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Existence of Nontrivial Solutions for a Class of Nonlinear Fractional Schrödinger-Poisson System

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Abstract This paper is concerned with the following fractional Schrödinger-Poisson system:

$$\begin{cases} (-\Delta)^s u + u + \phi u = \lambda f(u) \text{ in } \mathbb{R}^3, \\ (-\Delta)^\alpha \phi = u^2 \text{ in } \mathbb{R}^3, \end{cases}$$

where $s \in (\frac{3}{4}, 1), \alpha \in (0,1), \lambda$ is a positive parameter, $(-\Delta)^s, (-\Delta)^\alpha$ are fractional Laplacian operators. Under certain assumptions on f, we obtain the existence of at least one nontrivial solution of the system by using the methods of perturbation and Moser iterative method.

Keywords fractional Schrödinger-Poisson system; nontrivial solution; perturbation method; Moser iterative method

MR(2020) Subject Classification 35A15; 35R11; 35J60

1. Introduction

In this paper, we consider the following fractional Schrödinger-Poisson system:

$$\begin{cases} (-\Delta)^s u + u + \phi u = \lambda f(u) \text{ in } \mathbb{R}^3, \\ (-\Delta)^\alpha \phi = u^2 \text{ in } \mathbb{R}^3, \end{cases} \tag{1.1}$$

where $s \in (\frac{3}{4}, 1), \alpha \in (0,1), \lambda > 0$ is a real parameter. The $(-\Delta)^s$ is the fractional Laplacian operator

$$(-\Delta)^s u(x) = -\frac{C(3,s)}{2} \int_{\mathbb{R}^3} \frac{u(x+y) + u(x-y) - 2u(x)}{|y|^{3+2s}} \mathrm{d}y$$

for $u : \mathbb{R}^3 \to \mathbb{R}$ belonging to the Schwartz space $\mathcal{S}(\mathbb{R}^3)$ of rapidly decaying C^∞-functions in $\mathbb{R}^3$ where

$$C(3,s) = \Big(\int_{\mathbb{R}^N} \frac{1-\cos(x_1)}{|x|^{3+2s}} \Big)^{-1}$$

see for instance [1].

Fractional operators of elliptic type arise in a quite natural way in many different problems, such as the thin obstacle problem, optimization, finance, phase transitions, stratified materials, anomalous diffusion, crystal dislocation, soft thin films, semipermeable membranes, flame propagation, conservation laws, ultra-relativistic limits of quantum mechanics, quasi-geostrophic

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flows, multiple scattering, minimal surfaces, materials science, water waves and so on. The investigations of the problems involving these non-local operators are interesting and important both from pure mathematical research aspects and real-world applications; eg see [1,2] and references therein.

For a general form of fractional Schrödinger-Poisson system

$$\begin{cases} (-\Delta)^s u + V(x)u + \phi u = f(x,u) \text{ in } \mathbb{R}^N, \\ (-\Delta)^\alpha \phi = u^2 \text{ in } \mathbb{R}^N, \end{cases} \tag{1.2}$$

many existence and multiplicity results have been obtained by using different variational methods. If we only consider the first equation in (1.2), it reduces to a fractional Schrödinger equation when $\phi = 0$. Since this kind of problem is displayed on the whole space $\mathbb{R}^N$, the main difficulty for dealing with it by variational methods is the lack of compactness. To overcome this difficulty, the author in [3] makes the following assumptions:

(V_1) $V \in C(R^N)$ and $\inf V(x) > 0$;

(V_2) For any $M > 0$, there exists $r_0 > 0$ such that

$$\lim_{|y|\to\infty} \operatorname{meas}(\{x \in \mathbb{R}^N : |x-y| \leq r_0, V(x) \leq M\}) = 0,$$

where meas denotes the Lebesgue measure on the whole $\mathbb{R}^N$. These assumptions have been widely used in the study of fractional Schrödinger-Poisson system in [4–8] for instance. The nontrivial radial symmetric solution of the problem (1.2) has also been obtained under suitable assumptions in the radial symmetric space $H_r^s(\mathbb{R}^N)$, where $H_r^s(\mathbb{R}^N) = \{u \in H^s(\mathbb{R}^N), u(x) = u(|x|)\}$, because the embedding of $H_r^s(\mathbb{R}^N)$ into the space $L^p(\mathbb{R}^N)$ $(2 < p < 2_s^* = \frac{2N}{N-2s})$ is compact; see for instance [9].

Recently, a new approach namely the perturbation method has been proposed in [10, 11] to deal with quasi-linear elliptic equations. In [12], a perturbation method is used to study the Schrödinger-Poisson system

$$\begin{cases} -\Delta u + u + \phi u = f(x,u) \text{ in } \mathbb{R}^3, \\ (-\Delta)^{\frac{\alpha}{2}} \phi = u^2 \text{ in } \mathbb{R}^3, \end{cases} \tag{1.3}$$

where $\alpha \in (1,2]$. Under some conditions, the problem (1.3) possesses at least one nontrivial solution. Then the author in [13] studied the fractional Schrödinger-Poisson system

$$\begin{cases} (-\Delta)^s u + u + \phi u = f(x,u) \text{ in } \mathbb{R}^3, \\ (-\Delta)^\alpha \phi = u^2 \text{ in } \mathbb{R}^3, \end{cases} \tag{1.4}$$

where $s, \alpha \in (0,1], 2\alpha + 4s > 3$. The author proved that problem (1.4) possesses at least one nontrivial solution under some assumptions on f.

