Journal of Mathematical Research with Applications
Jul., 2022, Vol. 42, No. 4, pp. 374–380
DOI:10.3770/j.issn:2095-2651.2022.04.004
Http://jmre.dlut.edu.cn

Nonlinear Maps Preserving Mixed Jordan Triple Products on von Neumann Algebras

Dongfang ZHANG, Changjing LI*, Yuanyuan ZHAO
School of Mathematics and Statistics, Shandong Normal University, Shandong 250014*, P. R. China*

Abstract In this paper, we prove that if a bijective map Φ preserves mixed Jordan triple products between von Neumann algebras with no central abelian projections, then $\Phi(I)\Phi$ is the sum of a linear $*$-isomorphism and a conjugate linear $*$-isomorphism, where $\Phi(I)$ is a self-adjoint central element in the range with $\Phi(I)^2 = I$. Also, we give the structure of this map that preserves mixed Jordan triple products between factor von Neumann algebras.

Keywords mixed Jordan triple product; isomorphism; von Neumann algebras

MR(2020) Subject Classification 46L10; 47B49

1. Introduction

Let $\mathcal{A}$ and $\mathcal{B}$ be two $*$-algebras over the complex field $\mathbb{C}$. For $A, B \in \mathcal{A}$, define the Jordan product of A and B by $A \circ B = AB + BA$ and the Jordan $*$-product of A and B by $A \bullet B = AB + BA^*$. We say that a map $\Phi : \mathcal{A} \to \mathcal{B}$ preserves mixed Jordan triple product if $\Phi(A \bullet B \circ C) = \Phi(A) \bullet \Phi(B) \circ \Phi(C)$ for all $A, B, C \in \mathcal{A}$. This kind of maps are related to maps preserving Jordan product and maps preserving Jordan $*$-product which have been studied by many authors [1–7].

Recently, many authors studied the nonlinear maps preserving some mixed products [8–15]. For example, Li et al. studied the nonlinear maps preserving skew Lie triple products $[[A, B]_*, C]_*$ (see [9, 11]) and Jordan triple $*$-products $A \bullet B \bullet C$ (see [10, 15]) on von Neumann algebras. Yang and Zhang in [12, 13] studied the nonlinear maps preserving mixed skew Lie triple products $[[A, B]_*, C]$ and $[[A, B], C]_*$ on factor von Neumann algebras. In the present paper, we will establish the structure of nonlinear maps preserving mixed Jordan triple products $A \bullet B \circ C$ on von Neumann algebras.

Before stating the main results, we need some notations and preliminaries. A von Neumann algebra $\mathcal{A}$ is a weakly closed, self-adjoint algebra of operators on a Hilbert space H containing the identity operator I. The set $\mathcal{Z}(\mathcal{A}) = \{S \in \mathcal{A} : ST = TS \text{ for all } T \in \mathcal{A}\}$ is called the center of $\mathcal{A}$. A projection P is called a central abelian projection if $P \in \mathcal{Z}(\mathcal{A})$ and $P\mathcal{A}P$ is abelian.

Received September 24, 2021; Accepted February 19, 2022
Supported by the Natural Science Foundation of Shandong Province (Grant No. ZR2018BA003) and the National Natural Science Foundation of China (Grant No. 11801333).
* Corresponding author
E-mail address: 1776767307@qq.com (Dongfang ZHANG); lcjbxh@163.com (Changjing LI); 2934377003@qq.com (Yuanyuan ZHAO)

Recall that the central carrier of A, denoted by $\overline{A}$, is the smallest central projection P satisfying $PA = A$. It is not difficult to see that the central carrier of A is the projection onto the closed subspace spanned by $\{BA(x) : B \in \mathcal{A}, x \in H\}$. If A is self-adjoint, then the core of A, denoted by $\underline{A}$, is $\sup\{S \in \mathcal{Z}(\mathcal{A}) : S = S^*, S \leq A\}$. If P is a projection, it is clear that $\underline{P}$ is the largest central projection Q satisfying $Q \leq P$. A projection P is said to be core-free if $\underline{P} = 0$. It is easy to see that $\underline{P} = 0$ if and only if $\overline{I - P} = I$.

