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Fekete-Szegö Problem Associated with k-th Root Transformation for the Inverse of Univalent Functions Defined by Quasi-Subordination

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Abstract In this paper, we estimate the Fekete-Szegö functional with k-th root transform for the inverse of certain classes of analytic univalent functions using quasi-subordination.

Keywords univalent functions; Fekete-Szegö inequality; k-th root transformation; inverse; quasi-subordination

MR(2020) Subject Classification 30C45

1. Introduction

Let $\mathcal{H}$ denote the class of analytic functions in the unit disk $\Delta = \{z : |z| < 1\}$ on the complex $\mathbb{C}$. Let $\mathcal{A}$ denote the class of all analytic functions $f \in \mathcal{H}$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

in the open disk Δ normalized by $f(0) = 0$ and $f'(0) = 1$ and $\mathcal{S}$ be in $\mathcal{A}$ consisting of univalent functions in Δ.

Robertson [1] introduced the concept of quasi-subordination. Denote

$$\mathcal{B} = \{p(z) \in \mathcal{H} : |p(z)| < 1, |z| < 1\},$$

$$\mathcal{B}_0 = \{p(z) \in \mathcal{B} : p(0) = 0\}.$$

An analytic function $f(z)$ is quasi-subordination to an analytic function $g(z)$, in the open unit disk Δ if there exist analytic functions $h(z) \in \mathcal{B}$ and $p(z) \in \mathcal{B}_0$ such that $f(z) = h(z)g[p(z)]$. Then we write $f(z) \prec_q g(z)$. If $h(z) \equiv 1$, then the quasi-subordination reduces to be subordination. Also, if $p(z) \equiv z$, then $f(z) = h(z)g(z)$ and in this case we say that $f(z)$ is majorized by $g(z)$

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and it is written as $f(z) \ll g(z)$ in Δ. Consequently, it is clear that the quasi-subordination is the generalization of subordination as well as majorization.

EI-Ashwah and Kanas [2] introduced and studied the following two subclasses:

$$S_q^*(\gamma,\varphi) = \{f \in \mathcal{A} : \frac{1}{\gamma}(\frac{zf'(z)}{f(z)} - 1) \prec_q \varphi(z) - 1, z \in \Delta, \gamma \in \mathbb{C}\backslash\{0\}\}$$

and

$$C_q(\gamma,\varphi) = \{f \in \mathcal{A} : \frac{1}{\gamma}\frac{zf''(z)}{f'(z)} \prec_q \varphi(z) - 1, z \in \Delta, \gamma \in \mathbb{C}\backslash\{0\}\},$$

where

$$\varphi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \cdots \ (B_1 > 0). \tag{1.2}$$

We note that, when $h(z) \equiv 1$, the classes $S_q^*(\gamma,\varphi)$ and $C_q(\gamma,\varphi)$ reduce, respectively, to the familiar classes $S^*(\gamma,\varphi)$ and $C(\gamma,\varphi)$ of Ma-Minda starlike and convex functions of complex order γ in Δ (see [3]). For $\gamma = 1$, the classes $S_q^*(\gamma,\varphi)$ and $C_q(\gamma,\varphi)$ reduce to the classes $S_q^*(\varphi)$ and $C_q(\varphi)$ studied by Mohd and Darus [4].

The Koebe one quarter theorem states the image of Δ under every function $f \in \mathcal{S}$ contains a disk of radius $\frac{1}{4}$. Thus such univalent function has an inverse f^{-1} which satisfies

$$f^{-1}(f(z)) = z, \quad z \in \Delta$$

and

$$f(f^{-1}(\omega)) = \omega, \quad |\omega| < r_0(f), \ r_0(f) \geq \frac{1}{4}.$$

In fact the inverse function f^{-1} is given by

$$f^{-1}(\omega) = \omega - a_2\omega^2 + (2a_2^2 - a_3)\omega^3 - (5a_2^3 - 5a_2a_3 + a_4)\omega^4 + \cdots = \omega + \sum_{n=2}^{\infty} d_n\omega^n. \tag{1.3}$$

For a univalent function f^{-1} of the form (1.3), the k-th root transform is defined by

$$F(\omega) = [f^{-1}(\omega^k)]^{\frac{1}{k}} = \omega + \sum_{n=1}^{\infty} b_{kn+1}\omega^{kn+1}. \tag{1.4}$$

Remark 1.1 Set $k = 1$. Then the above expression reduces to the functional f^{-1} itself.