In [14], the authors studied a class of superlinear elliptic problems $-\Delta u = \lambda f(u)$ under the Dirichlet boundary condition on a bounded smooth domain in $\mathbb{R}^N$ where the nonlinearity $f(u)$ is superlinear in a neighborhood of $u = 0$. Then the problem has solutions for all λ sufficiently large by using the Moser iterative method. The Moser iterative method is used to study the supercritical situation recently; see for instance, [15,16] and [17,18]. Recently, the authors in [19] considered the following Schrödinger-Poisson system

$$\begin{cases} -\Delta u + V(x)u + \phi u = \lambda f(u) \text{ in } \mathbb{R}^3, \\ -\Delta\phi = u^2 \text{ in } \mathbb{R}^3, \end{cases} \tag{1.5}$$

and prove that the problem possesses a positive solution for large value of λ without any growth and Ambrosetti-Rabinowitz condition.

Motivated by the above facts, the main purpose of this paper is to consider the existence of nontrivial solutions for problem (1.1). Since $s \in (\frac{3}{4}, 1)$, we have $4 < 2_s^*$. To state our main results, we assume the following conditions.

(f_1) There is a $\tau \in (4, 2_s^*)$ such that $\limsup_{|u|\to 0} \frac{f(u)u}{|u|^\tau} < +\infty$, where $2_s^* = \frac{6}{3-2s}$;

(f_2) There is a $\beta \in (4, 2_s^*)$ such that $\liminf_{|u|\to 0} \frac{F(u)}{|u|^\beta} > 0$, where $F(u) = \int_0^u f(t)\mathrm{d}t$;

(f_3) There is a $\mu \in (4, 2_s^*)$ such that $uf(u) \geq \mu F(u) > 0$ for $|u| \neq 0$ small.

Theorem 1.1 *Assume that assumptions (f_1)–(f_3) hold. Then problem (1.1) has at least one non-trivial solution for all λ sufficiently large.*

The paper is organized as follows. In Section 2, we present some preliminary results. In Section 3, we prove Theorem 1.1.

2. Preliminaries and the variational setting

This section is concerned with the variational framework for fractional Schrödinger-Poisson system. Also, we collect some preliminary results.

As usual, $\|u\|_m = (\int_{\mathbb{R}^3} |u|^m \mathrm{d}x)^{\frac{1}{m}}$, $\forall 1 \leq m < \infty$.

For any $s \in (0,1)$, we define $\mathcal{D}^{s,2}(\mathbb{R}^3)$ as the completion of $C_c^\infty(\mathbb{R}^3)$ with respect to

$$[u]_s^2 = \int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \frac{|u(x)-u(y)|^2}{|x-y|^{3+2s}}\mathrm{d}x\mathrm{d}y.$$

Equivalently,

$$\mathcal{D}^{s,2}(\mathbb{R}^3) = \{u \in L^{2_s^*}(\mathbb{R}^3) : [u]_s < \infty\}.$$

The fractional space $H^s(\mathbb{R}^3)$ is defined by

$$H^s(\mathbb{R}^3) = \Big\{u \in L^2(\mathbb{R}^3) : \int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \frac{|u(x)-u(y)|^2}{|x-y|^{3+2s}}\mathrm{d}x\mathrm{d}y < \infty\Big\},$$

equipped with the norm

$$\|u\|_{H^s(\mathbb{R}^3)}^2 = [u]_s^2 + \|u\|_2^2 = \int_{\mathbb{R}^3} (|(-\Delta)^{\frac{s}{2}}u|^2 + |u|^2)\mathrm{d}x.$$

We recall the following embeddings of the fractional Sobolev spaces into Lebesgue spaces.

Lemma 2.1 ([1]) *Let $s \in (0,1)$. Then there exists a sharp constant $S_* = S(s) > 0$ such that for any $u \in \mathcal{D}^{s,2}(\mathbb{R}^3)$*

$$\|u\|_{2_s^*}^2 \leq S_*[u]_s^2. \tag{2.1}$$

Moreover, $H^s(\mathbb{R}^3)$ is continuously embedded in $L^p(\mathbb{R}^3)$ for any $p \in [2, 2_s^*]$ and compactly in $L^p_{\mathrm{loc}}(\mathbb{R}^3)$ for any $p \in [1, 2_s^*)$.

In the following, we denote by C, C_i the generic constant, which may change from line to line.

Using Hölder's inequality, it follows from Lemma 2.1 that for every $u \in H^s(\mathbb{R}^3)$, we have

$$\int_{\mathbb{R}^3} u^2 v \mathrm{d}x \le \Big(\int_{\mathbb{R}^3} |u|^{\frac{12}{3+2\alpha}}\Big)^{\frac{3+2\alpha}{6}} \Big(\int_{\mathbb{R}^3} |v|^{2^*_\alpha} \mathrm{d}x\Big)^{\frac{1}{2^*_\alpha}} \le C\|u\|^2_{H^s}\|v\|_{\mathcal{D}^{\alpha,2}} \tag{2.2}$$

where we used the embedding $H^s(\mathbb{R}^3) \hookrightarrow L^{\frac{12}{3+2\alpha}}(\mathbb{R}^3)$ and the constant C is not dependent on α. By Lax-Milgram theorem, there exists a unique $\phi^\alpha_u \in \mathcal{D}^{\alpha,2}(\mathbb{R}^3)$ such that

$$\int_{\mathbb{R}^3} v(-\Delta)^\alpha \phi^\alpha_u \mathrm{d}x = \int_{\mathbb{R}^3} (-\Delta)^{\frac{\alpha}{2}}\phi^\alpha_u (-\Delta)^{\frac{\alpha}{2}} v \mathrm{d}x = \int_{\mathbb{R}^3} u^2 v \mathrm{d}x, \quad v \in \mathcal{D}^{\alpha,2}(\mathbb{R}^3). \tag{2.3}$$