Lemma 1.1 ([16]) *Let $\mathcal{A}$ be a von Neumann algebra with no central abelian projections. Then there exists a projection $P \in \mathcal{A}$ such that $\underline{P} = 0$ and $\overline{P} = I$.*

Lemma 1.2 ([2]) *Let $\mathcal{A}$ be a von Neumann algebra and P be a projection in $\mathcal{A}$ with $\overline{P} = I$. If $ABP = 0$ for all $B \in \mathcal{A}$, then $A = 0$.*

Lemma 1.3 ([5]) *Let $\mathcal{A}$ be a von Neumann algebra and A be an element in $\mathcal{A}$. Then $AB + BA^* = 0$ for all $B \in \mathcal{A}$ implies that $A = -A^* \in \mathcal{Z}(\mathcal{A})$.*

2. Main results

Our main result in this paper reads as follows.

Theorem 2.1 *Let $\mathcal{A}$ and $\mathcal{B}$ be two von Neumann algebras with no central abelian projections. Suppose that a bijective map $\Phi : \mathcal{A} \to \mathcal{B}$ satisfies*

$$\Phi(A \bullet B \circ C) = \Phi(A) \bullet \Phi(B) \circ \Phi(C),$$

for all $A, B, C \in \mathcal{A}$. Then the map $\Phi(I)\Phi$ is the sum of a linear $*$-isomorphism and a conjugate linear $*$-isomorphism, where $\Phi(I)$ is a self-adjoint central element in $\mathcal{B}$ with $\Phi(I)^2 = I$.

Proof First we give a key technique. Suppose that $A_1, A_2, \ldots, A_n$ and T are in $\mathcal{A}$ such that $\Phi(T) = \sum_{i=1}^{n} \Phi(A_i)$. Then for all $S_1, S_2 \in \mathcal{A}$, we have

$$\Phi(S_1 \bullet S_2 \circ T) = \Phi(S_1) \bullet \Phi(S_2) \circ \Phi(T) = \sum_{i=1}^{n} \Phi(S_1 \bullet S_2 \circ A_i), \tag{2.1}$$

$$\Phi(S_1 \bullet T \circ S_2) = \Phi(S_1) \bullet \Phi(T) \circ \Phi(S_2) = \sum_{i=1}^{n} \Phi(S_1 \bullet A_i \circ S_2) \tag{2.2}$$

and

$$\Phi(T \bullet S_1 \circ S_2) = \Phi(T) \bullet \Phi(S_1) \circ \Phi(S_2) = \sum_{i=1}^{n} \Phi(A_i \bullet S_1 \circ S_2). \tag{2.3}$$

By Lemma 1.1, there exists a projection P such that $\underline{P} = 0$ and $\overline{P} = I$. Let $P_1 = P$ and $P_2 = I - P$. Denote $\mathcal{A}_{ij} = P_i \mathcal{A} P_j$. Then $\mathcal{A} = \sum_{i,j=1}^{2} \mathcal{A}_{ij}$. In all that follows, when we write A_{ij}, it indicates that $A_{ij} \in \mathcal{A}_{ij}$. The proof will be organized in some claims. In the following, we will show the additivity of Φ.

Claim 2.2 $\Phi(0) = 0$.

Since Φ is surjective, there exists $A \in \mathcal{A}$ such that $\Phi(A) = 0$. So

$$\Phi(0) = \Phi(0 \bullet A \circ A) = \Phi(0) \bullet 0 \circ 0 = 0.$$

Claim 2.3 For every $A_{12} \in \mathcal{A}_{12}$, $B_{21} \in \mathcal{A}_{21}$, we have

$$\Phi(A_{12} + B_{21}) = \Phi(A_{12}) + \Phi(B_{21}).$$

Choose $T = \sum_{i,j=1}^{2} T_{ij} \in \mathcal{A}$ such that $\Phi(T) = \Phi(A_{12}) + \Phi(B_{21})$. Since

