Journal of Mathematical Research with Applications
Jan., 2023, Vol. 43, No. 1, pp. 9–15
DOI:10.3770/j.issn:2095-2651.2023.01.002
Http://jmre.dlut.edu.cn

Least Common Multiple of Path, Star with Cartesian Product of Some Graphs

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Abstract A graph G without isolated vertices is a least common multiple of two graphs H_1 and H_2 if G is a smallest graph, in terms of number of edges, such that there exists a decomposition of G into edge disjoint copies of H_1 and H_2. The collection of all least common multiples of H_1 and H_2 is denoted by $\mathrm{LCM}(H_1, H_2)$ and the size of a least common multiple of H_1 and H_2 is denoted by $\mathrm{lcm}(H_1, H_2)$. In this paper $\mathrm{lcm}(P_4, P_m \,\Box\, P_n)$, $\mathrm{lcm}(P_4, C_m \,\Box\, C_n)$ and $\mathrm{lcm}(K_{1,3}, K_{1,m} \,\Box\, K_{1,n})$ are determined.

Keywords graph decomposition; least common multiple

MR(2020) Subject Classification 05C38; 05C51; 05C70

1. Introduction

All graphs considered in this paper are assumed to be simple and to have no isolated vertices. The number of vertices of a graph G denoted by $v(G)$, is called the order of G and the number of edges of G denoted by $e(G)$, is called the size of G.

A graph H is said to divide a graph G if there exists a set of subgraphs of G, each isomorphic to H, whose edge sets partition the edge set of G. Such a set of subgraphs is called an H-decomposition of G. If G has an H-decomposition, we say that G is H-decomposable and write $H|G$.

A graph is called a common multiple of two graphs H_1 and H_2 if both $H_1|G$ and $H_2|G$. A graph G is a least common multiple of H_1 and H_2 if G is a common multiple of H_1 and H_2 and no other common multiple has fewer edges. Several authors have investigated the problem of finding least common multiples of pairs of graphs H_1 and H_2; that is graphs of minimum size which are both H_1 and H_2 decomposable. The problem was introduced by Chartrand et al. in [1] and they showed that every two nonempty graphs have a least common multiple. The problem of finding the size of least common multiples of graphs has been studied for several pairs of graphs: cycles and stars [1–3], paths and complete graphs [4], pairs of complete graphs, complete graphs and a 4-cycle, paths and stars and pairs of cycles. Least common multiple of digraphs were considered in [5].

Received March 20, 2022; Accepted May 22, 2022
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If G is a common multiple of H_1 and H_2 and G has q edges, then we call G a (q, H_1, H_2) graph. An obvious necessary condition for the existence of a (q, H_1, H_2) graph is that $e(H_1)|q$ and $e(H_2)|q$. This obvious necessary condition is not always sufficient. Therefore, we may ask: Given two graphs H_1 and H_2, for which value of q does there exist a (q, H_1, H_2) graph? Adams, Bryant and Maenhaut [6] gave a complete solution to this problem in the case where H_1 is the 4-cycle and H_2 is a complete graph; Bryant and Maenhaut [7] gave a complete solution to this problem in the case where H_1 is the complete graph K_3 and H_2 is a complete graph. Thus the problem to find least common multiple of H_1 and H_2 is to find the least positive integer q such that there exists a (q, H_1, H_2) graph. We denote the set of all least common multiples of H_1 and H_2 by $\mathrm{LCM}(H_1, H_2)$. The size of a least common multiple of H_1 and H_2 is denoted by $\mathrm{lcm}(H_1, H_2)$. Since every two nonempty graphs have a least common multiple, $\mathrm{LCM}(H_1, H_2)$ is nonempty. For many pairs of graphs number of elements of $\mathrm{LCM}(H_1, H_2)$ is greater than one. For example both P_7 and C_6 are least common multiples of P_4 and P_3. In fact Chartrand et al. [8] proved that for every positive integer n there exist two graphs having exactly n least common multiples.

2. Preliminaries

The path P_n having vertex set $\{v_1, v_2, \ldots, v_n\}$ and edge set $\{e_1, e_2, \ldots, e_{n-1}\}$ will be denoted by $\langle e_1, e_2, \ldots, e_{n-1}\rangle$ and a star $K_{1,n}$ having vertex set $\{v_1, v_2, \ldots, v_{n+1}\}$, where v_1 is the hub vertex, and edge set $\{e_1, e_2, \ldots, e_n\}$ will be denoted by $[v_1; e_1, e_2, \ldots, e_n]$. The cartesian product of two graphs G and H denoted by $G \square H$ is a graph with vertex set $V(G) \times V(H)$ for which $\{(x, a), (y, b)\}$ is an edge if $x = y$ and $\{a, b\} \in E(H)$ or $\{x, y\} \in E(G)$ and $a = b$. $v(G \square H) = v(G)v(H)$ and $e(G \square H) = v(G)e(H) + v(H)e(G)$.