Hence, ϕ^α_u satisfies the Poisson equation

$$(-\Delta)^\alpha \phi^\alpha_u = u^2.$$

Moreover, ϕ^α_u has the following integral expression

$$\phi^\alpha_u = c_\alpha \int_{\mathbb{R}^3} \frac{u^2(y)}{|x-y|^{3-2\alpha}} \mathrm{d}y,$$

which is the Riesz potential [20], where

$$c_\alpha = \frac{\Gamma(\frac{3-2\alpha}{2})}{\pi^{\frac{3}{2}} 2^{2\alpha}\Gamma(\alpha)}.$$

Thus $\phi^\alpha_u \ge 0$ for a.e. $x \in \mathbb{R}^3$. From (2.2) and (2.3), we have

$$\|\phi^\alpha_u\|_{\mathcal{D}^{\alpha,2}} \le C\|u\|^2_{\frac{12}{3+2\alpha}} \le C\|u\|^2_{H^s}. \tag{2.4}$$

It follows that

$$\|\phi^\alpha_u\|^2_{\mathcal{D}^{\alpha,2}} = \int_{\mathbb{R}^3} u^2\phi^\alpha_u \mathrm{d}x \le \|u\|^2_{\frac{12}{3+2\alpha}} \|\phi^\alpha_u\|_{2^*_\alpha} \le C\|u\|^2_{\frac{12}{3+2\alpha}} \|\phi^\alpha_u\|_{\mathcal{D}^{\alpha,2}}. \tag{2.5}$$

Then

$$\|\phi^\alpha_u\|_{\mathcal{D}^{\alpha,2}} \le C\|u\|^2_{\frac{12}{3+2\alpha}}. \tag{2.6}$$

By conditions (f_1) and (f_2), there exist $0 < \delta < \frac{1}{2}$, $C_1 > 0$ and $C_2 > 0$ such that

$$F(u) \ge C_1|u|^\beta, \quad F(u) \le C_2|u|^\tau \tag{2.7}$$

and $uf(u) \ge \mu F(u) > 0$ for $0 < |u| \le 2\delta$. For the fixed $\delta > 0$, we consider a cut-off function $\rho(t)$ satisfying

$$\rho(t) = \begin{cases} 1, & \text{if } |t| \le \delta \\ 0, & \text{if } |t| \ge 2\delta, \end{cases}$$

$t\rho'(t) \le 0$, and $|t\rho'(t)| \le \frac{2}{\delta}$.

Lemma 2.2 ([14]) *Define $\tilde{F}(u) = \rho(u)F(u) + (1-\rho(u))F_\infty(u)$, where $F_\infty(u) := C_2|u|^\tau$. Then*

$$u\tilde{f}(u) \ge \theta \tilde{F}(u) > 0$$

for all $u \ne 0$, where $\theta = \min\{\mu, \tau\}$, $\tilde{f}(u) = \tilde{F}'(u)$.

So we consider the following modified fractional Schrödinger equation

$$(-\Delta)^s u + u + \phi_u^\alpha u = \lambda \tilde{f}(u), \quad x \in \mathbb{R}^3. \tag{2.8}$$

The energy functional $I_\lambda : H^s(\mathbb{R}^3) \to \mathbb{R}$ corresponding to problem (2.8) is defined by

$$I_\lambda(u) = \frac{1}{2}\int_{\mathbb{R}^3} (|(-\Delta)^{\frac{s}{2}} u|^2 + u^2)\mathrm{d}x + \frac{1}{4}\int_{\mathbb{R}^3} \phi_u^\alpha u^2 \mathrm{d}x - \lambda \int_{\mathbb{R}^3} \tilde{F}(u)\mathrm{d}x. \tag{2.9}$$

It is easy to see that I_λ is well defined in $H^s(\mathbb{R}^3)$, $I_\lambda \in C^1(H^s(\mathbb{R}^3), \mathbb{R})$ and

$$(I_\lambda^{'}(u), v) = \int_{\mathbb{R}^3} \big((-\Delta)^{\frac{s}{2}} u (-\Delta)^{\frac{s}{2}} v + uv + \phi_u^\alpha uv - \lambda \tilde{f}(u)v\big)\mathrm{d}x, \quad v \in H^s(\mathbb{R}^3). \tag{2.10}$$

We choose a potential $V(x)$ satisfying the following condition.

(V) $V \in C(\mathbb{R}^3, \mathbb{R})$ satisfies $\inf_{x\in\mathbb{R}^3} V(x) \geq V_0 > 0$ and for every $M > 0$, $\mathrm{meas}\{x \in \mathbb{R}^3 : V(x) \leq M\} < \infty$. Let

$$E = \{u \in H^s(\mathbb{R}^3) : \int_{\mathbb{R}^3} (|(-\Delta)^{\frac{s}{2}} u|^2 + V(x)u^2)\mathrm{d}x < \infty\}.$$

Then E is a Hilbert space with the inner product and norm

$$(u, v)_E = \int_{\mathbb{R}^3} ((-\Delta)^{\frac{s}{2}} u (-\Delta)^{\frac{s}{2}} v + V(x)uv)\mathrm{d}x, \quad \|u\|_E = (u, u)_E^{\frac{1}{2}}.$$

It is known that E is compactly embedded in $L^p(\mathbb{R}^3)$ for $2 \leq p < 2_s^*$ (see [3]).

For fixed $\sigma \in (0, 1]$, we introduce the following inner product

$$(u, v)_{H_\sigma^s} = \int_{\mathbb{R}^3} \big((-\Delta)^{\frac{s}{2}} u (-\Delta)^{\frac{s}{2}} v + \sigma V(x)uv\big)\mathrm{d}x$$

and the norm $\|u\|_{H_\sigma^s} = (u, u)_{H_\sigma^s}^{\frac{1}{2}}$. Let $E_\sigma = (E, \|.\|_{H_\sigma^s})$. Define the perturbed functional $I_{\sigma,\lambda} : E \to \mathbb{R}$:

$$I_{\sigma,\lambda}(u) = I_\lambda(u) + \frac{\sigma}{2}\int_{\mathbb{R}^3} V(x)u^2 \mathrm{d}x, \quad \sigma \in (0, 1]. \tag{2.11}$$

3. Proof of Theorem 1.1

Firstly, we will prove that for every fixed $\lambda \geq 1$, the problem (2.8) has at least one nontrivial solution.