$$(P_2 - P_1) \bullet I \circ A_{12} = (P_2 - P_1) \bullet I \circ B_{21} = 0,$$

it follows from Eq. (2.1) and Claim 2.2 that

$$\Phi((P_2 - P_1) \bullet I \circ T) = 0.$$

From this, we get $(P_2 - P_1) \bullet I \circ T = 0$. So $T_{11} = T_{22} = 0$. Since $A_{12} \bullet P_1 \circ I = 0$, it follows from Eq. (2.3) that

$$\Phi(T \bullet P_1 \circ I) = \Phi(B_{21} \bullet P_1 \circ I).$$

By the injectivity of Φ, we obtain

$$2(P_1 T^* + T P_1) = T \bullet P_1 \circ I = B_{21} \bullet P_1 \circ I = 2(B_{21}^* + B_{21}).$$

Hence $T_{21} = B_{21}$. Similarly, $T_{12} = A_{12}$, proving the claim.

Claim 2.4 For every $A_{11} \in \mathcal{A}_{11}$, $B_{12} \in \mathcal{A}_{12}$, $C_{21} \in \mathcal{A}_{21}$, $D_{22} \in \mathcal{A}_{22}$, we have

$$\Phi(A_{11} + B_{12} + C_{21}) = \Phi(A_{11}) + \Phi(B_{12}) + \Phi(C_{21})$$

and

$$\Phi(B_{12} + C_{21} + D_{22}) = \Phi(B_{12}) + \Phi(C_{21}) + \Phi(D_{22}).$$

Choose $T = \sum_{i,j=1}^{2} T_{ij} \in \mathcal{A}$ such that

$$\Phi(T) = \Phi(A_{11}) + \Phi(B_{12}) + \Phi(C_{21}).$$

It follows from Eq. (2.1) and Claim 2.3 that

$$\begin{aligned}\Phi(2(P_2 T + T P_2)) &= \Phi(P_2 \bullet I \circ T) \\ &= \Phi(P_2 \bullet I \circ A_{11}) + \Phi(P_2 \bullet I \circ B_{12}) + \Phi(P_2 \bullet I \circ C_{21}) \\ &= \Phi(2B_{12}) + \Phi(2C_{21}) = \Phi(2(B_{12} + C_{21})).\end{aligned}$$

Thus $P_2 T + T P_2 = B_{12} + C_{21}$, which implies that $T_{22} = 0$, $T_{12} = B_{12}$, $T_{21} = C_{21}$. Now we get $T = T_{11} + B_{12} + C_{21}$. Since

$$(P_2 - P_1) \bullet I \circ B_{12} = (P_2 - P_1) \bullet I \circ C_{21} = 0,$$

it follows from Eq. (2.1) that

$$\Phi((P_2 - P_1) \bullet I \circ T) = \Phi((P_2 - P_1) \bullet I \circ A_{11}),$$

from which we get $T_{11} = A_{11}$. Consequently, $\Phi(A_{11} + B_{12} + C_{21}) = \Phi(A_{11}) + \Phi(B_{12}) + \Phi(C_{21})$. Similarly, we can get that $\Phi(B_{12} + C_{21} + D_{22}) = \Phi(B_{12}) + \Phi(C_{21}) + \Phi(D_{22})$.

Claim 2.5 For every $A_{11} \in \mathcal{A}_{11}$, $B_{12} \in \mathcal{A}_{12}$, $C_{21} \in \mathcal{A}_{21}$, $D_{22} \in \mathcal{A}_{22}$, we have

$$\Phi(A_{11} + B_{12} + C_{21} + D_{22}) = \Phi(A_{11}) + \Phi(B_{12}) + \Phi(C_{21}) + \Phi(D_{22}).$$