Theorem 2.1 ([9]) *A nontrivial connected graph G is Eulerian if and only if every vertex of G has even degree.*

Theorem 2.2 ([9]) *Let G and H be nontrivial connected graphs. Then $G \square H$ is Eulerian if and only if both G and H are Eulerian or every vertex of G and H is odd.*

Theorem 2.3 ([9]) *A nontrivial graph G is a bipartite graph if and only if G contains no odd cycles.*

Theorem 2.4 ([10]) *Let E be an Eulerian circuit in a graph G. If $k_1, k_2, \ldots, k_m$ are positive integers such that $k_1 + k_2 + \cdots + k_m = e(G)$ and each less than $g(E)$, where $g(E)$ is the length of the minimal cycle contained in E, then G can be decomposed into paths of lengths $k_1, k_2, \ldots, k_m$.*

Theorem 2.5 ([11]) *A complete bipartite graph of size $q \equiv 0 \pmod 3$ is $K_{1,3}$-decomposable.*

Theorem 2.6 ([12]) *If the graphs G and H have an F-decomposition, then their cartesian product $G \square H$ also has an F-decomposition.*

Theorem 2.7 ([13]) *If the graphs G and H are bipartite, then $\mathrm{lcm}(G, H) \leq e(G).e(H)$ where* *equality holds if* $\gcd(e(G), e(H)) = 1$.

For a graph G, let G^t for $t = 1, 2, 3$ denote the t-th copy of G. Let v^t denote a vertex and e^t denote an edge in G^t.

3. Main results

In this section we compute

$$\text{lcm}(P_4, P_m \,\Box\, P_n),\ \text{lcm}(P_4, C_m \,\Box\, C_n) \text{ and } \text{lcm}(K_{1,3}, K_{1,m} \,\Box\, K_{1,n}).$$

lcm of P_4 and $P_m \,\Box\, P_n$

Let $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_n$ be the vertices of P_m and P_n, respectively. $P_m \times \{b_j\}$, $1 \le j \le n$ are the P_m-fibers and $\{a_i\} \times P_n$, $1 \le i \le m$ are the P_n-fibers in $P_m \,\Box\, P_n$. Label the vertices and edges of the j-th P_m-fiber, $P_m \times \{b_j\}$ as $\{v_{1,j}, v_{2,j}, \ldots, v_{m,j}\}$, $\{f_{1,j}, f_{2,j}, \ldots, f_{m-1,j}\}$ and that of the i-th P_n-fiber, $\{a_i\} \times P_n$ as $\{v_{i,1}, v_{i,2}, \ldots, v_{i,n}\}$, $\{e_{i,1}, e_{i,2}, \ldots, e_{i,n-1}\}$. A path on m vertices P_m has $m-1$ edges and it is P_n-decomposable if and only if $n-1$ divides $m-1$.

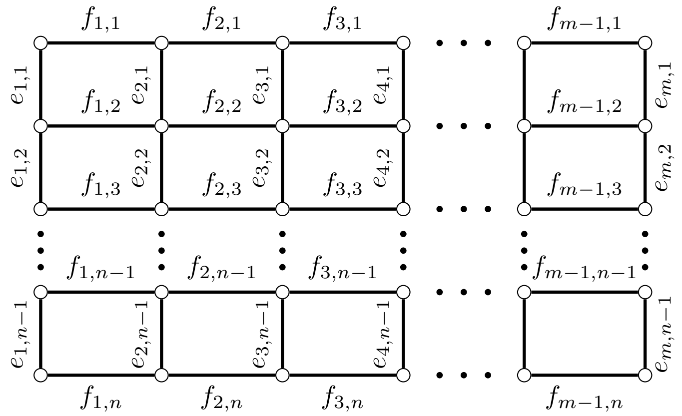

Figure 1 $\quad P_m \,\Box\, P_n$

Theorem 3.1 $\quad \text{lcm}(P_4, P_m \,\Box\, P_n) = \begin{cases} m(n-1) + n(m-1); & m, n \equiv 0 \pmod 3, \\ & m, n \equiv 1 \pmod 3, \\ 3(m(n-1) + n(m-1)); & \textit{otherwise.} \end{cases}$