Lemma 3.1 *For every fixed $\lambda \geq 1$ and fixed $\sigma \in (0, 1]$, there exist $\rho_\lambda > 0$, $\delta_\lambda > 0$ such that*

$$\inf_{u\in E, \|u\|_E = \rho_\lambda} I_{\sigma,\lambda}(u) > \delta_\lambda.$$

Proof It is clear that $\tilde{F}(u) \leq C_2|u|^\tau$.

$$\begin{aligned} I_{\sigma,\lambda}(u) &= \frac{1}{2}\int_{\mathbb{R}^3} (|(-\Delta)^{\frac{s}{2}} u|^2 + |u|^2)\mathrm{d}x + \frac{1}{4}\int_{\mathbb{R}^3} \phi_u^\alpha u^2 \mathrm{d}x + \frac{\sigma}{2}\int_{\mathbb{R}^3} V(x)u^2 \mathrm{d}x - \lambda \int_{\mathbb{R}^3} \tilde{F}(u)\mathrm{d}x \\ &\geq \frac{\sigma}{2}\|u\|_E^2 + \frac{1}{2}\int_{\mathbb{R}^3} u^2 \mathrm{d}x + \frac{1}{4}\int_{\mathbb{R}^3} \phi_u^\alpha u^2 \mathrm{d}x - \lambda \int_{\mathbb{R}^3} \tilde{F}(u)\mathrm{d}x \\ &\geq \frac{\sigma}{2}\|u\|_E^2 - \lambda C\|u\|_E^\tau = \|u\|_E^2(\frac{\sigma}{2} - \lambda C\|u\|_E^{\tau-2}). \end{aligned}$$

Let $\rho_\lambda = (\frac{\sigma}{4C\lambda})^{\frac{1}{\tau-2}}$. Then for $\|u\|_E = \rho_\lambda$, $I_{\sigma,\lambda}(u) \geq \frac{\sigma}{4}\rho_\lambda^2 = \delta_\lambda > 0$. □

Lemma 3.2 *For every fixed $\lambda \geq 1$, there exists $e \in E$ with $\|e\|_E \geq \rho_\lambda$ such that $I_{\sigma,\lambda}(e) < 0$ for fixed $\sigma \in (0,1]$.*

Proof By Lemma 2.2, there exists a constant $C > 0$ such that

$$\tilde{F}(u) \geq C|u|^\theta, \text{ for } |u| \text{ large.} \tag{3.1}$$

By (2.4) and (2.5),

$$\int_{\mathbb{R}^3} \phi_u^\alpha u^2 \mathrm{d}x = \|\phi_u^\alpha\|^2_{\mathcal{D}^{\alpha,2}} \leq C\|u\|^4_{H^s}. \tag{3.2}$$

Choosing a fixed nontrivial function $v \in C_0^\infty(\mathbb{R}^3)$, we have

$$\begin{aligned} I_{\sigma,\lambda}(tv) &= \frac{t^2}{2}\|v\|^2_{H^s_\sigma} + \frac{t^2}{2}\|v\|_2^2 + \frac{Ct^4}{4}\|v\|^4_{H^s} - \lambda\int_{\mathbb{R}^3}\tilde{F}(tv)\mathrm{d}x \\ &\leq \frac{t^2}{2}\|v\|^2_E + \frac{t^2}{2}\|v\|_2^2 + \frac{Ct^4}{4}\|v\|^4_{H^s} - Ct^\theta\|v\|_\theta^\theta \to -\infty \end{aligned}$$

as $t \to +\infty$. Let $T > 0$ and define a path $h : [0,1] \to E$ by $h(t) = tTv$. For $T > 0$ large enough, independent of σ and λ, we have

$$\|h(1)\|_E \geq (\frac{1}{4C})^{\frac{1}{\tau-2}} \geq \rho_\lambda, \quad I_{\sigma,\lambda}(h(1)) < 0.$$

By taking $e = h(1)$, we complete the proof. □

Lemma 3.3 *For every fixed $\lambda \geq 1$, $I_{\sigma,\lambda}$ satisfies the Palais-Smale condition on E for fixed $\sigma \in (0,1]$.*

Proof Let $\{u_{n,\lambda}\}$ be a sequence in E so that $I_{\sigma,\lambda}(u_{n,\lambda})$ is bounded and $I'_{\sigma,\lambda}(u_{n,\lambda}) \to 0$. Then

$$\begin{aligned} C_\lambda + C\|u_{n,\lambda}\| \geq & I_{\sigma,\lambda}(u_{n,\lambda}) - \frac{1}{\theta}(I'_{\sigma,\lambda}(u_{n,\lambda}), u_{n,\lambda}) \\ =&(\frac{1}{2}-\frac{1}{\theta})\|u_{n,\lambda}\|^2_{H^s_\sigma} + (\frac{1}{2}-\frac{1}{\theta})\|u_{n,\lambda}\|_2^2 + (\frac{1}{4}-\frac{1}{\theta})\int_{\mathbb{R}^3}\phi^\alpha_{u_{n,\lambda}}u^2_{n,\lambda}\mathrm{d}x + \\ &\lambda\int_{\mathbb{R}^3}(\frac{\tilde{f}(u_{n,\lambda})u_{n,\lambda}}{\theta} - \tilde{F}(u_{n,\lambda}))\mathrm{d}x \\ \geq&(\frac{1}{2}-\frac{1}{\theta})\sigma\|u_{n,\lambda}\|^2_E. \end{aligned}$$