Definition 1.2 ([5]) *A function $f \in \mathcal{A}$ given by (1.1) is said to be in the class $\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$, $0 \neq \gamma \in \mathbb{C}$, $\delta \geq 0$, if the following quasi-subordination condition is satisfied*

$$\frac{1}{\gamma}\big[(1-\delta)\frac{z\mathcal{F}_\lambda'(z)}{\mathcal{F}_\lambda(z)} + \delta(1 + \frac{z\mathcal{F}_\lambda''(z)}{\mathcal{F}_\lambda'(z)}) - 1\big] \prec_q \varphi(z) - 1, \quad z \in \Delta,$$

where

$$\mathcal{F}_\lambda(z) = (1-\lambda)f(z) + \lambda z f'(z), \quad 0 \leq \lambda \leq 1.$$

We note that

(1) $\mathcal{M}_q^{\delta,0}(\gamma,\varphi) = \mathcal{M}_q^{\delta}(\gamma,\varphi)$;
(2) $\mathcal{M}_q^{\delta}(1,\varphi) = \mathcal{M}_q^{\delta}(\varphi)$ (see [4, Definition 1.7]);
(3) $\mathcal{M}_q^{0,0}(\gamma,\varphi) = \mathcal{S}_q^*(\gamma,\varphi)$ (see [2, Definition 1.1]);
(4) $\mathcal{S}_q^*(1,\varphi) = \mathcal{S}_q^*(\varphi)$ (see [4, Definition 1.1]);

(5) $\mathcal{M}_q^{1,0}(\gamma,\varphi)=\mathcal{C}q(\gamma,\varphi)$ (see [2, Definition 1.3]);
(6) $\mathcal{C}q(1,\varphi)=\mathcal{C}q(\varphi)$ (see [4, Definition 1.3]);
(7) $\mathcal{M}_q^{1,\lambda}(\gamma,\varphi)=\mathcal{C}_q(\gamma,\lambda,\varphi)$ (see [5]);
(8) $\mathcal{M}_q^{0,\lambda}(\gamma,\varphi)=\mathcal{P}_q(\gamma,\lambda,\varphi)$ (see [5]);
(9) $\mathcal{M}_q^{0,\lambda}(1,\varphi)=\mathcal{M}_q^{*}(\lambda,\varphi)$ (see [6]).

We note that if $h(z)=1$, then the quasi-subordination reduces to the subordination.
(1) $\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)=\mathcal{M}^{\delta,\lambda}(\gamma,\varphi)$;
(2) $\mathcal{M}_q^{\lambda,0}(1,\varphi)=\mathcal{M}(\lambda,\varphi)$ (see [7]);
(3) $\mathcal{M}_q^{0,0}(1,\varphi)=\mathcal{S}^*(\varphi)$ and $\mathcal{M}_q^{1,1}(1,\varphi)=\mathcal{C}(\varphi)$ (see [8]).

Inspired by papers [5, 9–12], we obtain sharp bound for the Fekete-Szegö coefficient functional $|b_{2k+1}-\mu b_{k+1}^2|$ associated with the k-th root transform of the function f^{-1} belonging to $\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$. In order to derive our main results, we recall here the following lemmas.

Lemma 1.3 ([13]) *Let $p(z)=c_1z+c_2z^2+c_3z^3+\cdots$ be in the class $\mathcal{B}_0$. Then, for $t\in\mathbb{C}$*

$$|c_2-tc_1^2|\le\max\{1;|t|\}.$$

The result is sharp for the functions given by $p(z)=z^2$ or $p(z)=z$.

Lemma 1.4 ([13]) *Let $h(z)=h_0+h_1z+h_2z^2+\cdots$ be in the class $\mathcal{B}$. Then*

$$|h_0|\le 1 \text{ and } |h_n|\le 1-|h_0|^2\le 1,\quad n>0.$$

Lemma 1.5 ([14]) *Let $p(z)=c_1z+c_2z^2+c_3z^3+\cdots$ be in the class $\mathcal{B}_0$. Then*

$$|c_1|\le 1 \text{ and } |c_n|\le 1-|c_1|^2,\quad n\ge 2.$$

The result is sharp for the function given by $p(z)=z^2$ or $p(z)=z$.