Proof Least common multiple of P_4 and $P_m \,\Box\, P_n$ is the number of edges in the graph F of least size that is both P_4-decomposable and $P_m \,\Box\, P_n$-decomposable. Since $e(P_m \,\Box\, P_n) = m(n-1) + n(m-1)$, $e(F)$ must be a multiple of 3 and $m(n-1) + n(m-1)$. We consider various cases for m and n in modulo 3 and will construct in each case a graph of least size that is P_4-decomposable and $P_m \,\Box\, P_n$-decomposable. Let $G = P_m \,\Box\, P_n$.

Let $X = \{(a, b) : a, b \equiv 0 \pmod 3 \text{ or } a, b \equiv 1 \pmod 3\}$. Then $m(n-1) + n(m-1) \equiv 0 \pmod 3$ if and only if $(m, n) \in X$.

Case 1. $(m,n) \in X$.

Subcase 1.1. Let $m,n \equiv 0 \pmod 3$. The $m-1$ edges of the j-th P_m-fiber, where $1 \le j \le n-1$, together with the edge $e_{1,j}$ make a P_{m+1}, which is P_4-decomposable. Similarly, the $n-1$ edges of the i-th P_n-fiber, where $2 \le i \le m$, together with the edge $f_{i,n}$ will make a P_{n+1} and it is P_4-decomposable. Thus G is P_4-decomposable and hence $\text{lcm}(P_4, P_m \,\square\, P_n) = m(n-1) + n(m-1)$.

Subcase 1.2. Let $m,n \equiv 1 \pmod 3$. Then each P_m-fiber has $3k$ edges and each P_m-fiber has $3l$ edges for some positive integers k and l. So each fiber and hence $P_m \,\square\, P_n$ is P_4-decomposable. Thus $\text{lcm}(P_4, P_m \,\square\, P_n) = m(n-1) + n(m-1)$.

Case 2. $(m,n) \notin X$.

In this case $P_m \,\square\, P_n$ is not P_4-decomposable. Since P_4 and $P_m \,\square\, P_n$ have no odd cycles by Theorem 2.3, both are bipartite. Also $\gcd(3, m(n-1) + n(m-1)) = 1$. So by Theorem 2.7, $\text{lcm}(P_4, P_m \,\square\, P_n) = 3(m(n-1) + n(m-1))$. □

From Theorem 3.1 the following result is obtained, which is a subcase of the open problem: The P_4-decomposability of a graph.

Theorem 3.2 *$P_m \,\square\, P_n$ is P_4-decomposable if and only if $m \equiv 0 \pmod 3$ and $n \equiv 0 \pmod 3$ or $m \equiv 1 \pmod 3$ and $n \equiv 1 \pmod 3$.*

lcm of P_4 and $C_m \,\square\, C_n$

Let $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_n$ be the vertices of C_m and C_n, respectively. $C_m \times \{b_j\}$, $1 \le j \le n$ are the C_m-fibers and $\{a_i\} \times C_n$, $1 \le i \le m$ are the C_n-fibers in $C_m \,\square\, C_n$. Label the vertices and edges of the j-th C_m-fiber, $C_m \times \{b_j\}$ as $\{v_{1,j}, v_{2,j}, \ldots, v_{m,j}\}$, $\{f_{1,j}, f_{2,j}, \ldots, f_{m,j}\}$ and that of the i-th C_n-fiber, $\{a_i\} \times C_n$ as $\{v_{i,1}, v_{i,2}, \ldots, v_{i,n}\}$, $\{e_{i,1}, e_{i,2}, \ldots, e_{i,n}\}$.

Theorem 3.3 $\text{lcm}(P_4, C_m \,\square\, C_n) = \begin{cases} 2mn & \text{if } mn \equiv 0 \pmod 3, \\ 6mn & \text{otherwise.} \end{cases}$

Proof Least common multiple of P_4 and $C_m \,\square\, C_n$ is the number of edges in the graph F of least size that is both P_4-decomposable and $C_m \,\square\, C_n$-decomposable. Since $e(C_m \,\square\, C_n) = 2mn$, $e(F)$ must be a multiple of 3 and $2mn$.