Thus, $\{u_{n,\lambda}\}$ is bounded in E. Up to a subsequence, we can assume that $u_{n,\lambda} \rightharpoonup u_\lambda$ in E, $u_{n,\lambda} \to u_\lambda$ in $L^p(\mathbb{R}^3)$, $2 \leq p < 2_s^*$. Observe that

$$\begin{aligned} \|u_{n,\lambda} - u_\lambda\|^2_{H^s_\sigma} =&(I'_{\sigma,\lambda}(u_{n,\lambda}) - I'_{\sigma,\lambda}(u_\lambda), u_{n,\lambda} - u_\lambda) - \|u_{n,\lambda} - u_\lambda\|_2^2 - \\ &\int_{\mathbb{R}^3}(\phi^\alpha_{u_{n,\lambda}}u_{n,\lambda} - \phi^\alpha_{u_\lambda}u_\lambda)(u_{n,\lambda} - u_\lambda)\mathrm{d}x + \lambda\int_{\mathbb{R}^3}(\tilde{f}(u_{n,\lambda}) - \tilde{f}(u_\lambda))(u_{n,\lambda} - u_\lambda)\mathrm{d}x \\ =&J_1 + J_2 + J_3 + J_4. \end{aligned}$$

It is clear that $J_1 \to 0$ and $J_2 \to 0$ as $n \to \infty$. By (2.6),

$$\int_{\mathbb{R}^3}\phi^\alpha_{u_{n,\lambda}}u_{n,\lambda}(u_{n,\lambda} - u_\lambda)\mathrm{d}x \leq \|\phi^\alpha_{u_{n,\lambda}}\|_{2^*_\alpha}\|u_{n,\lambda}\|_{\frac{12}{3+2\alpha}}\|u_{n,\lambda} - u_\lambda\|_{\frac{12}{3+2\alpha}}$$

$$
\begin{aligned}
&\le C\|\phi^{\alpha}_{u_{n,\lambda}}\|_{\mathcal{D}^{\alpha,2}}\|u_{n,\lambda}\|_{\frac{12}{3+2\alpha}}\|u_{n,\lambda}-u_\lambda\|_{\frac{12}{3+2\alpha}}\\
&\le C\|u_{n,\lambda}\|^3_{\frac{12}{3+2\alpha}}\|u_{n,\lambda}-u_\lambda\|_{\frac{12}{3+2\alpha}}\\
&\le C\|u_{n,\lambda}\|^3_{E}\|u_{n,\lambda}-u_\lambda\|_{\frac{12}{3+2\alpha}}\to 0.
\end{aligned}
$$

Similarly, we have

$$\int_{\mathbb{R}^3}\phi^{\alpha}_{u_\lambda}u_\lambda(u_{n,\lambda}-u_\lambda)\mathrm{d}x\to 0,$$

thus $J_3\to 0$ as $n\to\infty$. Since the cut-off function $\rho(t)$ satisfies $|\rho'(t)t|\le \frac{2}{\delta}$, we have

$$|\tilde{f}(t)|\le C|t|^{\tau-1},$$

hence

$$
\begin{aligned}
\int_{\mathbb{R}^3}\tilde{f}(u_{n,\lambda})(u_{n,\lambda}-u_\lambda)\mathrm{d}x &\le C\int_{\mathbb{R}^3}|u_{n,\lambda}|^{\tau-1}|u_{n,\lambda}-u_\lambda|\mathrm{d}x\\
&\le C\|u_{n,\lambda}\|^{\tau-1}_{\tau}\|u_{n,\lambda}-u_\lambda\|_{\tau}\\
&\le C\|u_{n,\lambda}\|^{\tau-1}_{E}\|u_{n,\lambda}-u_\lambda\|_{\tau}\to 0.
\end{aligned}
$$

Similarly, we have

$$\int_{\mathbb{R}^3}\tilde{f}(u_\lambda)(u_{n,\lambda}-u_\lambda)\mathrm{d}x\to 0,$$

thus $J_4\to 0$ as $n\to\infty$. We see that

$$\sigma\|u_{n,\lambda}-u_\lambda\|_E\le\|u_{n,\lambda}-u_\lambda\|^2_{H^s_\sigma}\to 0,$$

therefore, $I_{\sigma,\lambda}$ satisfies the Palais-Smale condition on E for fixed $\sigma\in(0,1]$. □

Lemma 3.4 *For every fixed $\lambda\ge 1$, let $\sigma_n\to 0$ and $\{u_{n,\lambda}\}\subset E$ be a sequence of critical points of $I_{\sigma_n,\lambda}$ satisfying $I'_{\sigma_n,\lambda}(u_{n,\lambda})=0$ and $I_{\sigma_n,\lambda}(u_{n,\lambda})\le C_\lambda$ for some C_λ independent of n. Then, up to a subsequence $u_{n,\lambda}\rightharpoonup u_\lambda$ in $H^s(\mathbb{R}^3)$ as $n\to\infty$ and u_λ is a critical point of I_λ.*

Proof By $I'_{\sigma_n,\lambda}(u_{n,\lambda})=0$ and $I_{\sigma_n,\lambda}(u_{n,\lambda})\le C_\lambda$, we have

$$
\begin{aligned}
C_\lambda\ge& I_{\sigma_n,\lambda}(u_{n,\lambda})-\frac{1}{\theta}(I'_{\sigma_n,\lambda}(u_{n,\lambda}),u_{n,\lambda})\\
=&(\frac{1}{2}-\frac{1}{\theta})\|u_{n,\lambda}\|^2_{H^s_\sigma}+(\frac{1}{2}-\frac{1}{\theta})\|u_{n,\lambda}\|^2_2+\\
&(\frac{1}{4}-\frac{1}{\theta})\int_{\mathbb{R}^3}\phi^{\alpha}_{u_{n,\lambda}}u^2_{n,\lambda}\mathrm{d}x+\int_{\mathbb{R}^3}(\frac{u_{n,\lambda}\tilde{f}(u_{n,\lambda})}{\theta}-\tilde{F}(u_{n,\lambda}))\mathrm{d}x\\
\ge&(\frac{1}{2}-\frac{1}{\theta})\|u_{n,\lambda}\|^2_{H^s},
\end{aligned}
$$