2. Main results

Using the above lemmas, we obtain the following conclusions:

Theorem 2.1 *If $f\in\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$ and F is the k-th root transformation of f^{-1} given by (1.4), then*

$$|b_{k+1}|\le\frac{|\gamma|B_1}{k(1+\delta)(1+\lambda)},$$

$$|b_{2k+1}|\le\frac{|\gamma|\{B_1+\max\{B_1;|B_2|+\frac{|k\gamma(1+\lambda)^2(1+3\delta)-\gamma(1+2\lambda)(1+2\delta)(3k+1)|B_1^2}{k(1+\delta)^2(1+\lambda)^2}\}\}}{2k(1+2\delta)(1+2\lambda)}$$

and for $\tau\in\mathbb{C}$

$$|b_{2k+1}-\tau b_{k+1}^2|\le\frac{|\gamma|\{B_1+\max\{B_1;|B_2|+\frac{|k\gamma(1+\lambda)^2(1+3\delta)+\gamma(1+2\lambda)(1+2\delta)(2\tau-3k-1)|B_1^2}{k(1+\delta)^2(1+\lambda)^2}\}\}}{2k(1+2\delta)(1+2\lambda)}.$$

Proof Let $f\in\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$. Then, in view of Definition 1.2, there exist two analytic functions $h\in\mathcal{B}$ and $p\in\mathcal{B}_0$, such that

$$\frac{1}{\gamma}\big[(1-\delta)\frac{z\mathcal{F}_\lambda'(z)}{\mathcal{F}_\lambda(z)}+\delta(1+\frac{z\mathcal{F}_\lambda''(z)}{\mathcal{F}_\lambda'(z)})-1\big]=h(z)[\varphi(p(z))-1].\tag{2.1}$$

Since

$$\frac{1}{\gamma}[(1-\delta)\frac{z\mathcal{F}'_\lambda(z)}{\mathcal{F}_\lambda(z)}+\delta(1+\frac{z\mathcal{F}''_\lambda(z)}{\mathcal{F}'_\lambda(z)})-1]$$
$$=\frac{1}{\gamma}(1+\delta)(1+\lambda)a_2z+\frac{1}{\gamma}[2(1+2\delta)(1+2\lambda)a_3-(1+3\delta)(1+\lambda)^2a_2^2]z^2+\cdots$$

and

$$h(z)[\varphi(p(z))-1]=B_1h_0c_1z+[B_1h_1c_1+B_1h_0c_2+B_2h_0c_1^2]z^2+\cdots,$$

it follows from (2.1) that

$$\frac{1}{\gamma}(1+\delta)(1+\lambda)a_2=B_1h_0c_1 \tag{2.2}$$

and

$$\frac{1}{\gamma}[2(1+2\delta)(1+2\lambda)a_3-(1+3\delta)(1+\lambda)^2a_2^2]=B_1h_1c_1+B_1h_0c_2+B_2h_0c_1^2. \tag{2.3}$$

From (2.2) and (2.3), we get

$$a_2=\frac{\gamma h_0B_1c_1}{(1+\delta)(1+\lambda)} \tag{2.4}$$

and

$$a_3=\frac{\gamma}{2(1+2\delta)(1+2\lambda)}[h_1B_1c_1+h_0B_1c_2+(h_0B_2+\frac{\gamma(1+3\delta)h_0^2B_1^2}{(1+\delta)^2})c_1^2]. \tag{2.5}$$

For a function $f^{-1}\in\mathcal{S}$ given by (1.3), a computation shows that

$$\begin{aligned}F(\omega)=[f^{-1}(\omega^k)]^{\frac{1}{k}}&=\omega+\frac{d_2}{k}\omega^{k+1}+(\frac{d_3}{k}-\frac{k-1}{2k^2}d_2^2)\omega^{2k+1}+\cdots\\&=\omega+\sum_{n=1}^{\infty}b_{kn+1}\omega^{kn+1}.\end{aligned} \tag{2.6}$$