Let $G = C_m \,\square\, C_n$. If G is P_4-decomposable, then $G \in \text{LCM}(P_4, C_m \,\square\, C_n)$. Since $deg(v) = 2$ for all $v \in V(C_n)$, by Theorem 2.1, the cycle C_n is Eulerian for every n. So by Theorem 2.2, $G = C_m \,\square\, C_n$ is Eulerian. If m or n is a multiple of three, $e(C_m \,\square\, C_n) = 2mn$ is a multiple of three. Let $e(C_m \,\square\, C_n) = 3r$, for some $r \in \mathbb{Z}$. Then by Theorem 2.4, $C_m \,\square\, C_n$ can be decomposed into r copies of P_4 and hence $G = C_m \,\square\, C_n$ is P_4-decomposable. Thus $\text{lcm}(P_4, C_m \,\square\, C_n) = 2mn$, if $mn \equiv 0 \pmod 3$.

Suppose $mn \not\equiv 0 \pmod 3$. Then $C_m \,\square\, C_n$ is not P_4-decomposable and the least positive integer which is a multiple of 3 and $2mn$ is $6mn$. We will prove that $\text{lcm}(P_4, C_m \,\square\, C_n) = 6mn$ if $mn \not\equiv 0 \pmod 3$. For this consider various cases for m and n in modulo 3 and in each case we will construct a graph of size $6mn$ that is both P_4-decomposable and $C_m \,\square\, C_n$-decomposable. Let $G = C_m \,\square\, C_n$.

Case 1. $m = 3k + 1$, $n = 3l + 1$.

Let H be the graph obtained by identifying the vertex $v^1_{m,n}$ of G^1 with the vertex $v^2_{1,1}$ of G^2 and the vertex $v^2_{m,n}$ of G^2 with $v^3_{1,1}$ of G^3. Clearly, H is $C_m \,\square\, C_n$-decomposable. A P_4 decomposition of H is given below. In each G^t consider the C_{3k+1}-fibers and the C_{3l+1}-fibers except the first and last fibers. The first $3k$ edges in the C_{3k+1}-fiber makes a P_{3k+1} and the first $3l$ edges in the C_{3l+1}-fiber makes a P_{3l+1} and both are P_4-decomposable. A P_4-decomposition of the remaining edges of H is obtained as follows. For $1 \le i \le m-1$, $1 \le j \le n-1$ and $1 \le t \le 3$,

$$\langle e^t_{1,j}, f^t_{m,j+1}, e^t_{m,j}\rangle \quad \langle f^t_{i,1}, e^t_{i,n}, f^t_{i,n}\rangle \quad \langle f^1_{m,1}, e^1_{m,n}, f^2_{m,1}\rangle \quad \langle e^2_{m,n}, f^3_{m,1}, e^3_{m,n}\rangle$$

Thus H is P_4-decomposable.

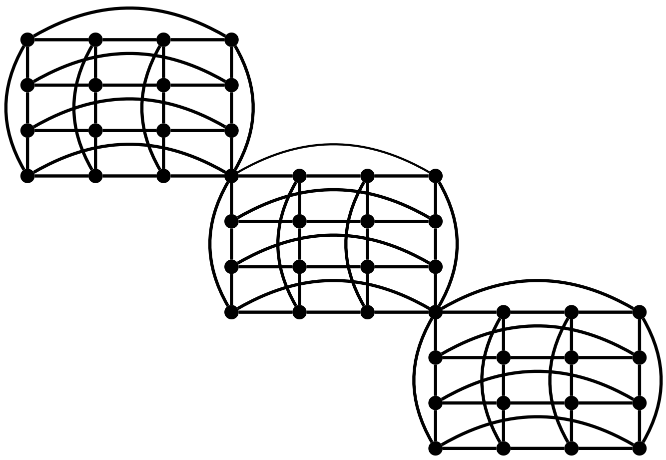

Figure 2 $m = 4$, $n = 4$

Case 2. $m = 3k + 2$, $n = 3l + 1$.

Let H be the graph obtained by identifying the vertex $v^1_{m,n}$ of G^1 with the vertex $v^2_{m,1}$ of G^2 and the vertex $v^2_{m,n}$ of G^2 with $v^3_{m,1}$ of G^3. Clearly, H is $C_m \,\square\, C_n$-decomposable. A P_4 decomposition of H is given below. In each G^t consider the C_{3k+2}-fibers and the C_{3l+1}-fibers except the first and last fibers. The first $3k$ edges in the C_{3k+2}-fiber makes a P_{3k+1} and the first $3l$ edges in the C_{3l+1}-fiber makes a P_{3l+1} and both are P_4-decomposable. A P_4-decomposition of the remaining edges of H is obtained as follows. For $1 \le i \le m-1$, $2 \le j \le n-1$ and $1 \le t \le 3$,

$$\langle e^t_{1,j}, f^t_{m-1,j}, f^t_{m,j}\rangle \qquad \langle f^t_{i,1}, e^t_{i,n}, f^t_{i,n}\rangle \qquad \langle e^t_{1,1}, f^t_{m,1}, e^t_{m,1}\rangle$$
$$\langle f^t_{m,n}, e^t_{m,n-2}, e^t_{m,n-1}\rangle \qquad \langle e^1_{m,n}, e^2_{m,n}, e^3_{m,n}\rangle$$