Choose $T = \sum_{i,j=1}^{2} T_{ij} \in \mathcal{A}$ such that

$$\Phi(T) = \Phi(A_{11}) + \Phi(B_{12}) + \Phi(C_{21}) + \Phi(D_{22}).$$

It follows from Eq. (2.1) and Claim 2.4 that

$$\begin{aligned}
\Phi(2(P_1T + TP_1)) &= \Phi(P_1 \bullet I \circ T) \\
&= \Phi(P_1 \bullet I \circ A_{11}) + \Phi(P_1 \bullet I \circ B_{12}) + \Phi(P_1 \bullet I \circ C_{21}) + \Phi(P_1 \bullet I \circ D_{22}) \\
&= \Phi(4A_{11}) + \Phi(2B_{12}) + \Phi(2C_{21}) \\
&= \Phi(2(2A_{11} + B_{12} + C_{21})).
\end{aligned}$$

Thus

$$P_1T + TP_1 = 2A_{11} + B_{12} + C_{21}$$

and then $T_{11} = A_{11}$, $T_{12} = B_{12}$, $T_{21} = C_{21}$. Similarly, we can get

$$\Phi(2(P_2T + TP_2)) = \Phi(2(B_{12} + C_{21} + 2D_{22})).$$

From this, we get $T_{22} = D_{22}$, proving the claim.

Claim 2.6 For every $A_{jk}, B_{jk} \in \mathcal{A}_{jk}$, $1 \le j \ne k \le 2$, we have

$$\Phi(A_{jk} + B_{jk}) = \Phi(A_{jk}) + \Phi(B_{jk}).$$

For every $A_{jk}, B_{jk} \in \mathcal{A}_{jk}$, since

$$\frac{I}{2} \bullet (P_j + A_{jk}) \circ (P_k + B_{jk}) = A_{jk} + B_{jk},$$

we get from Claim 2.5 that

$$\begin{aligned}
\Phi(A_{jk} + B_{jk}) &= \Phi(\frac{I}{2} \bullet (P_j + A_{jk}) \circ (P_k + B_{jk})) \\
&= \Phi(\frac{I}{2}) \bullet \Phi(P_j + A_{jk}) \circ \Phi(P_k + B_{jk}) \\
&= \Phi(\frac{I}{2}) \bullet (\Phi(P_j) + \Phi(A_{jk})) \circ (\Phi(P_k) + \Phi(B_{jk})) \\
&= \Phi(\frac{I}{2}) \bullet \Phi(P_j) \circ \Phi(P_k) + \Phi(\frac{I}{2}) \bullet \Phi(P_j) \circ \Phi(B_{jk}) + \\
&\quad \Phi(\frac{I}{2}) \bullet \Phi(A_{jk}) \circ \Phi(P_k) + \Phi(\frac{I}{2}) \bullet \Phi(A_{jk}) \circ \Phi(B_{jk}) \\
&= \Phi(B_{jk}) + \Phi(A_{jk}),
\end{aligned}$$

which implies that $\Phi(A_{jk} + B_{jk}) = \Phi(A_{jk}) + \Phi(B_{jk})$.

Claim 2.7 For every $A_{jj}, B_{jj} \in \mathcal{A}_{jj}$, $1 \le j \le 2$, we have

$$\Phi(A_{jj} + B_{jj}) = \Phi(A_{jj}) + \Phi(B_{jj}).$$

Let $T=\sum_{i,j=1}^{2}T_{ij}\in\mathcal{A}$ such that

$$\Phi(T)=\Phi(A_{jj})+\Phi(B_{jj}).$$

For $1\le j\ne k\le 2$, it follows from Eq. (2.1) that

$$\Phi(P_k\bullet I\circ T)=\Phi(P_k\bullet I\circ A_{jj})+\Phi(P_k\bullet I\circ B_{jj})=0.$$

Hence $P_kT+TP_k=0$, which implies $T_{jk}=T_{kj}=T_{kk}=0$. Now we get $T=T_{jj}$. For every $C_{jk}\in\mathcal{A}_{jk}$, $j\ne k$, it follows that

$$\begin{aligned}\Phi(2T_{jj}C_{jk})&=\Phi(P_j\bullet T_{jj}\circ C_{jk})\\&=\Phi(P_j\bullet A_{jj}\circ C_{jk})+\Phi(P_j\bullet B_{jj}\circ C_{jk})\\&=\Phi(2A_{jj}C_{jk})+\Phi(2B_{jj}C_{jk})\\&=\Phi(2(A_{jj}C_{jk}+B_{jj}C_{jk})).\end{aligned}$$