then up to a subsequence, we have $u_{n,\lambda}\rightharpoonup u_\lambda$ in $H^s(\mathbb{R}^3)$. Taking $v\in C_0^\infty(\mathbb{R}^3)$, by [21, Lemma 2.3], $\phi^{\alpha}_{u_{n,\lambda}}\rightharpoonup\phi^{\alpha}_{u_\lambda}$ in $\mathcal{D}^{\alpha,2}(\mathbb{R}^3)$, then we have

$$\int_{\mathbb{R}^3}\phi^{\alpha}_{u_{n,\lambda}}u_\lambda v\mathrm{d}x\to\int_{\mathbb{R}^3}\phi^{\alpha}_{u_\lambda}u_\lambda v\mathrm{d}x.$$

By Hölder inequality, it follows that

$$\Big|\int_{\mathbb{R}^3}\phi^{\alpha}_{u_{n,\lambda}}(u_n-u)v\mathrm{d}x\Big|\le\|\phi^{\alpha}_{u_{n,\lambda}}\|_{2^*_\alpha}\|u_n-u\|_{\frac{12}{3+2\alpha}(\Omega)}\|v\|_{\frac{12}{3+2\alpha}(\Omega)}\to 0$$

as $n \to \infty$, where Ω is the support of v. Then,

$$\begin{aligned}&\Big|\int_{\mathbb{R}^3}\phi^{\alpha}_{u_{n,\lambda}}u_{n,\lambda}v\mathrm{d}x-\int_{\mathbb{R}^3}\phi^{\alpha}_{u_{\lambda}}u_{\lambda}v\mathrm{d}x\Big|\\&\quad\le\Big|\int_{\mathbb{R}^3}\phi^{\alpha}_{u_{n,\lambda}}(u_{n,\lambda}-u_{\lambda})v\mathrm{d}x\Big|+\Big|\int_{\mathbb{R}^3}\phi^{\alpha}_{u_{n,\lambda}}(u_n-u)v\mathrm{d}x\Big|\to 0\end{aligned}$$

as $n \to \infty$ for all $v \in C_0^{\infty}(\mathbb{R}^3)$.

$$\begin{aligned}0=(I'_{\sigma_n,\lambda}(u_{n,\lambda}),v)=&\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}u_{n,\lambda}(-\Delta)^{\frac{s}{2}}v+u_{n,\lambda}v\mathrm{d}x\\&\int_{\mathbb{R}^3}(\phi^{\alpha}_{u_{n,\lambda}}u_{n,\lambda}v-\tilde{f}(u_{n,\lambda})v)\mathrm{d}x+\sigma_n\int_{\mathbb{R}^N}V(x)u_{n,\lambda}v\mathrm{d}x.\end{aligned}$$

We have

$$\sigma_n\int_{\mathbb{R}^3}V(x)u_{n,\lambda}v\mathrm{d}x\le\sigma_n^{\frac{1}{2}}\Big(\sigma_n\int_{\mathbb{R}^3}V(x)u_{n,\lambda}^2\mathrm{d}x\Big)^{\frac{1}{2}}\Big(\int_{\mathbb{R}^3}V(x)v^2\mathrm{d}x\Big)^{\frac{1}{2}}\le C_{\lambda}\sigma_n^{\frac{1}{2}}\to 0,$$

as $n \to \infty$. Thus by density, we see that $I'_{\lambda}(u_{\lambda})v = 0$ for all $v \in H^s(\mathbb{R}^3)$, u_{λ} is a critical point of I_{λ}. □

Lemma 3.5 ([22]) *Let $B_r(x)$ be the open ball in $\mathbb{R}^3$ of radius r centred at x. If $\{u_n\}$ is bounded in $H^s(\mathbb{R}^3)$ and for $2 \le q < 2_s^*$, we have*

$$\sup_{x\in\mathbb{R}^3}\int_{B_r(x)}|u_n|^q\mathrm{d}x\to 0\quad\text{as}\quad n\to\infty,$$

then $u_n \to 0$ in $L^w(\mathbb{R}^3)$ for $w \in (2, 2_s^*)$.

By Lemma 3.2, we have for fixed $v \in C_0^{\infty}(\mathbb{R}^3)$,

$$I_{\sigma,\lambda}(tv)\le\frac{t^2}{2}\|v\|_E^2+\frac{t^2}{2}\|v\|_2^2+\frac{Ct^4}{4}\|v\|_{H^s}^4-Ct^{\theta}\|v\|_{\theta}^{\theta}\to-\infty$$

as $t \to \infty$. We denote

$$c_{\sigma,\lambda}=\inf_{\gamma\in\Gamma}\max_{t\in[0,1]}I_{\sigma,\lambda}(\gamma(t)),$$

where $\Gamma = \{\gamma \in C([0,1], E) : \gamma(0) = 0, I_{\sigma,\lambda}(\gamma(1)) < 0\}$. Then we obtain

