and the comparison principle, it follows that

$$u_2(t, x, \phi_0) \geq \varsigma_0 u^{\rho_0}(t - (m+1)\omega, x, \bar{\psi}), \quad \forall t \geq (m+1)\omega, \ x \in [0, L].$$

Hence,

$$u_2(k\omega, x, \phi_0) = \varsigma_0 u^{\rho_0}((k-m-1)\omega, x, \bar{\psi}) = \varsigma_0(\Lambda^{(2)}_\rho)^{k-m-1}\bar{\psi}(0, x) \to \infty$$

as $k \to \infty$, which contradicts (4.11). Similarly, the following claim is valid.

Claim 4. E_2 is a uniform weak repeller for $\mathbf{Z}_0$.

It follows from the above claim that each of E_0, E_1, E_2 is an isolated invariant set for Ψ_ω in $\mathbf{Z}_0$, and $W^S(E_i) \cap \mathbf{Z}_0 = \emptyset$, where $W^S(E_i)$ is the stable set of E_i $(i = 0, 1, 2)$. By appealing to the acyclicity theorem on uniform persistence for maps [17, Theorem 1.3.1 and Remark 1.3.1], we have that $\Psi_\omega : C^+_\tau \to C^+_\tau$ is uniformly persistent with respect to $(\mathbf{Z}_0, \partial\mathbf{Z}_0)$. It then follows from [17, Theorem 3.1.1] that the periodic semiflow $\Phi_t : C^+_\tau \to C^+_\tau$ is also uniformly persistent with respect to $(\mathbf{Z}_0, \partial\mathbf{Z}_0)$. Since $\Psi_{n_0\omega}$ is compact, where $n_0 := \min\{n \in \mathbb{N}, n\omega > 2\tau\}$, it follows from [18, Theorem 4.5] that $\Psi_{n_0\omega}:\mathbf{Z}_0\to\mathbf{Z}_0$ has a global attractor $\mathcal{A}_0$ and system (1.1)–(1.3) has an $n_0\omega$-periodic solution $(\tilde{R}(t,x),\tilde{u}_1(t,x),\tilde{u}_2(t,x))$ with $(\tilde{R}_t(\cdot)(\cdot),\tilde{u}_{1t}(\cdot)(\cdot),\tilde{u}_{2t}(\cdot)(\cdot))\in\mathbf{Z}_0$.

In order to prove the practice uniform persistence function in conclusion (ii), we use the arguments similar to [19, Theorem 4.1]. Define a continuous function $g:C_\tau^+\to[0,\infty)$ by

$$g(\phi):=\min\{\min_{x\in[0,L]}\phi_2(0,x),\ \min_{x\in[0,L]}\phi_3(0,x)\},\quad \forall\phi=(\phi_1,\phi_2,\phi_3)\in C_\tau^+.$$

Since $\mathcal{A}_0=\Psi_{n_0\omega}(\mathcal{A}_0)$, we have that $\phi_2(0,\cdot)>0$ and $\phi_3(0,\cdot)>0$ for all $\phi\in\mathcal{A}_0$. Let

$$\mathbf{B}_0:=\cup_{t\in[0,n_0\omega]}\Psi_t(\mathcal{A}_0)\ \text{for}\ \forall\mathbf{B}_0\in\mathbf{Z}_0.$$

It then follows that $\mathbf{B}_0\subset\mathbf{W}_0$ and $\lim_{t\to\infty}d(\Phi_t(\phi),\mathbf{B}_0)=0$ for all $\phi\in\mathbf{W}_0$. Since $\mathbf{B}_0$ is a compact subset of $\mathbf{W}_0$, we have

$$\min_{\phi\in\mathbf{B}_0}g(\phi)>0.$$

Thus, there exists a $\xi^*>0$ such that

$$\liminf_{t\to\infty}u_1(t,\cdot,\phi)\ge\xi^*,\quad \liminf_{t\to\infty}u_2(t,\cdot,\phi)\ge\xi^*.$$

Furthermore, in view of Lemma 4.1, there exists $0<\xi<\xi^*$ such that

$$\liminf_{t\to\infty}(R(t,\cdot,\phi),u_1(t,\cdot,\phi),u_2(t,\cdot,\phi))\ge(\xi,\zeta,\xi),\quad\forall\phi\in\mathbf{Z}_0,$$

that is, the persistence statement in (ii) is proved. □

5. Discussion

In this paper, we study the single species resource-two competing populations model, we first investigate the existence and uniqueness of solution by appealing to the theory of semigroup. By virtue of the analysis of the spectral radius of the Poincaré map of the associated linear delayed reaction-advection-diffusion equation, we show the competitive exclusion principle for the model system. The results have enriched the theoretical study of chemostat model to some extent.

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