Hence

$$(T_{jj}-A_{jj}-B_{jj})C_{jk}=0,$$

for all $C_{jk}\in\mathcal{A}_{jk}$, that is, $(T_{jj}-A_{jj}-B_{jj})CP_k=0$ for all $C\in\mathcal{A}$. It follows from Lemma 1.2 that $T_{jj}=A_{jj}+B_{jj}$, proving the claim.

Claim 2.8 Φ is additive.

Let $A=\sum_{i,j=1}^{2}A_{ij}$, $B=\sum_{i,j=1}^{2}B_{ij}\in\mathcal{A}$. By Claims 2.5–2.7, we have

$$\begin{aligned}\Phi(A+B)&=\Phi\Big(\sum_{i,j=1}^{2}A_{ij}+\sum_{i,j=1}^{2}B_{ij}\Big)=\Phi\Big(\sum_{i,j=1}^{2}(A_{ij}+B_{ij})\Big)\\&=\sum_{i,j=1}^{2}\Phi(A_{ij}+B_{ij})=\sum_{i,j=1}^{2}\Phi(A_{ij})+\sum_{i,j=1}^{2}\Phi(B_{ij})\\&=\Phi\Big(\sum_{i,j=1}^{2}A_{ij}\Big)+\Phi\Big(\sum_{i,j=1}^{2}B_{ij}\Big)=\Phi(A)+\Phi(B).\end{aligned}$$

Claim 2.9 For each $A\in\mathcal{A}$, $A=-A^*$ if and only if $\Phi(A)=-\Phi(A)^*$.

Let $A\in\mathcal{A}$ be arbitrary. Since Φ is surjective, there exists $B\in\mathcal{A}$ such that $\Phi(B)=I$. Then

$$\begin{aligned}0&=\Phi(iI\bullet A\circ B)=\Phi(iI)\bullet\Phi(A)\circ I\\&=2(\Phi(iI)\Phi(A)+\Phi(A)\Phi(iI)^*)\end{aligned}$$

holds true for all $A\in\mathcal{A}$. So $\Phi(iI)C+C\Phi(iI)^*=0$ holds true for all $C\in\mathcal{B}$. It follows from Lemma 1.3 that $\Phi(iI)=-\Phi(iI)^*\in\mathcal{Z}(\mathcal{B})$. Similarly, $\Phi^{-1}(iI)\in\mathcal{Z}(\mathcal{A})$.

Let $A=-A^*\in\mathcal{A}$ and $\Phi(B)=I$. It follows that

$$0=\Phi(A\bullet\Phi^{-1}(iI)\circ B)=\Phi(A)\bullet(iI)\circ I=2i(\Phi(A)+\Phi(A)^*).$$

This implies that $\Phi(A)=-\Phi(A)^*$. Similarly, if $\Phi(A)=-\Phi(A)^*$, then

$$0=\Phi^{-1}(\Phi(A)\bullet\Phi(iI)\circ\Phi(I))=A\bullet(iI)\circ I=2i(A+A^*),$$

and so $A = -A^*$.

Claim 2.10 $\Phi(\mathcal{Z}(\mathcal{A})) = \mathcal{Z}(\mathcal{B})$.