$$\begin{aligned}c_{\sigma,\lambda}&=\inf_{\gamma\in\Gamma}\max_{t\in[0,1]}I_{\sigma,\lambda}(\gamma(t))\le\max_{t\in[0,1]}I_{1,\lambda}(tTv)\\&\le\max_{t\in[0,1]}\Big(\frac{t^2T^2}{2}\|v\|_E^2+\frac{t^2T^2}{2}\|v\|_2^2+\frac{t^4T^4}{4}\|v\|_{H^s}^4-\lambda\int_{\mathbb{R}^3}\tilde{F}(tTv)\mathrm{d}x\Big)\\&\le\max_{t\in[0,1]}\Big(\frac{t^2T^2}{2}\|v\|_E^2+\frac{t^2T^2}{2}\|v\|_2^2+\frac{t^2T^2}{4}\|v\|_{H^s}^4-C\lambda t^{\beta}T^{\beta}\int_{\mathbb{R}^3}|v|^{\beta}\mathrm{d}x\Big)\\&\le C\lambda^{-\frac{2}{\beta-2}},\end{aligned}$$

where T is given by Lemma 3.2, C is independent of σ and λ. By Mountain pass theorem, $c_{\sigma,\lambda}$ is a critical point of $I_{\sigma,\lambda}$. Then, we can choose a sequence $\sigma_n \to 0$, a sequence of critical points $\{u_{n,\lambda}\} \subset E$ satisfying

$$I'_{\sigma_n,\lambda}(u_{n,\lambda})=0,\quad I_{\sigma_n,\lambda}(u_{n,\lambda})\le C\lambda^{-\frac{2}{\beta-2}}.\tag{3.3}$$

By Lemma 3.4, u_λ is a critical point of I_λ. It remains to show that $u_\lambda \neq 0$. We derive

$$\begin{aligned}0 =&\|u_{n,\lambda}\|^2_{H^s(\mathbb{R}^3)} + \sigma_n \int_{\mathbb{R}^3} V(x)u^2_{n,\lambda}\mathrm{d}x+ \\ &\int_{\mathbb{R}^3} \phi^\alpha_{u_{n,\lambda}} u^2_{n,\lambda}\mathrm{d}x - \lambda \int_{\mathbb{R}^3} \tilde{f}(u_{n,\lambda})u_{n,\lambda}\mathrm{d}x \\ \geq&\|u_{n,\lambda}\|^2_{H^s(\mathbb{R}^3)} - C\lambda\|u_{n,\lambda}\|^\tau_\tau \\ \geq& C\|u_{n,\lambda}\|^2_\tau - C\lambda\|u_{n,\lambda}\|^\tau_\tau.\end{aligned}$$

Hence, we have $\|u_{n,\lambda}\|_\tau \geq (\frac{C}{\lambda})^{\frac{1}{\tau-2}}$. If $\{u_{n,\lambda}\}$ is vanishing, then $\|u_{n,\lambda}\|_\tau \to 0$ by Lemma 3.5, which is a contradiction. Therefore, we can obtain the existence of nontrivial critical point of I_λ for every $\lambda \geq 1$.

Next we shall study the L^∞-estimates for solution u_λ of problem (2.8).

Lemma 3.6 *Let $u_\lambda \in E$ be a weak solution of problem (2.8). Then $u_\lambda \in L^\infty(\mathbb{R}^3)$. Moreover,*

$$\|u_\lambda\|_\infty < C\lambda^{\frac{\beta-2^*_s}{(2^*_s-\tau)(\beta-2)}},$$

C is independent of λ.

Proof Let u_λ be a weak solution of

$$\begin{cases} (-\Delta)^s u + u + \phi u = \lambda\tilde{f}(u) \text{ in } \mathbb{R}^3, \\ (-\Delta)^\alpha \phi = u^2 \text{ in } \mathbb{R}^3. \end{cases} \tag{3.4}$$

For $L > 0$, set $u_{\lambda,L} = \min\{u_\lambda, L\}$ and $\Upsilon(u_\lambda) := \Upsilon_{\lambda,L}(u_\lambda) = u_\lambda|u_{\lambda,L}|^{2(\eta-1)}$ with $\eta > 1$ to be determined later. Let $\Phi(t) = \frac{1}{2}|t|^2$ and $\Gamma(t) = \int_0^t (\Upsilon'(t))^{\frac{1}{2}}$. Then, if $a > b$, we have

$$\begin{aligned}\Phi'(a-b)(\Upsilon(a)-\Upsilon(b)) = (a-b)(\Upsilon(a)-\Upsilon(b)) = (a-b)\int_b^a (\Gamma'(t))^2\mathrm{d}x \\ \geq \Big(\int_b^a \Gamma'(t)\mathrm{d}x\Big)^2 = |\Gamma(a)-\Gamma(b)|^2.\end{aligned}$$

If $a \leq b$, we can use a similar argument to obtain the conclusion. It follows that

$$\Phi'(a-b)(\Upsilon(a)-\Upsilon(b)) \geq |\Gamma(a)-\Gamma(b)|^2,$$

for every $a, b \in \mathbb{R}$, which implies that

$$|\Gamma(u_\lambda(x)) - \Gamma(u_\lambda(y))|^2 \leq [u_\lambda(x) - u_\lambda(y)][(u_\lambda|u_{\lambda,L}|^{2(\eta-1)})(x) - (u_\lambda|u_{\lambda,L}|^{2(\eta-1)})(y)].$$

Choosing $\Upsilon(u_\lambda)$ as a test function in (2.10), we obtain

$$\begin{aligned}&\int_{\mathbb{R}^3} |(-\Delta)^{\frac{s}{2}}\Gamma(u_\lambda)|^2\mathrm{d}x + \int_{\mathbb{R}^3} u^2_\lambda|u_{\lambda,L}|^{2(\eta-1)}\mathrm{d}x + \int_{\mathbb{R}^3} \phi^\alpha_{u_\lambda} u^2_\lambda|u_{\lambda,L}|^{2(\eta-1)}\mathrm{d}x \\ &\leq \int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \frac{[u_\lambda(x)-u_\lambda(y)][(u_\lambda|u_{\lambda,L}|^{2(\eta-1)})(x) - (u_\lambda|u_{\lambda,L}|^{2(\eta-1)})(y)]}{|x-y|^{3+2s}}\mathrm{d}x\mathrm{d}y+ \\ &\quad \int_{\mathbb{R}^3} u^2_\lambda|u_{\lambda,L}|^{2(\eta-1)}\mathrm{d}x + \int_{\mathbb{R}^3} \phi^\alpha_{u_\lambda} u^2_\lambda|u_{\lambda,L}|^{2(\eta-1)}\mathrm{d}x \\ &= \lambda\int_{\mathbb{R}^3} \tilde{f}(u_\lambda)u_\lambda|u_{\lambda,L}|^{2(\eta-1)}\mathrm{d}x,\end{aligned}$$