The edges $\{e^t_{m,2}, e^t_{m,3}, \ldots, e^t_{m,n-3}\}$ makes a P_{3l-2} and it has $3l-3$ edges which is P_4-decomposable. Thus H is P_4-decomposable.

Case 3. $m = 3k + 2$, $n = 3l + 2$.

Let H be the graph obtained by identifying the vertex $v^1_{m,1}$ of G^1 with the vertex $v^2_{1,n}$ of G^2 and the vertex $v^2_{m,1}$ of G^2 with $v^3_{1,n}$ of G^3. Clearly, H is $C_m \,\square\, C_n$-decomposable. A P_4 decomposition of H is given below. In each G^t consider the C_{3k+2}-fibers except the last fiber and the C_{3l+2}-fibers except the first and last fibers. The $3k$ edges in the C_{3k+2}-fiber except the edges $\{f_{1,j}, f_{m,j}; 1 \le j \le n-1\}$ makes a P_{3k+1} and the $3l$ edges in the C_{3l+2}-fiber except the edges $\{e_{i.1}, e_{i.n}; 2 \le i \le m-1\}$ makes a P_{3l+1} and both are P_4-decomposable. A P_4-decomposition of the remaining edges of H is obtained as follows. For $2 \le i \le m$, $2 \le j \le n-1$ and $1 \le t \le 3$,

$$\langle f^t_{1,j}, f^t_{m,j}, e^t_{m,j}\rangle \quad \langle e^t_{i,1}, e^t_{i,n}, f^t_{i-1,n}\rangle \quad \langle f^t_{1,1}, e^t_{1,1}, e^t_{1,2}\rangle$$
$$\langle e^t_{1,n-2}, e^t_{1,n-1}, f^t_{m,n}\rangle \quad \langle e^1_{1,n}, f^1_{m,1}, e^2_{1,n}\rangle \quad \langle f^2_{m,1}, e^3_{1,n}, f^3_{m,1}\rangle$$

The edges $\{e^t_{1,3}, e^t_{1,4}, \ldots, e^t_{1,n-3}\}$ makes a P_{3l-2} and it has $3l-3$ edges which is P_4-decomposable. Thus H is P_4-decomposable.

In all three cases $e(H) = 6mn$ and $H \in \mathrm{LCM}(P_4, C_m \,\square\, C_n)$. Thus $\mathrm{lcm}(P_4, C_m \,\square\, C_n) = 6mn$ if $mn \not\equiv 0 \pmod 3$. □

From Theorem 3.3, the following result is obtained.

Theorem 3.4 *$C_m \,\square\, C_n$ is P_4-decomposable if and only if the number of vertices of $C_m \,\square\, C_n$ is a multiple of three.*

lcm of $K_{1,3}$ and $K_{1,m} \,\square\, K_{1,n}$.