Let $Z \in \mathcal{Z}(\mathcal{A})$ be arbitrary and $\Phi(B) = I$. For every $A = -A^* \in \mathcal{A}$, we have

$$0 = \Phi(A \bullet Z \circ B) = \Phi(A) \bullet \Phi(Z) \circ I = 2(\Phi(A)\Phi(Z) + \Phi(Z)\Phi(A)^*).$$

That is $\Phi(A)\Phi(Z) = -\Phi(Z)\Phi(A)^*$ holds true for all $A = -A^* \in \mathcal{A}$. Since Φ preservers conjugate self-adjoint elements, it follows that $C\Phi(Z) = \Phi(Z)C$ holds true for all $C = -C^* \in \mathcal{B}$. Since for every $C \in \mathcal{B}$, we have $C = C_1 + iC_2$, where $C_1 = \frac{C-C^*}{2}$ and $C_2 = \frac{C+C^*}{2i}$ are conjugate self-adjoint elements. Hence $C\Phi(Z) = \Phi(Z)C$ holds true for all $C \in \mathcal{A}$. Then $\Phi(Z) \in \mathcal{Z}(\mathcal{B})$, which implies that $\Phi(\mathcal{Z}(\mathcal{A})) \subseteq \mathcal{Z}(\mathcal{B})$. Thus $\Phi(\mathcal{Z}(\mathcal{A})) = \mathcal{Z}(\mathcal{B})$ by considering Φ^{-1}.

Claim 2.11 $\Phi(I)$ is a self-adjoint central element in $\mathcal{B}$ with $\Phi(I)^2 = I$.

Let $\Phi(B) = I$. Since $\Phi(I) \in \mathcal{Z}(\mathcal{B})$, by Claim 2.8, we have

$$4I = 4\Phi(B) = \Phi(I \bullet I \circ B) = \Phi(I) \bullet \Phi(I) \circ I = 2\Phi(I)(\Phi(I) + \Phi(I)^*),$$

that is $\Phi(I)(\Phi(I) + \Phi(I)^*) = 2I$. Taking the adjoint, we have $\Phi(I)^*(\Phi(I) + \Phi(I)^*) = 2I$. Subtracting the above two equations, we get $(\Phi(I) - \Phi(I)^*)(\Phi(I) + \Phi(I)^*) = 0$. Note that $\Phi(I) + \Phi(I)^*$ is invertible, we get $\Phi(I) = \Phi(I)^*$. Also, since $\Phi(I)(\Phi(I) + \Phi(I)^*) = 2I$, we obtain $\Phi(I)^2 = I$.

Now, defining a map $\phi : \mathcal{A} \to \mathcal{B}$ by $\phi(A) = \Phi(I)\Phi(A)$ for all $A \in \mathcal{A}$. Then $\phi(I) = I$. For all $A, B \in \mathcal{A}$, by Claim 2.8, we have

$$2\phi(A \bullet B) = \phi(A \bullet B \circ I) = \phi(A) \bullet \phi(B) \circ I = 2\phi(A) \bullet \phi(B).$$

This implies that

$$\phi(A \bullet B) = \phi(A) \bullet \phi(B),$$

for all $A, B \in \mathcal{A}$. Now by the main theorem in [2], we have that ϕ is a sum of a linear $*$-isomorphism and a conjugate linear $*$-isomorphism. So $\Phi(I)\Phi$ is a sum of a linear $*$-isomorphism and a conjugate linear $*$-isomorphism. $\square$

$\mathcal{A}$ is a factor von Neumann algebra means that its center only contains the scalar operators. It is well known that the factor von Neumann algebra $\mathcal{A}$ is prime, in the sense that $A\mathcal{A}B = 0$ for $A, B \in \mathcal{A}$ implies either $A = 0$ or $B = 0$.

Corollary 2.12 Let $\mathcal{A}$ and $\mathcal{B}$ be two factor von Neumann algebras with $\dim\mathcal{A} \geq 2$. Suppose that a bijective map $\Phi : \mathcal{A} \to \mathcal{B}$ satisfies

$$\Phi(A \bullet B \circ C) = \Phi(A) \bullet \Phi(B) \circ \Phi(C),$$

for all $A, B, C \in \mathcal{A}$. Then Φ is a linear $*$-isomorphism, or a conjugate linear $*$-isomorphism, or the negative of a linear $*$-isomorphism, or the negative of a conjugate linear $*$-isomorphism.