we get

$$\int_{\mathbb{R}^3} |(-\Delta)^{\frac{s}{2}}\Gamma(u_\lambda)|^2 \mathrm{d}x \le C\lambda \int_{\mathbb{R}^3} |u_\lambda|^\tau |u_{\lambda,L}|^{2(\eta-1)}\mathrm{d}x.$$

Since $|\Gamma(u_\lambda)| \ge \frac{1}{\eta}|u_\lambda||u_{\lambda,L}|^{\eta-1}$, we have

$$\int_{\mathbb{R}^3} |(-\Delta)^{\frac{s}{2}}\Gamma(u_\lambda)|^2 \mathrm{d}x \ge C\Big(\int_{\mathbb{R}^3} |\Gamma(u_\lambda)|^{2_s^*}\mathrm{d}x\Big)^{\frac{2}{2_s^*}} \ge \frac{C}{\eta^2}\|u_\lambda|u_{\lambda,L}|^{\eta-1}\|_{2_s^*}^2.$$

Therefore,

$$\|u_\lambda|u_{\lambda,L}|^{\eta-1}\|_{2_s^\alpha}^2 \le C\eta^2\lambda \int_{\mathbb{R}^3} |u_\lambda|^\tau |u_{\lambda,L}|^{2(\eta-1)}\mathrm{d}x.$$

Let $\tau_s^* = \frac{22_s^*}{2_s^*-\tau+2}$. We have

$$\begin{aligned}\|u_\lambda|u_{\lambda,L}|^{\eta-1}\|_{2_s^*}^2 &\le C\eta^2\lambda \int_{\mathbb{R}^3} |u_\lambda|^\tau |u_{\lambda,L}|^{2(\eta-1)}\mathrm{d}x \\ &\le C\eta^2\lambda \int_{\mathbb{R}^3} |u_\lambda|^{\tau-2} u_\lambda^2 |u_{\lambda,L}|^{2(\eta-1)}\mathrm{d}x \\ &\le C\eta^2\lambda \|u_\lambda\|_{2_s^*}^{\tau-2}\Big(\int_{\mathbb{R}^3} |u_\lambda|u_{\lambda,L}|^{\eta-1}|^{\tau_s^*}\mathrm{d}x\Big)^{\frac{2}{\tau_s^*}} \\ &\le C\eta^2\lambda\|u_\lambda\|^{\tau-2}\|u_\lambda\|_{\eta\tau_s^*}^{2\eta}.\end{aligned}$$

Using the Fatou's lemma, letting $L \to \infty$, it follows that

$$\|u_\lambda\|_{2_s^*\eta} \le (C\eta^2\lambda\|u_\lambda\|^{\tau-2})^{\frac{1}{2\eta}}\|u_\lambda\|_{\eta\tau_s^*}.$$

Define $\eta_{n+1}\tau_s^* = 2_s^*\eta_n$, where $n = 1, 2, \dots$ and $\eta_1 = \frac{2_s^*+2-\tau}{2}$. We have

$$\begin{aligned}\|u_\lambda\|_{2_s^*\eta_2} &\le (C\eta_2^2\lambda\|u_\lambda\|^{\tau-2})^{\frac{1}{2\eta_2}}\|u_\lambda\|_{2_s^*\eta_1} \\ &\le (C\lambda\|u_\lambda\|^{\tau-2})^{\frac{1}{2\eta_1}+\frac{1}{2\eta_2}}\eta_1^{\frac{1}{\eta_1}}\eta_2^{\frac{1}{\eta_2}}\|u_\lambda\|_{2_s^*}.\end{aligned}$$

By the elementary calculus, we know that

$$\sum_{i=1}^{\infty}\frac{1}{\eta_i} = \frac{1}{\eta_1 - 1}, \quad \sum_{i=1}^{\infty}\frac{i}{\eta_1^i} = \eta_1^{\frac{\eta_1}{(\eta_1-1)^2}}.$$

By iteration we have $\|u_\lambda\|_\infty \le C\lambda^{\frac{1}{2_s^*-\tau}}\|u_\lambda\|_{H^s}^{\frac{2_s^*-2}{2_s^*-\tau}}$. Since u_λ is a weak solution of (2.8), by (3.3),

$$\|u_\lambda\|_{H^s}^2 \le \liminf_{n\to\infty}\|u_{n,\lambda}\|_{H^s}^2 \le C(I_{\sigma_n,\lambda}(u_{n,\lambda}) - \frac{1}{\theta}(I'_{\sigma_n,\lambda}(u_{n,\lambda}), u_{n,\lambda})) \le C\lambda^{-\frac{2}{\beta-2}},$$

then we have $\|u_\lambda\|_{H^s} \le C\lambda^{-\frac{1}{\beta-2}}$. Thus,

$$\|u_\lambda\|_\infty \le C\lambda^{\frac{\beta-2_s^*}{(2_s^*-\tau)(\beta-2)}},$$

we complete the proof. □

Proof of Theorem 1.1 By Lemma 3.6, there exists $\lambda_1 \ge 1$ such that for all $\lambda > \lambda_1$,

$$\|u_\lambda\|_\infty \le \delta,$$

where δ is fixed in (2.7). Thus, u_λ is a nontrivial solution of the original problem (1.1). □

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