The Eqs. (1.3) and (2.6) yield:

$$b_{k+1}=-\frac{1}{k}a_2 \tag{2.7}$$

and

$$b_{2k+1}=\frac{1}{2k^2}[(3k+1)a_2^2-2ka_3]. \tag{2.8}$$

Using (2.4) and (2.5) in (2.7) and (2.8) gives

$$b_{k+1}=-\frac{\gamma h_0B_1c_1}{k(1+\delta)(1+\lambda)} \tag{2.9}$$

and

$$b_{2k+1}=-\frac{\gamma B_1\{h_1c_1+h_0[c_2+(\frac{B_2}{B_1}+\frac{[k\gamma(1+\lambda)^2(1+3\delta)-\gamma(1+2\delta)(1+2\lambda)(3k+1)]h_0B_1}{k(1+\delta)^2(1+\lambda)^2})c_1^2]\}}{2k(1+2\delta)(1+2\lambda)}.$$

Also for $\tau\in\mathbb{C}$

$$\begin{aligned}b_{2k+1}-\tau b_{k+1}^2=-&\frac{\gamma B_1}{2k(1+2\delta)(1+2\lambda)}\{h_1c_1+h_0[c_2-(\frac{-B_2}{B_1}-\\&\frac{[k\gamma(1+\lambda)^2(1+3\delta)+\gamma(1+2\delta)(1+2\lambda)(2\tau-3k-1)]h_0B_1}{k(1+\delta)^2(1+\lambda)^2})c_1^2]\}.\end{aligned}$$

By Lemmas 1.4 and 1.5, we obtain

$$|b_{k+1}| \leq \frac{|\gamma|B_1}{k(1+\delta)(1+\lambda)}$$

and

$$|b_{2k+1}-\tau b_{k+1}^2| \leq \frac{|\gamma|B_1}{2k(1+2\delta)(1+2\lambda)}\{1+|c_2-(\frac{-B_2}{B_1}- \frac{[k\gamma(1+\lambda)^2(1+3\delta)+\gamma(1+2\delta)(1+2\lambda)(2\tau-3k-1)]h_0B_1}{k(1+\delta)^2(1+\lambda)^2})c_1^2|\}.$$

In view of Lemma 1.3, we have

$$|b_{2k+1}-\tau b_{k+1}^2| \leq \frac{|\gamma|\{B_1+\max\{B_1;|B_2|+\frac{|k\gamma(1+\lambda)^2(1+3\delta)+\gamma(1+2\lambda)(1+2\delta)(2\tau-3k-1)|B_1^2}{k(1+\delta)^2(1+\lambda)^2}\}\}}{2k(1+2\delta)(1+2\lambda)}.$$

When $\tau=0$, we have

$$|b_{2k+1}| \leq \frac{|\gamma|\{B_1+\max\{B_1;|B_2|+\frac{|k\gamma(1+\lambda)^2(1+3\delta)-\gamma(1+2\lambda)(1+2\delta)(3k+1)|B_1^2}{k(1+\delta)^2(1+\lambda)^2}\}\}}{2k(1+2\delta)(1+2\lambda)} \quad \Box.$$

By taking $h(z)=1$ in the proof of Theorem 2.1, we have the next result.

Theorem 2.2 *If $f\in\mathcal{M}^{\delta,\lambda}(\gamma,\varphi)$ and F is the k-th root transformation of f^{-1} given by (1.4), then*

$$|b_{k+1}| \leq \frac{|\gamma|B_1}{k(1+\delta)(1+\lambda)},$$

$$|b_{2k+1}| \leq \frac{|\gamma|\max\{B_1;|B_2+\frac{[k\gamma(1+\lambda)^2(1+3\delta)-\gamma(1+2\lambda)(1+2\delta)(3k+1)]B_1^2}{k(1+\delta)^2(1+\lambda)^2}|\}}{2k(1+2\delta)(1+2\lambda)}$$

and for $\tau\in\mathbb{C}$

$$|b_{2k+1}-\tau b_{k+1}^2| \leq \frac{|\gamma|\max\{B_1;|B_2+\frac{[k\gamma(1+\lambda)^2(1+3\delta)+\gamma(1+2\lambda)(1+2\delta)(2\tau-3k-1)]B_1^2}{k(1+\delta)^2(1+\lambda)^2}|\}}{2k(1+2\delta)(1+2\lambda)}.$$

Remark 2.3 In the special case when $\gamma=k=1$, $\lambda=0$, $\delta=\alpha$, $B_1=1$, $B_2=\frac{1}{2}$, the $|b_{2k+1}-\tau b_{k+1}^2|$ in Theorem 2.2 reduces to the one in Theorem 4.1 studied by Sharma et al. [15].