Proof Let P be a nontrivial projection in $\mathcal{A}$. Since A is prime, $ABP = 0$ for all $B \in \mathcal{A}$ implies $A = 0$. So Lemma 1.2 holds true for factor von Neumann algebras. It is easy to check that all claims of Theorem 2.1 hold true for factor von Neumann algebras. Since $\Phi(I)$ is a self-adjoint central element and $\Phi(I)^2 = I$, we get $\Phi(I) = I$ or $\Phi(I) = -I$. So Φ or $-\Phi$ is a map preserving the product $A \bullet B$ on factor von Neumann algebras. Now, by the main result of [5], we have that Φ or $-\Phi$ is a $*$-ring isomorphism. It is easy to show that Φ or $-\Phi$ is a map preserving the absolute value. By [17, Theorem 2.5], Φ or $-\Phi$ is a linear $*$-isomorphism or a conjugate linear $*$-isomorphism. Now, we have proved the corollary. $\square$

Acknowledgements We thank the referees for their time and comments.

References

[1] M. BREŠAR. *Jodran mappings of semiprime rings*. J. Algebra, 1989, **127**: 218–228.

[2] Liqing DAI, Fangyan LU. *Nonlinear maps preserving Jordan $*$-products*. J. Math. Anal. Appl., 2014, **409**(1): 180–188.

[3] I. N. HERSTEIN. *Jordan homomorphisms*. Trans. Amer. Math. Soc., 1956, **81**: 331–341.

[4] Peisheng JI, Zhongyan LIU. *Additivity of Jordan maps on standard Jordan operator algebras*. Linear Algebra Appl., 2009, **430**(1): 335–343.

[5] Changjing LI, Fangyan LU. *Mappings preserving new product $XY + YX^*$ on factor von Neumann algebras*. Linear Algebra Appl., 2013, **438**: 2339–2345.

[6] Fangyan LU. *Additivity of Jordan maps on standard operator algebras*. Linear Algebra Appl., 2002, **357**: 123–131.

[7] Fangyan LU. *Jordan maps on associative algebras*. Comm. Algebra, 2003, **31**(5): 2273–2286.

[8] Donghua HUO, Baodong ZHENG, Hongyu LIU. *Nonlinear maps preserving Jordan triple η-$*$-products*. J. Math. Anal. Appl., 2015, **430**(2): 830–844.

[9] Changjing LI, Quanyuan CHEN, Ting WANG. *Nonlinear maps preserving the Jordan triple $*$-product on factors*. Chin. Ann. Math. Ser. B, 2018, **39**(4): 633–642.

[10] Changjing LI, Fangyan LU. *Nonlinear maps preserving the Jordan triple 1-$*$-product on von Neumann algebras*. Complex Anal. Oper. Theory, 2017, **11**(1): 109–117.

[11] Changjing LI, Fangyan LU. *Nonlinear maps preserving the Jordan triple $*$-product on von Neumann algebras*. Ann. Funct. Anal., 2016, **7**(3): 496–507.

[12] Zhujun YANG, Jinhua ZHANG. *Nonlinear maps preserving the mixed skew Lie triple product on factor von Neumann algebras*. Ann. Funct. Anal., 2019, **10**: 325–336.

[13] Zhujun YANG, Jinhua ZHANG. *Nonlinear maps preserving the second mixed skew Lie triple product on factor von Neumann algebras*. Linear Multilinear Algebra, 2020, **68**(2): 377–390.

[14] Yuanyuan ZHAO, Changjing LI, Quanyuan CHEN. *Nonlinear maps preserving the mixed product on factors*. Bull. Iran. Math. Soc., 2021, **47**: 1325–1335.

[15] Fangfang ZHAO, Changjing LI. *Nonlinear maps preserving the Jordan triple $*$-product between factors*. Indag. Math. (N.S.), 2018, **29**(2): 619–627.

[16] C. R. MIERS. *Lie homomorphisms of operator algebras*. Pacific J. Math., 1971, **38**: 717–735.

[17] A. TAGHAVI. *Additive mappings on C^*-algebras preseving absolute value*. Linear Multilinear Algebra, 2012, **60**: 33–38.