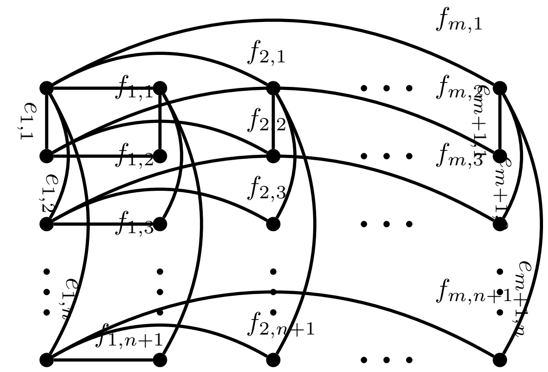

Figure 3 $K_{1,m} \,\square\, K_{1,n}$

Let $a_1, a_2, \ldots, a_{m+1}$ and $b_1, b_2, \ldots, b_{n+1}$ be the vertices of $K_{1,m}$ and $K_{1,n}$ respectively. $K_{1,m} \times \{b_j\}$, $1 \le j \le n+1$ are the $K_{1,m}$-fibers and $\{a_i\} \times K_{1,n}$, $1 \le i \le m+1$ are the $K_{1,n}$-fibers in $K_{1,m} \,\square\, K_{1,n}$. Label the vertices and edges of the j-th $K_{1,m}$-fiber, $K_{1,m} \times \{b_j\}$ as $\{v_{1,j}, v_{2,j}, \ldots, v_{m+1,j}\}$, $\{f_{1,j}, f_{2,j}, \ldots, f_{m,j}\}$ and that of the i-th $K_{1,n}$-fiber, $\{a_i\} \times K_{1,n}$ as $\{v_{i,1}, v_{i,2}, \ldots, v_{i,n+1}\}$, $\{e_{i,1}, e_{i,2}, \ldots, e_{i,n}\}$.

Theorem 3.5 $$\mathrm{lcm}(K_{1,3}, K_{1,m} \,\square\, K_{1,n}) = \begin{cases} (m+1)n + m(n+1); & m, n \equiv 0 \pmod 3, \\ & m, n \equiv 2 \pmod 3, \\ 3((m+1)n + m(n+1)); & \textit{otherwise}. \end{cases}$$

Proof Least common multiple of $K_{1,3}$ and $K_{1,m} \,\square\, K_{1,n}$ is the number of edges in the graph of least size that is both $K_{1,3}$-decomposable and $K_{1,m} \,\square\, K_{1,n}$-decomposable. Let $G = K_{1,m} \,\square\, K_{1,n}$. Then $e(G) = n(m+1) + m(n+1) = 2mn + m + n$ and $e(G)$ is a multiple of 3 if and only if $m, n \equiv 0 \pmod 3$ or $m, n \equiv 2 \pmod 3$. Let $X = \{(a, b) : m, n \equiv 0 \pmod 3 \text{ or } m, n \equiv 2 \pmod 3\}$.

Case 1. $(m, n) \in X$.

Subcase 1.1. If $m, n \equiv 0 \pmod 3$, by Theorem 2.5, $K_{1,m}$ and $K_{1,n}$ are $K_{1,3}$-decomposable. Then by Theorem 2.6, $G = K_{1,m} \square K_{1,n}$ is $K_{1,3}$-decomposable. So $G \in \mathrm{LCM}(K_{1,3}, K_{1,m} \square K_{1,n})$ and $\mathrm{lcm}(K_{1,3}, K_{1,m} \square K_{1,n}) = (m+1)n + m(n+1)$.

Subcase 1.2. If $m, n \equiv 2 \pmod 3$, in G consider the first $3k$ edges of $K_{1,m}$-fibers and the first $3l$ edges of the $K_{1,n}$-fibers, except the first $K_{1,m}$ and $K_{1,n}$-fibers. These edges will have a $K_{1,3}$-decomposition. In any $K_{1,m}$-fiber the edges $f_{3k+1,j}, f_{3k+2,j}$ for $2 \leq j \leq n+1$, in any $K_{1,n}$-fiber the edges $e_{i,3l+1}, e_{i,3l+2}$ for $2 \leq i \leq m+1$ and the edges in the first $K_{1,m}$-fiber and $K_{1,n}$-fiber remains. A $K_{1,3}$-decomposition of these edges are given below

$$[v_{i,1}; f_{i-1,1}, e_{i,3l+1}, e_{i,3l+2}]_{i=2}^{m+1} \qquad [v_{1,j}; e_{1,j-1}, f_{3k+1,j}, f_{3k+2,j}]_{j=2}^{n+1}.$$

Thus G is $K_{1,3}$-decomposable.

Case 2. $(m, n) \notin X$.

In this case $K_{1,m} \square K_{1,n}$ is not P_4-decomposable. Since P_4 and $K_{1,m} \square K_{1,n}$ have no odd cycles by Theorem 2.3, both are bipartite. Also $\gcd(3, (m+1)n + m(n+1)) = 1$. So by Theorem 2.7, $\mathrm{lcm}(P_4, K_{1,m} \square K_{1,n}) = 3((m+1)n + m(n+1))$. □

Theorem 3.6 *$K_{1,m} \square K_{1,n}$ is $K_{1,3}$-decomposable if and only if number of edges of $K_{1,m} \square K_{1,n}$ is a multiple of three.*

Proof From Theorem 3.5 the result follows. □

Acknowledgements The authors are grateful to the anonymous referees for their valuable comments.

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