Remark 2.4 In the special case when $\gamma=k=\delta=1$, $\lambda=0$, $B_1=2\beta$, $B_2=2\beta^2$, the $|b_{2k+1}-\mu b_{k+1}^2|$ in Theorem 2.2 reduces to the one in Theorem 6.2 studied by Thomas and Verma. [16].

3. Corollaries

Setting $\lambda=0$ in Theorem 2.1, we get the following corollary.

Corollary 3.1 *If $f\in\mathcal{M}_q^{\delta}(\gamma,\varphi)$ and F is the k-th root transformation of f^{-1} given by (1.4), then*

$$|b_{k+1}| \leq \frac{|\gamma|B_1}{k(1+\delta)},$$

$$|b_{2k+1}| \leq \frac{|\gamma|\{B_1 + \max\{B_1; |B_2| + \frac{|k\gamma(1+3\delta)-\gamma(1+2\delta)(3k+1)|B_1^2}{k(1+\delta)^2}\}\}}{2k(1+2\delta)}$$

and for $\tau \in \mathbb{C}$

$$|b_{2k+1} - \tau b_{k+1}^2| \leq \frac{|\gamma|\{B_1 + \max\{B_1; |B_2| + \frac{|k\gamma(1+3\delta)+\gamma(1+2\delta)(2\tau-3k-1)|B_1^2}{k(1+\delta)^2}\}\}}{2k(1+2\delta)}.$$

Setting $\delta = 0$ in Theorem 2.1, we get the following corollary.

Corollary 3.2 *If* $f \in \mathcal{M}_q^{0,\lambda}(\gamma, \varphi) = \mathcal{P}_q(\gamma, \lambda, \varphi)$ *and* F *is the* k*-th root transformation of* f^{-1} *given by (1.4), then*

$$|b_{k+1}| \leq \frac{|\gamma|B_1}{k(1+\lambda)},$$

$$|b_{2k+1}| \leq \frac{|\gamma|\{B_1 + \max\{B_1; |B_2| + \frac{|k\gamma(1+\lambda)^2-\gamma(1+2\lambda)(3k+1)|B_1^2}{k(1+\lambda)^2}\}\}}{2k(1+2\lambda)}$$

and for $\tau \in \mathbb{C}$

$$|b_{2k+1} - \tau b_{k+1}^2| \leq \frac{|\gamma|\{B_1 + \max\{B_1; |B_2| + \frac{|k\gamma(1+\lambda)^2+\gamma(1+2\lambda)(2\tau-3k-1)|B_1^2}{k(1+\lambda)^2}\}\}}{2k(1+2\lambda)}.$$

Setting $\delta = 1$ in Theorem 2.1, we get the following corollary.

Corollary 3.3 *If* $f \in \mathcal{M}_q^{1,\lambda}(\gamma, \varphi) = \mathcal{C}_q(\gamma, \lambda, \varphi)$ *and* F *is the* k*-th root transformation of* f^{-1} *given by (1.4), then*

$$|b_{k+1}| \leq \frac{|\gamma|B_1}{2k(1+\lambda)},$$

$$|b_{2k+1}| \leq \frac{|\gamma|\{B_1 + \max\{B_1; |B_2| + \frac{|4k\gamma(1+\lambda)^2-3\gamma(1+2\lambda)(3k+1)|B_1^2}{4k(1+\lambda)^2}\}\}}{6k(1+2\lambda)}.$$

and for $\tau \in \mathbb{C}$

$$|b_{2k+1} - \tau b_{k+1}^2| \leq \frac{|\gamma|\{B_1 + \max\{B_1; |B_2| + \frac{|4k\gamma(1+\lambda)^2+3\gamma(1+2\lambda)(2\tau-3k-1)|B_1^2}{4k(1+\lambda)^2}\}\}}{6k(1+2\lambda)}.$$

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